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Public and Private Money Creation for Distributed Ledgers: Stablecoins, Tokenized Deposits, or Central Bank Digital Currencies?

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Abstract

This paper explores the implications of introducing digital public and private monies (e.g. tokenized central bank digital currency [CBDC] or tokenized deposits) for stablecoins and illicit crypto transactions. When they pay a high interest rate and guarantee a high degree of anonymity, these tokenized currencies crowd out stablecoins as payment methods in the crypto space. Conversely, with low anonymity and low interest rates, tokenized currencies become collateral, promoting the development of stablecoins. CBDCs dominate tokenized deposits because a central bank can better economize on scarce collateral assets and internalize the social costs of crypto activities. Prohibiting tokenized deposits may be necessary to implement the optimal CBDC design.

Topics: Digital currencies and fintech; Financial stability; Monetary policy
JEL codes: E50, E58

Résumé

Cette étude se penche sur les conséquences qu'aurait le lancement d'une monnaie numérique publique ou privée, telle qu'une monnaie numérique de banque centrale (MNBC) jetonisée ou des dépôts jetonisés, sur le développement des cryptomonnaies stables et les opérations illicites en cryptomonnaies. Lorsque ces monnaies jetonisées paient des intérêts élevés et garantissent l'anonymat de leurs usagers, elles supplantent les cryptomonnaies stables comme moyens de paiement dans le secteur des cryptomonnaies. En revanche, si la garantie d'anonymat et les intérêts produits sont faibles, les monnaies jetonisées sont utilisées comme sûretés par les cryptomonnaies stables, facilitant ainsi le développement de ces cryptomonnaies stables. La MNBC est préférable aux dépôts jetonisés, car la banque centrale utilise moins de sûretés que les banques commerciales et internalise aussi les coûts sociaux des opérations en cryptomonnaies. Ainsi, l'interdiction des dépôts jetonisés peut être une condition nécessaire pour permettre l'utilisation optimale de la MNBC.

Sujets : Monnaies numériques et technologies financières; Stabilité financière; Politique monétaire
Codes JEL : E50, E58

1 Introduction

Recent years have witnessed a surge in blockchain-based innovations in finance. Since the introduction of the Bitcoin technology in 2008, thousands of "crypto" projects have been developed. In 2015, Ethereum was launched to support smart contracts running on a blockchain. Since then, blockchain-based crypto assets have been used to finance start-up projects (e.g., initial coin offerings, or ICOs), facilitate institutional governance (e.g., decentralized autonomous organization, or DAO), offer financial services (e.g., decentralized finance, or DeFi), and manage asset ownership (e.g., non-fungible tokens, or NFTs).

A safe payment device is, however, needed for this new ecosystem to unleash all of its promises. Popular cryptocurrencies like Bitcoin and Ether exhibit high price volatility, making them less appealing as a store of value or unit of account. As a result, many consider that they are too volatile to be a good means of payment in the crypto sphere. To fill the gap left by traditional payment methods that are not readily accessible in the crypto space due to regulatory and technical constraints, private initiatives have proposed various recipes for a stable crypto money (a.k.a. stablecoins). Stablecoins are, however, stable to different degrees.[1] The market capitalization of top stablecoins is about US$125 billion in September 2023.

The growth of stablecoins and the issues surrounding their management have prompted regulators and policymakers to express concerns regarding the adverse impacts of stablecoins and crypto activities on illicit finance and financial stability.[2] Concurrently, some central bankers argue that the issuance of central bank digital currencies (CBDCs) can offer a solution to these problems.[3] The fundamental

[1]Stablecoins are basically cryptocurrencies that peg their value to a stable asset, such as the US dollar. Stablecoins can have different designs. First, the issuance can be centralized or decentralized. Centralized stablecoins such as Tether (USDT) and USD Coin (USDC) are issued by third parties that offer off-chain custody of the reserve assets. Decentralized stablecoins such as Dai are minted through smart contracts that also hold the reserve assets on-chain, limiting the need for a trusted third party. Second, the choice and management of reserve assets differ. Stablecoins can be backed by fiat currencies, financial assets (e.g., corporate bonds), commodities (e.g., gold), or cryptocurrencies. Stablecoins backed by cryptocurrencies (e.g., Dai) are often over-collateralized due to the price volatility of the reserve asset. Algorithmic stablecoins also exist, which are not backed explicitly by reserve assets but follow an algorithm to adjust the coin supply to maintain the peg (e.g., TerraUSD).

[2]For instance, a Financial Stability Board report highlighted potential risks to financial stability if stablecoins are widely adopted (FSB, 2020). The Biden Crypto Executive Order also stressed the importance of mitigating illicit finance and national security risks associated with the misuse of digital assets.

[3]According to a Bank for International Settlements report, CBDC exploration is underway in over 100 countries. As of January 2023, four retail CBDCs are operational in the Bahamas, Eastern Caribbean, Nigeria, and Jamaica. Additionally,

idea is that high-quality public money (i.e., CBDC) can crowd out low-quality private money (i.e., stablecoins).[4] Others suggest that private digital money issued by banks, in the form of tokenized deposits, is an alternative and viable option.[5]

Economic research on this matter is, however, scant. This paper develops a general equilibrium monetary model to inform this discussion and seeks to answer the following questions: Should a public digital money (e.g., CBDC) or private digital monies (e.g., tokenized deposits) be issued to serve the crypto space? Would their issuance be a curse or a blessing for crypto assets, stablecoins, and illicit transactions? How do the answers depend on the design of these tokenized public and private monies?

Our model economy consists of two sectors: a traditional sector (referred to as TradFi) and a crypto sector (referred to as DeFi). In the TradFi sector, households employ bank deposits, which are issued by traditional banks, as a means of payment. An important feature is that banks are susceptible to an agency problem. Consequently, they must secure their deposit issuance with collateral (i.e., government securities). In the DeFi sector, deposits are not feasible. Therefore households use stablecoins issued by crypto banks as a means of payment.

The DeFi sector differs from the TradFi sector in three aspects. First, some transactions may be less desirable from the perspective of a social planner (e.g., those involving criminal activities). Second, crypto banks may be less regulated compared to traditional banks, making it easier for them to run away with their collateral. Third, crypto banks are allowed to hold crypto assets (e.g., Bitcoin, Ether) as collateral, whereas traditional banks cannot. Consequently, the regulation of crypto banks and the characteristics of crypto assets determine the tightness of the incentive constraints of crypto banks.

We then examine the impact of introducing tokenized money, which can take the form of a public

pilots involving both wholesale and retail CBDCs are being conducted in 34 jurisdictions.

[4]Jerome Powell, chair of the Federal Reserve, argued that the introduction of a CBDC would eliminate the use case for cryptocurrencies. He stated, "You wouldn't need stablecoins; you wouldn't need cryptocurrencies if you had a digital U.S. currency. I think that's one of the stronger arguments in its favour." (URL: shorturl.at/aWZ15). Reserve Bank of India Deputy Governor T. Rabi Sankar also believed that CBDC would "kill whatever little case there could be" for cryptocurrencies (URL: shorturl.at/kGO18). Academics such as Gary Gorton argue that the government can "introduce a central bank digital currency and tax private stablecoins out of existence" (Gorton and Zhang, 2023).

[5]Swiss Bank Association published a white paper on deposit tokens in March 2023. In July 2023, Bank of England Governor Andrew Bailey gave a speech to encourage private banks to consider tokenizing bank deposits. In September 2023, JP Morgan reportedly could launch blockchain-based deposit tokens relatively quickly after regulator approval (source: https://fortune.com/crypto/2023/09/08/jpm-jpmorgan-chase-deposit-token-blockchain/).

digital money that we call CBDC, issued by a central bank, or private digital money, in the form of tokenized deposits issued by private banks. We focus on two key design features: (i) the level of privacy when tokenized money is used for transactions in the crypto space and (ii) the interest rate paid on tokenized money.

We demonstrate that the issuance of tokenized money can either be a curse or a blessing for stablecoins. Specifically, the effect depends on whether tokenized money is used as a means of payment or as a collateral asset in the DeFi sector—an equilibrium outcome that, in turn, hinges on the design of the tokenized money. We find that tokenized money is used as a means of payment when privacy is high and the interest rate is moderate. In such cases, tokenized money tends to be a curse for stablecoins ("crowding-out" effects). When the associated interest rate is high and the degree of privacy is low, tokenized money can serve as collateral for stablecoin issuance, and its introduction can be a blessing for stablecoins ("crowding-in" effects). Additionally, we observe that decreasing privacy can sometimes crowd in stablecoins—an outcome that may surprise policymakers.

Next, we delve into the optimal policy regarding the issuance of tokenized money. We demonstrate that when a CBDC is available, its design should prevent stablecoin issuers from using it as collateral. This implies that it is optimal that CBDC preserves the anonymity of its users by setting the highest degree of privacy. Concerning the optimal choice between CBDC and tokenized deposits, CBDCs can outperform tokenized deposits in terms of social welfare for at least two reasons. First, private bankers face incentive problems, whereas the central bank is considered trustworthy. Second, private bankers may either over-issue or under-issue tokenized money balances since they do not fully internalize the impacts of privacy and illicit activities on society. We show that tokenized deposits need to be prohibited to implement the optimal CBDC design when illicit activities are too many: otherwise the central bank cannot set the interest rate on CBDC at the optimally low level because private banks will find it profitable to issue tokenized deposits when they can pay this low rate on their deposits.

Our paper contributes to several lines of research in the literature. First of all, several papers such as Ahnert et al. (2022), Andolfatto (2020), Chiu and Davoodalhosseini (2021), Chiu et al. (2023a), Keister and Sanches (2023), and Williamson (2022a) study the effects of CBDC issuance on traditional bank intermediation in normal times. In addition, Fernandez-Villaverde et al. (2020), Keister and Monnet (2022), Monnet et al. (2020), Schilling et al. (2020), and Williamson (2022b) examine the effects on bank stability in crisis times. Wang (2023) studies the implications of money laundering for the optimal design of a CBDC. None of these papers model the crypto sector and study its response to a CBDC issuance.[6] Furthermore, most papers in the existing literature focus on the interest rate as the main design feature of a CBDC. Our paper examines two other important design features, namely, tokenization and the degree of privacy, generating implications useful for practical policy discussion.

Our work is also related to the emerging literature on stablecoin and decentralized finance. Theoretical papers by D'Avernas, Bourany, and Vandeweyer (2021), Li and Mayer (2021), and Kozhan and Viswanath-Natraj (2021) study decentralized stablecoins such as Dai issued by the MakerDAO. Other DeFi platforms are also actively studied. For example, Aoyagi and Itoy (2021), Capponi and Jia (2021), and Lehar and Parlour (2021) study decentralized exchanges in the form of automated market makers (e.g., Uniswap), while Chiu et al. (2022) and Lehar and Parlour (2022) focus on lending platforms. Bertsch (2023) analyzes the consequences of stablecoins on the fragility of the DeFi ecosystem. Cong and Mayer (2022) study the competition among national fiat currencies, cryptocurrencies, and central bank digital currencies. Chiu et al. (2023b) use a network model to investigate the relationships among different crypto tokens and DeFi activities. None of these papers evaluate the general-equilibrium impact of a CBDC and tokenized deposits on crypto activities, which is the main contribution of our analysis.

This paper is organized as follows. Section 2 presents the model environment. Section 3 studies the banks' decision problems and the equilibrium payment choice in the crypto sector. Section 4 considers CBDC and its optimal design and analyzes whether CBDC is better than tokenized deposits with respect to a welfare function. Section 5 examines various extensions of the basic model, and Section 6 concludes.

2 Environment

Time is discrete and continues forever: $t = 0, 1, 2, ...$ Each period consists of two sub-periods with two alternating markets, à la Lagos and Wright (2005). In the first sub-period, a frictional market opens, and we call it AM. In the second sub-period, a Walrasian centralized market opens, and we call it PM. The discount factor between the PM and the next AM is $\beta < 1$.

The economy is peopled with a measure 2 of infinitely lived buyers and sellers, as well as a large

[6] See Auer et al. (2022) for a review of some policy issues and the academic literature.

measure of short-lived banks, that enter the economy in each PM and exit in the following PM. Banks, buyers, and sellers can be of two types: "traditional" and "crypto." These types are fixed, and there is an equal measure of both types.

All agents trade a common numeraire good in the PM. All buyers and sellers derive utility $U(y)$ from consuming y units of the numeraire and suffer a linear cost $-h$ from producing h units of the numeraire. We assume $U'(y) > 0$ and $U''(y) \leq 0$ and $U(0) = 0$. Banks have a linear utility from consuming the numeraire.

The AM market consists of two segmented sectors: TradFi and DeFi. Traditional agents trade a traditional good in TradFi, while crypto agents trade a crypto good in DeFi. Buyers derive utility $u(x_T)$ from consuming x_T units of the traditional good and $u(x_D)$ from consuming x_D units of the crypto good. We assume $u'(x) > 0$ and $u''(x) \leq 0$ and $u(0) = 0$. Sellers incur a cost of $-x$ when producing x units of their (traditional or crypto) good. There is a crypto asset that gives a return R^e in the PM. Only crypto agents are entitled to hold the crypto asset.

The role of the banks in each sector is to provide buyers with a means to pay. We consider a digital and cashless economy where there is no supply of physical cash but only digital money.

TradFi transactions are facilitated by deposits. Traditional banks issue deposits d. However, they cannot commit to repaying their outstanding claims, and they need to back them by holding government bonds b. Banks can run away with a fraction $1 - \rho$ of reserve assets, so we call ρ the pledgeability parameter of traditional banks.

DeFi transactions have to be facilitated by some forms of digital money. We will consider three types of digital money: stablecoins issued by crypto banks in the DeFi sector, tokenized deposits issued by traditional banks, and (tokenized) CBDC issued by the central bank. We go through each type of digital money in turn.

- Crypto banks can issue **stablecoins** s backed by crypto assets e which pay a real return $R^e < 1/\beta$ in the PM each period, or by other assets available in the DeFi sector, such as tokenized money. As traditional banks, crypto banks cannot commit, and they can run away with a fraction $1 - \kappa$ of their assets. So, their pledgeability parameter is κ. The case of $\kappa < \rho$ captures the idea that, unlike traditional banks, crypto banks are unregulated. Note that regulation precludes traditional banks from holding crypto assets, as in reality.

- The central bank can issue **CBDC**, M. The central bank can fully commit but backs the issuance of CBDC with government bonds b_C. The CBDC has two design features: First, the CBDC can pay a real rate R^m. Second, the central bank can control the degree of privacy that CBDC allows. Specifically, sellers are perfectly anonymous when they trade with stablecoins, but maybe not as much when they accept CBDC, as the government may be able to trace back the transaction. Therefore, crypto sellers only value 1 (real) unit of CBDC at $\mu \in [0, 1]$, so that a lower μ implies a lower degree of privacy.

- Traditional banks can issue **tokenized deposits**, denoted by D. These are "normal" deposits—therefore backed by bonds—with the feature that they can be recorded and transferred on a blockchain, facilitating DeFi transactions. Tokenized deposits are subject to the same degree of privacy as CBDC, as the same compliance rule is applied to both tokenized monies.

We refer to "tokenized money" as comprising both tokenized deposits and CBDC (which is a tokenized form of traditional money). Both the CBDC and the tokenized deposits can be used by DeFi households as a means of payment and as a reserve asset held by crypto banks.

Finally, in each period in the PM the government issues a fixed supply, B, of illiquid, one-period bonds trading at a price q (in terms of the numeraire good), which is endogenously determined. Each unit of bonds pays one unit of numeraire good in the following PM, financed by lump-sum taxation in the PM.

We allow social welfare to assign different weights on consumption in different sectors to capture the fact that some activities conducted in the DeFi sector are undesirable. With a weight $\omega \in [0, 1]$ on DeFi transactions, the social welfare function is

$$W = u(x_T) - x_T + \omega[u(x_D) - x_D].$$

Hence, the regulator may want to discourage DeFi transactions when $\omega < 1$. We denote by x^* the efficient level of consumption in each sector that satisfies $u'(x^*) = 1$.

The timeline is the following: In the PM, traditional banks acquire government bonds by issuing deposits to buyers, and crypto banks acquire collateral – either crypto assets, CBDC, or tokenized deposits – by issuing stablecoins to buyers. Crypto banks can also acquire tokenized money to be used directly by crypto buyers on their behalf. In the next AM, buyers and sellers trade x_i in each sector i

using the means to pay of choice. In the following PM, sellers redeem their deposits, tokenized money, or stablecoins with the respective issuer. Figure 1 is a succinct representation of our economy.[7]

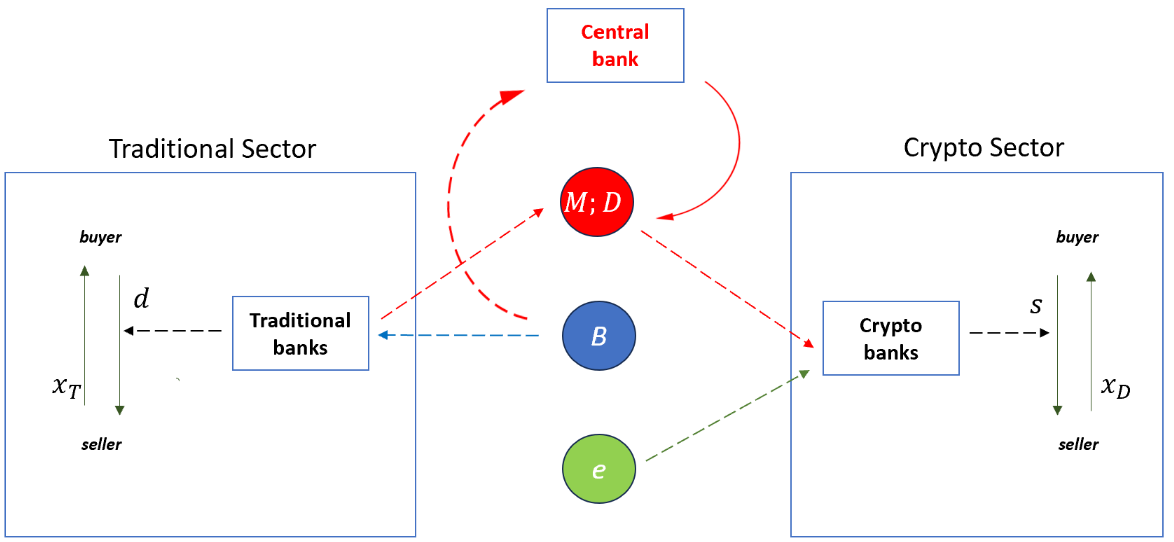

Figure 1: Model Setup

3 The banks' problems

We assume that crypto and traditional banks maximize the value of their users in the respective sectors. Therefore, their problems conveniently embed all the actions of buyers and sellers in both DeFi and TradFi, so it is sufficient to consider the banks' problems to characterize the equilibrium. In this section, we first study the problem of crypto banks, then the one for traditional banks, and finally, the central bank's problem.

[7]To keep the model tractable and to capture real-world features, we have made some assumptions to simplify the benchmark model. In Section 5 and in the appendices, we study various extensions, such as introducing bank credit, endogenizing crypto asset returns, allowing crypto banks to hold bonds, and studying government regulations. See the appendices for the details.

3.1 Crypto banks' problem

It is convenient first to analyze the optimal problem of a crypto bank, taking as given the supply of tokenized money, including its remuneration R^m and the degree of privacy μ. In the latter sections, we endogenize the supply of tokenized money by considering the choice of traditional banks to offer tokenized deposits and the supply of CBDC by the central bank.

In the DeFi sector, a buyer can use either stablecoins s or tokenized money $\tilde{m}$ as a means of payment to buy consumption goods. Since tokenized money is subject to compliance, the seller applies the discount μ to payment in tokenized money.[8]

Given μ, R^m, and R^e, a crypto bank maximizes its users' payoff by choosing the users' investment into the bank a, the quantity of tokenized money $\tilde{m}$ and stablecoins s directly held by its users, as well as the reserves of tokenized money m and crypto assets e that the crypto bank will hold to back its issuance of stablecoins,[9]

$$\max_{a,s,e,m,\tilde{m}} \left[-a - \tilde{m} + \beta u(s + \mu R^m \tilde{m})\right]$$

subject to the net worth of the bank being positive,

$$a - m - e + \beta \left[R^e e + R^m m - s\right] \geq 0, \qquad (PC)$$

and the pledgeability constraint

$$\kappa \left(R^e e + R^m m\right) \geq s, \qquad (IC)$$

where $\kappa \in (0,1)$ denotes the pledgeability parameter of assets for the crypto bank. Here, PC is the crypto bank's participation constraint, which requires a positive expected payoff, and IC is the incentive constraint, which ensures that the banker chooses not to run away after issuing the stablecoins. It is obvious that (PC) will bind. Also, let $c = m, e$ denote the asset held by the crypto as collateral. If $\beta R^c < 1$ the crypto bank only holds asset $c = m, e$ if it relaxes IC, and it is indifferent when $\beta R^c = 1$. Also, the crypto bank holds the cheapest asset to satisfy its IC, that is, it will hold m whenever $R^m > R^e$, e whenever $R^e > R^m$, and it will be indifferent otherwise. With this understanding and replacing a using

[8] We assume that private agents holding these balances will incur this "privacy cost" when they enter the PM. In equilibrium, these are sellers.

[9] This problem is equivalent to maximizing the bank's payoff subject to the user's participation constraint.

the bank's PC at equality, we can rewrite the problem of the crypto bank as

$$\max_{c,s,\tilde{m}} \{-c + \beta [R^c c - s] - \tilde{m} + \beta u(s + \mu R^m \tilde{m})\} + \beta\lambda [\kappa R^c c - s]$$

where $\beta\lambda$ is the Lagrange multiplier on the crypto bank's IC. The first-order conditions are

$$\begin{aligned} s: \quad u'(s + \mu R^m \tilde{m}) &\leq 1 + \lambda, \\ c: \quad (1 + \lambda\kappa)\beta R^c &\leq 1, \\ \tilde{m}: \quad \mu\beta R^m u'(s + \mu R^m \tilde{m}) &\leq 1. \end{aligned}$$

From these first-order conditions, we can derive three regions for the parameter space (Figure 2).

i) Region $\mathcal{A}_e$ (with only stablecoins, which are secured with crypto assets): $e > 0$ and $m = \tilde{m} = 0$, where

$$\begin{aligned} u'(z) &= \frac{1 - (1 - \kappa)\beta R^e}{\kappa\beta R^e}, \\ R^m &< \min\left\{\frac{\kappa R^e}{\mu [1 - (1 - \kappa)\beta R^e]}, R^e\right\} \equiv R_1^m. \end{aligned}$$

Under these parameters, the level of privacy μ and the return on tokenized money R^m are so low that sellers (and the crypto bank) do not value tokenized money much. At the same time, the return on crypto assets is large enough for the crypto bank to issue stablecoins backed only by crypto assets.

ii) Region $\mathcal{A}_m$ (with only stablecoins, which are secured with tokenized money): $m > 0$ and $e = \tilde{m} = 0$ where

$$\begin{aligned} u'(x_D) &= \frac{1 - (1 - \kappa)\beta R^m}{\kappa\beta R^m}, \\ R^m &> \max\left\{\frac{\mu - \kappa}{\beta\mu(1 - \kappa)}, R^e\right\} \equiv R_2^m. \end{aligned}$$

Under these parameters, the level of privacy is still too low that sellers do not value tokenized money much. At the same time, the return on tokenized money is now sufficiently high for the crypto bank to issue stablecoins backed by tokenized money. Therefore, buyers only trade with stablecoins. Notice that $R_2^m \geq R_1^m$.

iii) Region $\mathcal{A}_{\tilde{m}}$ (with only tokenized money and no stablecoins) : $\tilde{m} > 0$ and $e = m = 0$ where

$$\mu R^m u'(\mu R^m \tilde{m}) = \frac{1}{\beta},$$
$$\frac{\kappa R^e}{\mu[1-(1-\kappa)R^e\beta]} < R^m < \frac{\mu-\kappa}{\beta\mu(1-\kappa)}.$$

Under these parameters, the level of privacy is high enough that sellers value tokenized money. However, the return on tokenized money or crypto assets is too low for the crypto bank to issue stablecoins. Therefore, buyers trade directly with tokenized money, and the crypto bank does not issue stablecoins.

In the Appendix, we derive all the possible equilibria and the conditions for their existence, including the knife-edge cases.[10] The following proposition is a direct consequence of the equilibrium characterization:

Proposition 1. *Introducing tokenized money has the following effects:*

a) tokenized money crowds out stablecoins for $R^m \in (R_1^m, R_2^m)$

b) tokenized money crowds in stablecoins for $R^m \geq R_2^m$,

c) tokenized money promotes crypto consumption for $R^m > R_1^m$.

All proofs are in Appendix A. Proposition 1 has a simple intuition: In region $\mathcal{A}_e$, for $R^m < R_1^m$, there is no role for tokenized money because stablecoins backed by crypto assets are used as means to pay in DeFi. Increasing R^m while holding μ constant may bring the crypto bank in the region $\mathcal{A}_{\tilde{m}}$ where crypto buyers only use tokenized money. Therefore, introducing tokenized money with a relatively large μ and $R^m \in (R_1^m, R_2^m)$ will crowd out stablecoins (effect a in Proposition 1). However, starting from a point in region $\mathcal{A}_{\tilde{m}}$, increasing R^m to a level above R_2^m can bring the crypto bank in region $\mathcal{A}_m$ where tokenized money has such a high return that it is used as collateral by the crypto bank that now issues stablecoins. So, introducing tokenized money with a relatively large rate of return can crowd in stablecoins (effect b in Proposition 1). Regarding DeFi consumption (effect c), for $R^m < R_1^m$, DeFi consumption and

[10] There are also knife-edge cases where the crypto bank backs its stablecoins with both crypto assets and tokenized money (when $R^e = R^m$), or when buyers are indifferent between using both stablecoins and tokenized money (when $\mu\beta R^m \left[1 + \frac{1-\beta R^c}{\kappa\beta R^c}\right] = 1$ and $R^c = R^e$ if crypto assets are backing the stablecoins or $R^c = R^m$ if tokenized money are backing the stablecoins). We consider these knife-edge cases in all the proofs but do not consider them in the text.

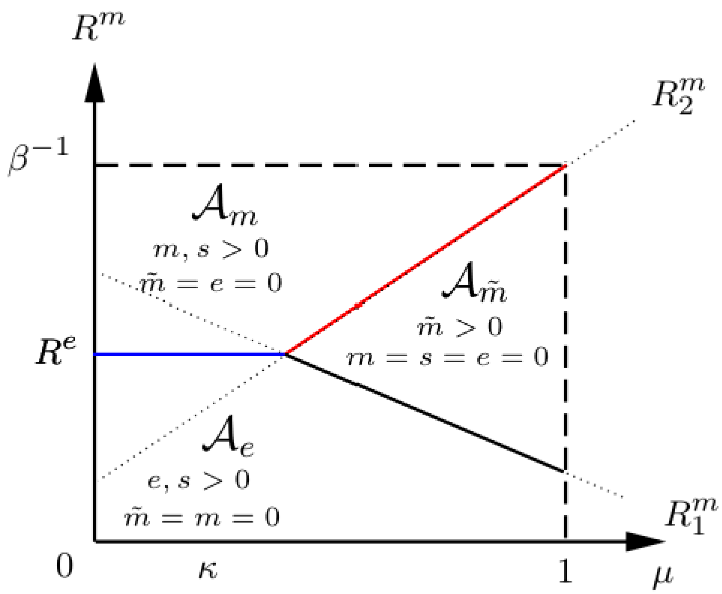

Figure 2: Distribution of Equilibria in the (R^m, μ) Space

stablecoin issuance are independent of R^m since only the return on crypto assets matters. However, for $R^m \in (R_1^m, R_2^m)$, DeFi consumption is increasing in R^m, while the crypto bank does not react to R^m since it is not issuing stablecoins. Finally, for $R^m \in (R_2^m, \beta^{-1})$, both DeFi consumption and the issuance of stablecoins are increasing in R^m: the crypto bank uses tokenized money as collateral and the higher interest rate R^m relaxes the IC of the crypto bank, allowing it to issue more stablecoins. In turn, this increases the consumption of crypto bank users.

Proposition 2. *Decreasing the degree of privacy can crowd in stablecoins.*

The intuition for Proposition 2 is straightforward: Suppose the crypto bank is not issuing stablecoins (in region $\mathcal{A}_m$). Decreasing the degree of privacy (i.e., lowering μ) directly lowers the payoff of buyers from using tokenized money to trade with sellers because sellers will discount the sales as the government may confiscate their profit. This encourages buyers to use stablecoins instead, which are not subject to privacy loss.

3.2 Traditional banks' problem

In this section, we consider the problem of a traditional bank in TradFi. Traditional banks issue deposits d to TradFi buyers, but they can also tokenize deposits D to be used in the DeFi sector, either directly by crypto buyers or as collateral by crypto banks.

We assume the traditional banks have to secure all their deposits by holding government bonds, with pledgeability ρ. Recall that the government issues B units of one-period bonds, and q is the price of newly issued bonds. A traditional bank takes q and the equilibrium interest rate on tokenized deposits R^m as given. Therefore, the traditional bank's problem is

$$\max_{a,d,D,b} [-a + \beta u(x_T)]$$

subject to $d = x_T$, its net worth being positive

$$a + D - qb + \beta [b - R^m D - d] \quad \geq \quad 0, \qquad (PC)$$

and the collateral constraint

$$\rho b \quad \geq \quad R^m D + d. \qquad (IC)$$

Note that D is an income for the bank when the tokenized deposit is created (hence relaxing the PC) and is an expense when it is redeemed (hence tightening the PC and IC). The PC necessarily binds, and when the IC also binds, the first-order conditions are

$$\begin{aligned} D: & \qquad R^m u'(\rho b - R^m D) \geq \frac{1}{\beta}, \quad (= \text{if } D > 0) \\ b: & \qquad \rho\beta u'(\rho b - R^m D) + \beta(1-\rho) = q. \end{aligned}$$

The intuition for the first-order condition with respect to b is that it costs q to acquire one additional unit of bonds, a fraction ρ of it can be used as collateral to boost consumption, which has value $u'(x_T)$, and the remaining fraction $1 - \rho$ is held to maturity, bringing a unit (discounted) payoff of β.

3.3 Central bank's problem

Before we state the definition of our equilibrium, we need to specify the problem of the central bank when issuing its CBDC.

For simplicity, we assume that traditional banks are not allowed to hold CBDC as reserves. Then, there are two main differences between CBDC and tokenized deposits: First, the central bank sets the interest rate on CBDC, implying that R^m becomes a policy rate rather than an equilibrium object that traditional banks take as given. Second, the central bank will never run away, and thus it is not subject to an incentive constraint. Still, in reality, the central bank must back the issuance of CBDC with government bonds. When it does so, it competes with the traditional banks that also have to back their deposits with government bonds.

Suppose the central bank issues M units of CBDC bearing an interest rate R^m, backed by bonds b_C. Then, the central bank's balance sheet constraint requires

$$R^m M \leq b_C.$$

When $R^m M < b_C$, the central bank earns a profit, which will be redistributed to the private agents with a lump-sum transfer. In addition, the revenue from CBDC issuance needs to be sufficient to buy the bonds to back the CBDC balances:

$$M \geq q b_C$$

These two constraints together imply that $R^m \leq 1/q$

4 Equilibrium

In this section, we start by defining an equilibrium with CBDC and tokenized deposits. Then, we show that an equilibrium with CBDC dominates an equilibrium with tokenized deposits and that traditional banks should not be allowed to issue tokenized deposits. Finally, and given these results, we show that full privacy is optimal when CBDC is the only form of tokenized money and that the interest rate on CBDC should be set such that CBDC is not used as collateral by crypto banks in DeFi but as a direct means to pay by crypto buyers. Finally, we characterize the optimal interest rate paid on the CBDC and the optimal decision on CBDC issuance.

Definition 1. *An equilibrium with CBDC and tokenized deposits is a list* $(x_T, d, D, b, x_D, e, m, \tilde{m}, b, b_C, M, q, R^m)$ *such that given prices* (q, R^m, R^e) *the traditional bank optimally chooses* (x_T, d, D, b)*, the crypto bank optimally chooses* $(x_D, e, m, \tilde{m})$*, the central bank sets* R^m *and issues* M *backed by bonds* b_C*, and both bond and money markets clear, so that* $B = (d + R^m D)/\rho + R^m M$ *and* $D + M = m + \tilde{m}$*.*

4.1 CBDC dominates tokenized deposits

Starting with any equilibrium where traditional banks issue tokenized deposits $D > 0$, the following proposition states that replacing D by M units of CBDC is welfare improving:

Proposition 3. *Replacing tokenized deposits by a CBDC can (weakly) increase welfare.*

There are three reasons why CBDC can dominate tokenized deposits. First, when the pledgeability parameter of traditional banks is $\rho < 1$, traditional banks are not as good as the central bank in providing tokenized money to DeFi. Hence, replacing tokenized deposits with a CBDC can be welfare-enhancing. Second, when $\omega < 1$, the central bank internalizes the social cost of DeFi consumption while private agents do not. Hence DeFi consumption x_D is inefficiently high. In this case, replacing D by M with a lower interest rate can improve welfare. Third, when $\mu < 1$, privacy concerns induce private agents to consume too little DeFi goods. In this case, replacing D by M with a higher interest rate can improve welfare.

Given the result above, it is natural to expect that prohibiting tokenized deposits will improve welfare. Indeed, the following proposition provides a condition under which it is necessary to prohibit tokenized deposits in order to enable the optimal design of the CBDC:

Proposition 4. *Suppose $\omega < \rho$. The optimal CBDC design requires the prohibition of tokenized deposits.*

When ω is low, DeFi trades are less socially desirable, and for a given level of privacy the central bank should set a relatively low remuneration rate on CBDC to limit DeFi activities. However, when R^m is low, traditional banks find it profitable to issue tokenized deposits at this cheap funding cost, leading to over-issuance of tokenized money. This drives the interest rate up, thus distorting the equilibrium allocation. In other words, when market forces are allowed to play, they will always constrain the central bank. Therefore, optimality requires that tokenized deposits be prohibited so that the central bank can set the interest rate at the optimal level.

Given this result, from now on, we will focus on the case where traditional banks are not allowed to issue tokenized deposits. For completeness, we describe the equilibrium with tokenized deposits in Appendix C.

4.2 Equilibrium with CBDC only

This subsection focuses on the case where only tokenized money takes the form of a central bank digital currency and traditional banks are not allowed to issue tokenized deposits or to hold CBDC as reserves.

Suppose the central bank issues M units of CBDC bearing an interest rate R^m, backed by bonds. The problem of a traditional bank is the same as above, except that $D = 0$ since it cannot issue tokenized deposits. When both the PC and IC are binding, the problem of traditional banks gives their bond demand b as the solution to

$$\beta\rho u'(\rho b) + \beta(1-\rho) = q. \tag{1}$$

When the central bank issues M units of CBDC, its balance sheet constraint requires

$$R^m M = b_C,$$

and $R^m \leq 1/q$. The market clearing condition for bonds is $b + b_C = B$. From the solution to the crypto bank problem, we know that the demand for CBDC is

$$R^m M = \begin{cases} 0, & R^m \in (0, R_1^m) \\ \frac{1}{\mu} u'^{-1}\left[\frac{1}{\beta\mu R^m}\right], & R^m \in (R_1^m, R_2^m) \\ \frac{1}{\kappa} u'^{-1}\left[\frac{1-(1-\kappa)\beta R^m}{\kappa\beta R^m}\right], & R^m \in (R_2^m, \beta^{-1}) \end{cases} \tag{2}$$

Hence, we have the following result:

Proposition 5. *With $D = 0$, an equilibrium with positive CBDC, $M > 0$, exists iff $1/q \geq R^m \geq R_1^m$.*

Notice that for the existence, it only requires that the crypto bank demands CBDC, which requires that the central bank sets $1/q \geq R^m \geq R_1^m$. Then, the central bank will supply $R^m M$ as given by (2), and the price of the government bond q is given by (1) with $b = B - R^m M$.

4.2.1 Optimal degree of privacy μ

With the central bank issuing a CBDC, the question of privacy is looming large. On the one hand, a central bank may not want to encourage illicit trade and tax evasion by providing an anonymous means to pay. On the other hand, broad adoption of CBDC requires that it preserves some level of privacy for its users. We first analyze the optimal degree of privacy μ and find the following result:

Proposition 6. *Full privacy (i.e., $\mu = 1$) is optimal when the central bank issues CBDC.*

The intuition for Proposition 6 is that government bonds are not used efficiently whenever $\mu < 1$. Recall that bonds are necessary to back the deposits of traditional banks as well as CBDC. When bonds are in scarce supply, CBDC takes resources away from traditional banks and distorts their allocation. Setting $\mu < 1$ implies that DeFi sellers value CBDC at a discount, increasing the distortion in the traditional sector. Setting $\mu = 1$ can eliminate this inefficiency. If it is important that sellers value CBDC "less," the central bank can achieve the same target value for CBDC (μR^m) by increasing μ and reducing R^m. This improves social welfare.

Note that when $\mu = 1$, CBDC is not used as collateral by crypto banks. Hence, we have the following result:

Corollary 1. *It is optimal to design CBDC so that it is not used as collateral in DeFi.*

4.2.2 Optimal CBDC rate

Given the optimality of setting $\mu = 1$, we now characterize the optimal CBDC rate and the optimal issuance policy. Assuming that the constraint $R^m \leq 1/q$ is satisfied and that the DeFi sector adopts the CBDC, the optimal rate solves

$$\max_{R^m} W = u(x_T) - x_T + \omega[u(x_D) - x_D]$$

$$\text{subject to } \beta R^m u'(x_D) = 1, \text{ and } \quad x_T = \rho(B - x_D).$$

Hence, the solution is characterized by the first-order condition

$$\rho[u'(x_T) - 1] = \omega[u'(x_D) - 1].$$

Let $\hat{R}^m$ be the rate satisfying the first-order condition, and $(\hat{x}_T, \hat{x}_D)$ be the associated consumption allocation. It is straightforward to show that $\hat{R}^m$ increases in ω, B and decreases in ρ. Using the first-order conditions of banks in the two sectors, we can verify the assumption that $R^m \leq 1/q$:

$$1/\beta R^m - 1 = u'(x_D) - 1 \geq \rho[u'(x_T) - 1] = q/\beta - 1. \tag{3}$$

So far, we assume that the CBDC is used by the DeFi sector. As we have shown, CBDC will not be adopted by the DeFi sector when $R^m < R_1^m$. In that case, the equilibrium allocation is given by TradFi consumption $\bar{x}_T \equiv \rho B$ as well as DeFi consumption $\bar{x}_D$ that satisfies

$$u'(\bar{x}_D) = \frac{1 - (1-\kappa)\beta R^e}{\kappa \beta R^e}.$$

Should the central bank set $R^m > R_1^m$ to induce CBDC adoption in the DeFi sector? This depends on the value of $\hat{R}^m$ relative to R_1^m. When $\hat{R}^m < R_1^m$, the best CBDC rate consistent with adoption is $R^m = R_1^m$ because raising $R^m > R_1^m$ will lower welfare since $\hat{R}^m < R_1^m$. But when $R^m = R_1^m$, we have $\bar{x}_T = \rho B > \hat{x}_T$ while $\bar{x}_D = \hat{x}_D$. The reason is that CBDC issuance at a higher rate requires more bonds, which takes resources away from the TradFi sector. Hence, it is optimal not to introduce a CBDC in this case. This means that $\hat{R}^m$ needs to be sufficiently high relative to R_1^m to make CBDC welfare-improving, and the optimal CBDC rate must satisfy $R^m > R_1^m$. By examining the effects of parameters on $\hat{R}^m$ and R_1^m, we can obtain the following proposition about the optimal CBDC issuance policy:

Proposition 7. *It is optimal to introduce a CBDC when B and ω are sufficiently high and κ and R^e are sufficiently low.*

Intuitively, introducing a CBDC to serve the DeFi sector is optimal when DeFi transactions are more socially desirable (ω is high), the provision of stablecoins is less efficient (κ and R^e are low), and the spillover effects on the TradFi sector are lower (B is high).

5 Extensions and discussion

In this section, we examine a few extensions of the model to generalize our results.

5.1 Private credit creation by banks

Our benchmark model considers an environment where there are no real benefits of issuing tokenized deposits. The reason is that anything a traditional bank does, the central bank can do better. In Appendix D, we examine an extension where traditional banks extend loans to firms and can use these loans to back the creation of deposits. It is important to consider lending to the real sector because the CBDC policy affects the interest rate, impacting the lending decisions of traditional banks.

We show that, as long as B is not too low, CBDC still dominates tokenized deposits, as in our benchmark model. However, when B is sufficiently scarce and DeFi consumption is sufficiently socially desirable, then the optimal CBDC policy will imply that all bonds are used to create CBDC for the crypto sector. Indeed, the central bank would like to supply more CBDC, but it is constrained by the shortage of reserve collateral. In this case, we show that allowing banks to create tokenized deposits can improve welfare as it creates corporate loans to relax the shortage of collateral. Under certain conditions, this will improve both production efficiency and consumption allocation. Of course, this result holds only under the assumption that the central bank cannot back CBDC with corporate loans.

5.2 Endogenous crypto asset returns R^e

The benchmark model takes the crypto asset return rate R^e as given. Appendix E endogenizes the return by assuming that there is a fixed supply, E, of crypto assets. The basic results hold. In addition, we show that crypto assets can carry a collateral premium, as their prices can go above their "fundamental" value. In that case, offering a higher interest rate on the CBDC can discourage crypto banks from holding crypto assets as a reserve and hence drive down the equilibrium price of crypto assets.

5.3 Supervision of crypto banks ($\kappa \uparrow$)

In the benchmark model, we assume that κ is a parameter. In reality, κ can likely be affected by government regulations. For example, supervision may raise the value of κ. This consideration is relevant as there is currently a debate on whether authorities ought to spend resources regulating crypto shadow banks. It is believed that regulation can make these banks more trustworthy so that the crypto sector can develop in a safe and sound way; abstaining from regulating them, it is thought that crypto banks cannot stand the test of time. While some economists argue that regulators should not act and just "let it burn" (e.g., Cecchetti and Schoenholtz, 2022), others point out that some regulations could be socially desirable (Waller, 2022). To shed some light on that debate, Appendix F analyzes the optimal level of pledgeability κ. The basic idea is that a strictly regulated crypto bank will have a higher pledgeability parameter because the regulation should make it more difficult for the crypto bank to run away with their assets. We derive conditions under which setting the highest degree of regulation, $\kappa = 1$, is optimal. In general, we show that the optimal regulation depends on (i) the severity of crypto banks' incentive problem, (ii) whether and how traditional bank liabilities are demanded in the crypto space, and (iii) the response of such demand to the regulation.

5.4 Outright ban of crypto banks ($\kappa = 0$)

Instead of driving up κ through supervision, regulators may also drive κ to zero by banning crypto banks outright. When $\omega > 0$, setting $\kappa = 0$ to ban crypto banks cannot improve welfare. In region $\mathcal{A}_m$, it does not matter. In the other two regions, welfare drops as x_D becomes zero. When $\omega < 0$, however, it is optimal to drive x_D to zero by setting $\kappa = 0$ and to stop the issuance of CBDC and tokenized deposits.

5.5 CBDC for the traditional sector

Many papers have already studied the competition between CBDC and bank deposits (see the literature review in Section 1). In the benchmark environment, if CBDC circulates only in the traditional sector, then it is optimal to crowd out traditional banks as central banks are more efficient in creating money whenever $\rho < 1$. We can restore a welfare trade-off when only banks can offer loans to firms, as modelled above.

5.6 Privacy concerns for stablecoins

In our model, CBDC balances lead to privacy loss, while stablecoins backed by CBDC do not. The reason is that using CBDC as a means of payment requires processing all payment transactions directly on the CBDC ledger, which the central bank can monitor directly. By contrast, using CBDC-backed stablecoins as a means of payment merely requires the issuers to hold some CBDC balances, with all the payments processed on the issuer's ledger. The central bank can potentially prohibit people from sending CBDC to the issuer's account address. However, as long as the issuer can easily create a new address to hold the CBDC, it is very hard for the central bank to completely prevent stablecoins from using CBDC as a reserve. In a sense, stablecoins are just vehicles for people to preserve privacy and use CBDC indirectly to conduct crypto transactions rather than use CBDC directly as a payment instrument. It should be straightforward to generalize the model so that stablecoins are also subject to a privacy loss $1 - \mu_m$. We expect that the basic results hold as long as stablecoins are subject to lower loss relative to a CBDC, i.e., $\mu_m > \mu$.

6 Conclusion

This paper develops a monetary model to study money creation for the crypto space and assess how introducing tokenized public and private money can affect the issuance and circulation of stablecoins.

We can now give an answer to the questions we posed in the introduction. Should public digital money (e.g., CBDC) or private digital money (e.g., tokenized deposits) be issued to serve the crypto space? In most cases, tokenized deposits are suboptimal because traditional banks face incentive problems and do not internalize the societal cost of activities, generating negative externalities. As a result, private banks can over- or under-supply tokenized deposits, hindering the implementation of optimal policy. Would the issuance of CBDC or tokenized deposits be a curse or a blessing for crypto assets, stablecoins, and illicit transactions? Both outcomes are possible, depending on the design of CBDC. Under the optimal design, a CBDC should preserve anonymity and be remunerated correctly so that it is used as a means of payment in the crypto sphere rather than as collateral by crypto banks. In addition, the optimal interest rate needs to reflect its spillover effects on the traditional sector.

A lot of our results concerning the optimal CBDC design depend on the preference of the planner for crypto activities. Therefore, our findings suggest that policymakers must clarify their objectives, pay attention to multiple channels, and consider several design features before deciding whether CBDC issuance is an appropriate response to crypto sector development.

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Appendices

A. Proofs

Proof of proposition 1:

Proof. Denote the equilibrium DeFi consumption by $x_D = s + \mu R^m \tilde{m}$. Before introducing tokenized money,

$$x_D = u'^{-1}\left[\frac{1-(1-\kappa)\beta R^e}{\kappa\beta R^e}\right], \text{ and } \quad s = x_D \tag{4}$$

When $R^m \in (1, R^m_1)$, tokenized money is not used and hence s and x_D stay unchanged and they are independent of R^m. When $R^m \in (R^m_1, R^m_2)$,

$$x_D = u'^{-1}\left[\frac{1}{\beta\mu R^m}\right], \text{ and } s = 0. \tag{5}$$

Relative to the case without tokenized money, x_D is higher (by definition of $\mathcal{A}_{\tilde{m}}$), and s is lower. Also, x_D is increasing in R^m while $s = 0$ is independent of R^m. When $R^m \geq R^m_2$,

$$x_D = u'^{-1}\left[\frac{1-(1-\kappa)\beta R^m}{\kappa\beta R^m}\right], \text{ and } s = x_D. \tag{6}$$

Relative to the case without tokenized money, x_D is higher (by definition of $\mathcal{A}_m$), and s is higher too. Also, both are increasing in R^m. □

Proof of Proposition 2:

Proof. Inside the regions $\mathcal{A}_e$, $\mathcal{A}_m$, and $\mathcal{A}_{\tilde{m}}$, an increasing μ has no effect on s. Moving from $\mathcal{A}_e$, $\mathcal{A}_m$, into $\mathcal{A}_{\tilde{m}}$, however, lowers s. □

Proof of Proposition 3:

Proof. When banks with $\rho < 1$ tokenize deposits, collateral is not used efficiently in the creation of tokenized deposits. Replacing tokenized deposits by CBDC that are traded at the same rate can support better allocation. We use superscript "0" to denote variables associated with the original equilibrium where tokenized deposits are allowed, and use "1" to denote the new equilibrium where tokenized deposits are replaced by a CBDC with the original interest rate R^m. In particular, $M^1 = D^0 + M^0, D^1 = 0$. The welfare levels associated with the two equilibria are

$$W^1 = u(x_T^1) - x_T^1 + \omega\left[u(x_D^1) - x_D^1\right]$$
$$W^0 = u(x_T^0) - x_T^0 + \omega\left[u(x_D^0) - x_D^0\right]$$

where $x_D^1 = x_D^0$, $x_T^0 = \rho(B - R^m M^0) - R^m D^0$, and $x_T^1 = \rho(B - R^m M^0 - R^m D^0)$. Obviously $x_T^1 > x_T^0$, implying that $W^1 \geq W^0$.

We now turn to the case where $\rho \leq 1$ and show that CBDC can still dominate a tokenized deposit by offering a higher or a lower R^m. Consider $R^{m,0} \in (R_1^m, R_2^m)$ when tokenized money $D^0 + M^0$ are used as a means of payments (region $\mathcal{A}_{\tilde{m}}$). The central bank can offer a CBDC with rate $R^{m,1}$ to maximize welfare:

$$\max_{R^{m,1}} W^1 = u(x_T^1) - x_T^1 + w[u(x_D^1) - x_D^1]$$
$$\text{subject to:} \qquad u'(x_D^1) = \frac{1}{\mu\beta R^{m,1}},$$

and $x_T^1 = \rho(B - x_D^1/\mu)$. The second condition (for market clearing) implies that $\partial x_T^1/\partial R^{m,1} = -\rho\left(\partial x_D^1/\partial R^{m,1}\right)/\mu$. Note that we can set the interest rate at the original level, $R^{m,1} = R^{m,0}$, and replace the tokenized deposits by CBDC (i.e., $M^1 = D^0 + M^0, D^1 = 0$) to support exactly the same allocations (i.e., $x_T^1 = x_T^0, x_D^1 = x_D^0$). Note also that the first-order conditions for DeFi and TradFi consumption imply that $u'(x_T^0) = \mu u'(x_D^0)$. Starting from the original equilibrium rate, the marginal effect of changing $R^{m,1}$ is

$$\frac{dW^1}{dR^{m,1}} = [u'(x_T^1) - 1]\frac{\partial x_T^1}{\partial R^{m,1}} + \omega[u'(x_D^1) - 1]\frac{\partial x_D^1}{\partial R^{m,1}}.$$

Using the fact that $u'(x_T^1) = \mu u'(x_D^1)$ when $R^{m,1} = R^{m,0}$, we can rewrite this expression as

$$\frac{dW^1}{dR^{m,1}}|_{R^{m,1}=R^{m,0}} = \left\{\frac{\rho}{\mu} - \omega - (\rho - \omega)\frac{1}{\mu\beta R^{m,1}}\right\}\frac{\partial x_D}{\partial R^{m,1}}. \qquad (7)$$

In general, offering a CBDC can improve welfare by varying R^m. The result is intuitive: The central bank prefers to reduce R^m to make tokenized money less valuable when ω is low and μ is high. Otherwise, the central bank prefers to raise R^m to make tokenized money more valuable when ω is high and μ is low. Note that raising R^m is feasible only when $R^m < 1/q$. We can show that this is true when $\rho < 1$. When

$\rho = 1$ and it is not feasible to raise R^m to improve welfare, CBDC cannot strictly dominate tokenized deposits.

We now consider the case with $R^{m,0} \geq R_2^m$, where tokenized money are used as collateral (region $\mathcal{A}_m$), with $\rho = 1$, the central bank solves

$$\max_{R^{m,1}} W^1 = u(x_T^1) - x_T^1 + \omega[u(x_D^1) - x_D^1]$$

$$\text{subject to:} \qquad u'(x_D^1) = \frac{1-(1-\kappa)\beta R^{m,1}}{\kappa\beta R^{m,1}},$$

and feasibility requires $x_T^1 = B - x_D^1/\kappa$. The feasibility constraint implies that $\partial x_T^1/\partial R^{m,1} = -\left(\partial x_D^1/\partial R^{m,1}\right)/\kappa$. Note that, in this region, the original equilibrium with tokenized deposits satisfies $u'(x_T^0) = (1-\kappa) + \kappa u'(x_D^0)$. Hence the marginal effect of changing $R^{m,1}$ is

$$\begin{aligned}\frac{dW^1}{dR^m}|_{R^{m,1}=R^{m,0}} &= [u'(x_T^1) - 1]\frac{\partial x_T^1}{\partial R^{m,1}} + \omega[u'(x_D^1) - 1]\frac{\partial x_D^1}{\partial R^{m,1}} \\ &= (\omega - 1)\left[u'(x_D^0) - 1\right]\frac{\partial x_D^1}{\partial R^{m,1}}\end{aligned}$$

Therefore, $dW/dR^m \leq 0$ whenever $\omega < 1$: The central bank wants to reduce the interest rate to lower the value of tokenized money when it is used as collateral, inducing crypto banks to issue less stablecoins. If $\omega = 1$, the central bank finds the allocation with tokenized deposits optimal and issuing CBDC with the same interest rate achieves the same welfare. □

Proof of Proposition 4:

Proof. The optimal CBDC design then solves

$$\max_{R^m} W = u(x_T) - x_T + \omega[u(x_D) - x_D]$$

$$\text{subject to:} \qquad u'(x_D) = \frac{1}{\beta R^m}, \text{ and } \quad x_T = \rho(B - \frac{x_D}{\mu})$$

Hence the first-order condition satisfies

$$[u'(x_T) - 1]\frac{\rho}{\mu} = \omega[u'(x_D) - 1].$$

Note that the traditional bank has an incentive to issue an infinite amount of tokenized deposits when

$$u'(x_T) < \frac{1}{\beta R^m} = u'(x_D),$$

which is satisfied because, according to the above first-order condition,

$$\begin{aligned} & u'(x_T) - u'(x_D) \\ = \quad & [u'(x_D) - 1] \frac{\mu\omega - \rho}{\rho} < 0. \end{aligned}$$

Hence, for any $\mu \in [0, 1]$, when $\omega < \rho$, the optimal allocation cannot be supported as an equilibrium if traditional banks can tokenize deposits.

□

Proof of Proposition 6

Proof. 6

Suppose $\mu < 1$. Welfare can be (weakly) improved by increasing μ as long as $x_T < x_T^*$ is defined as $u'(x_T^*) = 1$. Consider first the three interior equilibria:

regions	bounds	$R^m M$	x_D	x_T
$\mathcal{A}_e$	$R^m \leq \min\left\{\frac{\kappa R^e}{\mu[1-\beta(1-\kappa)R^e]}, R^e\right\} = R_1^m$	0	$u'(x_D) = \frac{1-(1-\kappa)\beta R^e}{\kappa\beta R^e}$	ρB
$\mathcal{A}_{\tilde{m}}$	$R_1^m \leq R^m \leq R_2^m$	$\frac{1}{\mu} u'^{-1}[u'(x_D)]$	$u'(x_D) = \frac{1}{\beta\mu R^m}$	$\rho(B - R^m M)$
$\mathcal{A}_m$	$R^m \geq \max\left\{\frac{\mu-\kappa}{\beta\mu(1-\kappa)}, R^e\right\} = R_2^m$	$\frac{1}{\kappa} u'^{-1}[u'(x_D)]$	$u'(x_D) = \frac{1-(1-\kappa)\beta R^m}{\kappa\beta R^m}$	$\rho(B - R^m M)$

In the first and the third cases, $\tilde{m} = 0$ and hence increasing μ has no effects.

In the second case, setting $\mu' > \mu$ and $R^{m\prime} = \frac{R^m \mu}{\mu'} < R^m$ will keep x_D unchanged while x_T is increased, since $R^m M$ will decline. This is welfare-improving whenever $x_T < x_T^*$.

For the four corner solutions:

	R^m	x_D	x_T
$e > 0, m > 0$	$R^m = R^e$	$u'(x_D) = \frac{1-(1-\kappa)\beta R^e}{\kappa\beta R^e}$	$\rho(B - mR^m)$
$m > 0, \tilde{m} > 0$	$\beta\mu R^m = \frac{\mu-\kappa}{1-\kappa}$	$u'(x_D) = \frac{1-\kappa}{\mu-\kappa}$	$\rho(B - (m + \tilde{m})R^m)$
$e > 0, \tilde{m} > 0$	$\beta\mu R^m = \frac{\beta\kappa R^e}{[1-(1-\kappa)\beta R^e]}$	$u'(x_D) = \frac{1-(1-\kappa)\beta R^e}{\kappa\beta R^e}$	$\rho(B - \tilde{m}R^m)$
$e > 0, \tilde{m} > 0, m > 0$	$\beta\mu R^m = \frac{\beta\kappa R^e}{[1-(1-\kappa)\beta R^e]} = \frac{\kappa R^m}{[1-(1-\kappa)\beta R^m]}$	$u'(x_D) = \frac{1-(1-\kappa)\beta R^e}{\kappa\beta R^e}$	$\rho(B - (m + \tilde{m})R^m)$

When $e > 0, m > \tilde{m} = 0$: Marginally increasing μ will maintain the indifference condition, having no effects on the equilibrium allocation.

When $e = 0, m > 0, \tilde{m} > 0$: Marginally increasing μ to μ' and lowering R^m to $R^{m\prime}$ will enter the $\mathcal{A}_{\tilde{m}}$ region. This will keep x_D unchanged at $u'(x_D) = \frac{1-\kappa}{\mu-\kappa}$ while x_T will go up as R^m declines.

When $e > 0, \tilde{m} > m = 0$: Marginally increasing μ to μ' and lowering R^m to $R^{m\prime}$ will maintain the indifference condition. This will keep x_D unchanged at $u'(x_D) = \frac{[1-(1-\kappa)\beta R^e]}{\beta\kappa R^e}$ while x_T goes up as R^m declines.

When $e > 0, \tilde{m} > m = 0$: Marginally increasing μ to μ' and lowering R^m to $R^{m\prime}$ will induce an equilibrium with $e = 0, m > 0, \tilde{m} > 0$. By setting $e' = e$ and $\tilde{m}' = \tilde{m} + m$, we can support the same x_D while x_T goes up as R^m declines. $\square$

B. Crypto bank problem: all equilibrium cases

Using (PC) rewrite as

$$\max_{s,e,m,\tilde{m}} -m - e + \beta\left[R^e e + R^m m - s\right] - \tilde{m} + \beta u(s + \mu R^m \tilde{m})$$
$$+\beta\lambda\left[\kappa\left(R^e e + R^m m\right) - s\right]$$

First-order condition:

$$\begin{aligned}
s: &\quad u'(s+\mu R^m \tilde{m}) &= 1+\lambda \\
e: &\quad (1+\lambda\kappa)\beta R^e &= 1 \\
m: &\quad (1+\lambda\kappa)\beta R^m &\leq 1 \\
\tilde{m}: &\quad u'(s+\mu R^m\tilde{m})\beta\mu R^m &\leq 1
\end{aligned}$$

We will focus on the parameter space where (IC) is binding. **We will provide conditions under which $\lambda > 0$.** Hence the problem becomes

$$\max_{e,m,\tilde{m}} -m - \tilde{m} - e + \beta(1-\kappa)\left[R^e e + R^m m\right] + \beta u\underbrace{\left(\kappa R^e e + \kappa R^m m + \mu R^m \tilde{m}\right)}_{=z}$$

First-order condition:

$$\begin{aligned}
e: &\quad (1-\kappa)R^e + \kappa R^e u'(z) &\leq \frac{1}{\beta} \\
m: &\quad (1-\kappa)R^m + \kappa R^m u'(z) &\leq \frac{1}{\beta} \\
\tilde{m}: &\quad \mu R^m u'(z) &\leq \frac{1}{\beta}
\end{aligned}$$

When $e > 0$, the consumption is given by

$$u'(z) \quad = \quad \frac{1-(1-\kappa)\beta R^e}{\kappa\beta R^e}.$$

Note that z is below the first-best level unless $\kappa = 1$ or $\beta R^e = 1$. The solution is $e, m, \tilde{m} > 0$ iff

$$R^m = R^e = \frac{(\mu-\kappa)}{\beta\mu(1-\kappa)},$$

which is a knife-edge case. We first consider three conditions. First, $m > 0$ and $\tilde{m} = 0$ iff

$$R^m \geq \mathbf{C}_{m\tilde{m}}(\mu, \kappa) \equiv \frac{(\mu - \kappa)}{\beta\mu(1 - \kappa)},$$

with the right-hand side increasing in μ and decreasing in κ. Also, $\mathbf{C}_{m\tilde{m}}(1, \kappa) = \beta^{-1}$. Second, $e > 0$ and $m = 0$ iff

$$R^m \leq \mathbf{C}_{em}(R^e) \equiv R^e.$$

Third, $e > 0$ and $\tilde{m} = 0$ iff

$$R^m \leq \mathbf{C}_{e\tilde{m}}(\mu, \kappa, R^e) \equiv \frac{\kappa R^e}{\mu\left[1 - (1 - \kappa)\beta R^e\right]},$$

with the right-hand side increasing in $R^e$and κ and decreasing in μ. We plot the three conditions in the (R^m, μ) space in Figure 2. Note that the lines defined by these conditions intersect at the point $R^m = R^e$ and $\mu = \frac{\kappa}{1 - R^e\beta(1-\kappa)} < \kappa$. Three equilibrium regions are identified:

(i) $\mathcal{A}_e$ with $e > 0$ $(m = \tilde{m} = 0)$ where

$$u'(z) = \frac{1 - (1 - \kappa)\beta R^e}{\kappa\beta R^e},$$
$$R^m < \min\left\{R^e, \frac{\kappa R^e}{\mu\left[1 - (1 - \kappa)\beta R^e\right]}\right\}.$$

(ii) $\mathcal{A}_m$ with $m > 0$ $(e = \tilde{m} = 0)$ where

$$u'(z) = \frac{1 - (1 - \kappa)\beta R^m}{\kappa\beta R^m},$$
$$\max([\frac{(\mu - \kappa)}{\beta\mu(1 - \kappa)}, R^e]) < R^m.$$

(iii) $\mathcal{A}_{\tilde{m}}$ with $\tilde{m} > 0$ $(e = m = 0)$ where

$$\mu R^m u'(\mu R^m \tilde{m}) = \frac{1}{\beta},$$
$$\frac{\kappa R^e}{\mu[1 - (1 - \kappa)R^e\beta]} < R^m < \frac{\mu - \kappa}{\beta\mu(1 - \kappa)}.$$

The following table lists the complete set of potential equilibria:

	e	m	$\tilde{m}$
(a)	> 0	0	0
(b)	0	> 0	0
(c)	0	0	> 0
(d)	> 0	> 0	0
(e)	0	> 0	> 0
(f)	> 0	0	> 0
(g)	> 0	> 0	> 0

The conditions for their existence are derived below.

Case (a): $e > 0, m = \tilde{m} = 0$**:**

$$\begin{aligned} e: \quad (1-\kappa)R^e + \kappa R^e u'(z) &= \frac{1}{\beta} \\ m: \quad (1-\kappa)R^m + \kappa R^m u'(z) &< \frac{1}{\beta} \\ \tilde{m}: \quad \mu R^m u'(z) &< \frac{1}{\beta} \end{aligned}$$

$$u'(z) = \frac{1-(1-\kappa)\beta R^e}{\kappa\beta R^e}$$

and

$$R^m < R^e$$

and

$$\begin{aligned} \mu R^m u'(z) = \mu R^m \frac{1-(1-\kappa)\beta R^e}{\kappa\beta R^e} &< \frac{1}{\beta}, \\ R^m &< \frac{\kappa R^e}{\mu\left[1-(1-\kappa)\beta R^e\right]}. \end{aligned}$$

This equilibrium exists in area $\mathcal{A}_e$.

Case (b): $e = 0, m > 0, \tilde{m} = 0$**:**

$$\begin{aligned} e: \quad (1-\kappa)R^e + \kappa R^e u'(z) &< \frac{1}{\beta} \\ m: \quad (1-\kappa)R^m + \kappa R^m u'(z) &= \frac{1}{\beta} \\ \tilde{m}: \quad \mu R^m u'(z) &< \frac{1}{\beta} \end{aligned}$$

and $R^e < R^m$ and

$$u'(z) = \frac{1-(1-\kappa)\beta R^m}{\kappa\beta R^m}$$

$$\begin{aligned} \mu u'(z) &< (1-\kappa)+\kappa u'(z) \\ \frac{(\mu-\kappa)}{\beta\mu(1-\kappa)} &< R^m \end{aligned}$$

This equilibrium exists in area $\mathcal{A}_m$.

Case (c): $e = 0, m = 0, \tilde{m} > 0$**:**

$$\begin{aligned} e: \quad (1-\kappa)R^e + \kappa R^e u'(z) &< \frac{1}{\beta} \\ m: \quad (1-\kappa)R^m + \kappa R^m u'(z) &< \frac{1}{\beta} \\ \tilde{m}: \quad \mu R^m u'(z) &= \frac{1}{\beta} \end{aligned}$$

and

$$\mu R^m u'(\mu R^m \tilde{m}) = \frac{1}{\beta}$$

which requires

$$\begin{aligned} (1-\kappa)R^m + \frac{\kappa}{\mu}\frac{1}{\beta} &< \frac{1}{\beta} \\ \rightarrow R^m &< \frac{\mu-\kappa}{\beta\mu(1-\kappa)} \end{aligned}$$

$$\begin{aligned} &(1-\kappa)R^e + \kappa R^e \frac{1}{\beta\mu R^m} < \frac{1}{\beta} \\ &\rightarrow \frac{\kappa R^e}{\mu[1-(1-\kappa)R^e\beta]} < R^m \end{aligned}$$

This equilibrium exists in area $\mathcal{A}_{\tilde{m}}$.

Case (d): $e > 0, m > \tilde{m} = 0$**:**

$$\begin{aligned} e: \quad (1-\kappa)R^e + \kappa R^e u'(z) &= \frac{1}{\beta} \\ m: \quad (1-\kappa)R^m + \kappa R^m u'(z) &= \frac{1}{\beta} \\ \tilde{m}: \quad \mu R^m u'(z) &< \frac{1}{\beta} \end{aligned}$$

Hence $R^m = R^e$ and

$$\begin{aligned}
\mu R^m u'(z) &< \frac{1}{\beta} \\
\mu R^m \frac{1-(1-\kappa)\beta R^e}{\kappa\beta R^e} &< \frac{1}{\beta} \\
\frac{\mu-\kappa}{\beta\mu(1-\kappa)} &< R^e = R^m
\end{aligned}$$

This equilibrium exists along the line defined by $R^m = \mathbf{C}_{em}$.

Case (e): $e = 0, m > 0, \tilde{m} > 0$**:**

$$\begin{aligned}
e: \quad (1-\kappa)R^e + \kappa R^e u'(z) &< \frac{1}{\beta} \\
m: \quad (1-\kappa)R^m + \kappa R^m u'(z) &= \frac{1}{\beta} \\
\tilde{m}: \quad \mu R^m u'(z) &= \frac{1}{\beta}
\end{aligned}$$

then $R^e < R^m$ and from the last two

$$R^m = \frac{\mu-\kappa}{\beta\mu(1-\kappa)}$$

so that

$$u'(z) = \frac{1-(1-\kappa)\beta\frac{\mu-\kappa}{\beta\mu(1-\kappa)}}{\kappa\beta\frac{\mu-\kappa}{\beta\mu(1-\kappa)}} = \frac{1-\kappa}{\mu-\kappa}.$$

This equilibrium exists along the line defined by $R^m = \mathbf{C}_{m\tilde{m}}$.

Case (f): $e > 0, \tilde{m} > m = 0$**:**

$$\begin{aligned}
e: \quad (1-\kappa)R^e + \kappa R^e u'(z) &= \frac{1}{\beta} \\
m: \quad (1-\kappa)R^m + \kappa R^m u'(z) &< \frac{1}{\beta} \\
\tilde{m}: \quad \mu R^m u'(z) &= \frac{1}{\beta}
\end{aligned}$$

Hence

$$R^m < R^e$$

and

$$(1-\kappa)R^e + \kappa R^e \frac{1}{\mu R^m \beta} = \frac{1}{\beta}$$

so that

$$R^m = \frac{\kappa R^e}{\mu\left[1-(1-\kappa)\beta R^e\right]}$$

Since $R^m < R^e$,

$$\begin{aligned} \frac{\kappa R^e}{\mu\left[1-(1-\kappa)\beta R^e\right]} &< R^e \\ R^e &< \frac{\mu-\kappa}{\beta\mu(1-\kappa)} \end{aligned}$$

This equilibrium exists along the line defined by $R^m = \mathbf{C}_{e\tilde{m}}$.

Case (g): $e, m, \tilde{m} > 0$

$R^m = R^e$ and

$$\begin{aligned} (1-\kappa)R^m + \kappa R^m u'(z) &= \mu R^m u'(z) \\ u'(z) &= \frac{(1-\kappa)}{(\mu-\kappa)} \end{aligned}$$

and

$$\begin{aligned} \mu R^m u'(z) &= \frac{1}{\beta} \\ R^m &= \frac{(\mu-\kappa)}{\beta\mu(1-\kappa)} \end{aligned}$$

which is a knife-edge case given by the interaction point of the three lines.

C. Equilibrium with tokenized deposits

The solution of the traditional bank's problem can potentially lead to two outcomes. First, the traditional bank chooses optimally not to issue tokenized deposits (i.e., $D = 0$). Then the consumption of traditional depositors is $x_T = \rho B$ and the price of government bonds q is given by

$$q = \rho\beta u'(\rho B) + \beta(1-\rho).$$

This is an equilibrium if

$$R^m u'(\rho B) \geq \frac{1}{\beta}.$$

Second, the traditional bank issues tokenized deposits $D > 0$ to the crypto bank. Then the consumption of traditional depositors is reduced to $x_T = \rho B - R^m D$, and the supply of tokenized deposits solves

$$\frac{1}{R^m} = \beta u'(\rho B - R^m D)$$

while (given ρ and B) the price of the government bonds is increased to

$$q = \rho\beta u'(\rho B - R^m D) + \beta(1-\rho) = \rho\frac{1}{R^m} + \beta(1-\rho).$$

This is an equilibrium if it is cheap to issue tokenized deposits (R^m is low) and/or there is an abundance of government bonds

$$R^m u'(\rho B) < \frac{1}{\beta}.$$

We first define an equilibrium with tokenized deposits.

Definition 2. *An equilibrium with tokenized deposits is a list* $(x_T, d, D, b, x_D, e, m, \tilde{m}, b, q, R^m)$ *such that given prices* (q, R^m, R^e) *the traditional bank optimally chooses* (x_T, d, D, b)*, and the crypto bank optimally chooses* $(x_D, e, m, \tilde{m})$*, and markets clear, so that* $B = (d + R^m D)/\rho$ *and* $D = m + \tilde{m}$*.*

Proposition 8. *An equilibrium with positive tokenized deposits* ($D > 0$) *exists iff*

$$u'(\rho B)\beta R^e \min\left\{1, \frac{\kappa}{\mu\left[1-(1-\kappa)\beta R^e\right]}\right\} < 1.$$

Proof. Recall that the crypto bank demands tokenized money whenever $R_1^m \leq R^m$, while the traditional bank supplies tokenized deposits whenever $u'(\rho B)\beta R^m < 1$. The result follows from combining both conditions and the definition of R_1^m. □

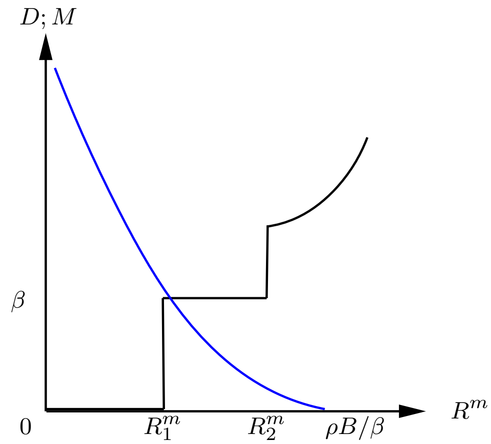

Figure 3: Tokenized Deposit Supply and Demand (with log utility)

Proposition 8 shows that an equilibrium with tokenized deposits exists whenever the supply of government bonds and their pledgeability is large, while the return on crypto assets and their pledgeability is small. Also, a high degree of privacy (a high μ) makes the existence of that equilibrium more likely because it increases the exchange value of tokenized deposits.

Figure 3 shows the demand and supply of tokenized deposits in the case of log utility. In this case, there exists a unique equilibrium. Deposits are tokenized iff

$$\rho B > \beta R^e \min \left\{ 1, \frac{\kappa}{\mu \left[1 - (1-\kappa)\beta R^e \right]} \right\}.$$

D. Private credit creation by banks

We model bank credit by assuming that traditional banks can purchase bonds from a firm that represents the aggregate productive sector. Each bond has price q and gives a real redemption value of 1. The representative firm's production function is $F(k)$ and the firms' profit is $F(k) - (1+r)k$, where k is the capital invested by the firm. In equilibrium, when the firm issues L bonds, it will be able to invest $k = qL$ and its profit will be $F(qL) - L$.

Hence the supply of corporate bonds L_s is given by the solution to the firm's problem

$$\max_{L_s} F(qL_s) - L_s$$

and the first-order condition gives $qF'(qL_s) = 1$. Therefore if $F(.)$ is very concave such that $-qL_s \frac{F''(qL_s)}{F'(qL_s)} > 1$, then $\partial L_s/\partial q < 0$. Otherwise, $\partial L_s/\partial q > 0$. We assume the latter so that increasing q (decreasing the interest rate) increases the supply of corporate bonds. For example, if $F(k) = Ak^\alpha$, the first-order condition gives $A\alpha q\,(qL)^{\alpha-1} = 1$ so $L^{1-\alpha} = A\alpha q^\alpha$ and L is increasing with q whenever $\alpha < 1$.

The problem of the traditional bank then is

$$\max_{a,d,b,L} \left[-a + \beta u(x_T)\right]$$

subject to $x_T = d$ and

$$\begin{aligned} a + D - q(b+L) + \beta\left(b + L - R^m D - d\right) &\geq 0 \\ \rho(b+L) &\geq d \end{aligned}$$

Notice that the problem is now a function of $\tilde{b} = b + L$. So when both constraints bind, an equilibrium with $D > 0$ is characterized by $x_T = \rho(B + L_s(q)) - R^m D$ and

$$\begin{aligned} u'(x_T) &= \frac{1}{\beta R^m} \\ \rho\beta u'(x_T) + \beta(1-\rho) &= q \end{aligned}$$

So $\partial x_T/\partial R^m > 0$ and $\partial q/\partial R^m < 0$, and D is given by the demand for tokenized deposits, as in the previous section.

Welfare is

$$W^T = u(x_T) - x_T + F(qL) - L + \omega\left[u(x_D) - x_D\right]$$

If $\rho < 1$ we have the same result as before (CBDC does better than tokenized deposits because it uses collateral more efficiently). Let us set $\rho = 1$. A change in R^m affects welfare in the following way:

$$\frac{\partial W^T}{\partial R^m} = [u'(x_T) - 1]\frac{\partial x_T}{\partial R^m} + \underbrace{[qF'(qL) - 1]}_{=0}\frac{\partial L}{\partial R^m} + LF'(qL)\frac{\partial q}{\partial R^m} + \omega\,[u'(x_D) - 1]\frac{\partial x_D}{\partial R^m}$$

In particular, increasing R^m now reduces firms' production. Also, from market clearing, $x_T = B + L_s(q) - x_D/\nu$ (where $\nu = \mu$ in region $\mathcal{A}_{\tilde{m}}$ and $\nu = \kappa$ in region $\mathcal{A}_m$). Therefore, it requires

$$\frac{\partial x_T}{\partial R^m} = L_s'(q)\frac{\partial q}{\partial R^m} - \frac{1}{\nu}\frac{\partial x_D}{\partial R^m}$$

and $\partial x_T/\partial R^m$ is affected by the change in q (since traditional banks are the only ones that purchase corporate bonds). It is straightforward to show that $\mu = 1$ using the same steps as in the previous section. The reason is that the previous proof only requires that R^m be reduced, which here plays to increase q and so L.

Then, following again the same steps as before, it is easy to show that, starting from an equilibrium with tokenized deposits, the central bank can achieve a better allocation by issuing a CBDC at a lower rate R^m_{cbdc} than the prevailing equilibrium interest rate, as long as $F(\cdot)$ is not too concave (so that L is increasing with q). However, a necessary condition is that the supply of government bonds B is large enough to allow the central bank to issue the sufficient amount of CBDC. Otherwise it is not clear that CBDC will dominate tokenized deposits. We summarize in the following result:

Corollary 2. *Suppose B is large enough or that the central bank can back CBDC with real assets. Then, in a lending economy ($L > 0$), replacing tokenized deposits by a CBDC can (weakly) increase welfare.*

Finally, we illustrate why legalizing tokenized deposits can be welfare-improving when B is small. Consider first an economy without tokenized deposits. Suppose $B \to 0$ and $R^e \to 0$. Then it is optimal for the central bank to devote all the government bonds B to issue CBDC for the DeFi sector if

$$[u'(\rho L) - 1]\rho < \omega[u'(B) - 1]$$

and the price of corporate bonds is $q_L < 1$. The former condition ensures that allocating government bonds to CBDC creation dominates using bonds to create deposits, and the latter condition ensures that this does not drive the corporate bond price so high that there is over-production. Now if banks are allowed to issue some tokenized deposits, they will have an incentive to do so by purchasing more corporate bonds and allocating more consumption to the DeFi sector as long as

$$[u'(\rho L) - 1] < [u'(B) - 1].$$

A sufficient condition is $\rho > \omega$. One can then show that, at the margin, permitting some tokenized deposits can improve social welfare by improving (i) consumption allocation between the two sectors and (ii) production efficiency.

E. Endogenous crypto asset returns R^e

Suppose there is a fixed supply E of the crypto assets. These assets pay a dividend δ each period. These assets are initially held by risk-neutral agents that discount the future at rate β. These agents are active in the PM and crypto AM, and they can trade their asset there to obtain utility $v(x)$. Assume $v'(0)$ is finite, so that these agents may want to sell all their assets. Also they can work to produce the PM good with a linear technology. The problem of these agents in the PM is to sell $e^s \leq E$ to maximize

$$\max_{e^s,x} p_t^e e^s + \beta v(x) + \beta(\delta + p_{t+1}^e)\left[E - e^s - x\right]$$

subject to

$$x \leq (p_{t+1}^e + \delta)\left(E - e^s\right).$$

The assumption here is that sellers acquire the crypto asset in the AM by producing x, and sell these assets back in the crypto PM. So the value of one unit of the crypto asset for sellers is $(p_{t+1}^e + \delta)$. So bringing $E - e^s$ in the crypto PM, crypto consumers can get at most $(p_{t+1}^e + \delta)\left(E - e^s\right)$ of the crypto good. The first-order condition with respect to x gives

$$\beta v'(x) - \beta(\delta + p_{t+1}^e) - \lambda_x = 0.$$

Hence if these agents constraint binds in the AM, $x = (p_{t+1}^e + \delta)\left(E - e^s\right)$, the first-order condition gives

$$p_t^e = \beta v'\left((\delta + p_{t+1}^e)\left(E - e^s\right)\right) > \beta(\delta + p_{t+1}^e)$$

while if the constraint does not bind, then

$$p_t^e = \beta(\delta + p_{t+1}^e).$$

So the "natural" price of these assets after they have paid the dividend δ is

$$p_t^e = \beta\left(\delta + p_{t+1}^e\right)$$

and in steady state,

$$p^e = \frac{\beta\delta}{1-\beta}.$$

If the constraint binds, $p_t^e > \beta(\delta + p_{t+1}^e)$.

To be consistent with the model notation, define the return at time t from the asset as

$$R^e = \frac{(\delta + p^e_{t+1})}{p^e_t}$$

and in steady state the equilibrium return gives the price,

$$\beta R^e = \frac{\beta(\delta + p^e)}{p^e} \leq 1,$$

where $\beta R^e < 1$ if the selling constraint binds. Notice that

$$p^e = \frac{\delta}{R^e - 1}.$$

Let's analyze an equilibrium in region $\mathcal{A}_e$, where we have $e > 0$ and $m = \tilde{m} = 0$—the equilibrium in the other two regions is straightforward because the crypto bank does not demand any e, so that $e^s = 0$ in equilibrium—where

$$\begin{aligned} u'(\kappa R^e e) &= \frac{1-(1-\kappa)\beta R^e}{\kappa \beta R^e}, \\ R^m &< \min\left\{\frac{\kappa R^e}{\mu\left[1-(1-\kappa)\beta R^e\right]}, R^e\right\} \equiv R^m_1 \end{aligned}$$

Case 1. $\beta R^e = 1$ and $e < E$. Then the equilibrium in the crypto sector is given by

$$u'\left(\kappa \frac{e}{\beta}\right) = 1$$

and market clearing gives $e = e^s$ such that

$$p^e = \frac{\beta\delta}{1-\beta} = \beta v'\left((\delta + p^e)(E - e)\right).$$

So this is a knife-edge case. This is an equilibrium if

$$R^m < \min\left\{\frac{1}{\beta\mu}, \frac{1}{\beta}\right\} \equiv R^m_1.$$

Case 2. $\beta R^e < 1$ and $e < E$. Then the equilibrium in the crypto sector is given by

$$u'(\kappa R^e e) = \frac{1-(1-\kappa)\beta R^e}{\kappa\beta R^e} > 1,$$

and using market clearing

$$p^e = \frac{\delta}{R^e - 1} = \beta v'\left((\delta + p^e)(E - e)\right).$$

These two equations give the equilibrium (R^e, e). This is an equilibrium iff

$$R^m < \min\left\{\frac{\kappa R^e}{\mu\left[1-(1-\kappa)\beta R^e\right]}, R^e\right\} \equiv R_1^m.$$

Case 3. $\beta R^e < 1$ and $e = E$. Then the equilibrium rate of return in the crypto sector R^e is given by

$$u'(\kappa R^e E) = \frac{1-(1-\kappa)\beta R^e}{\kappa\beta R^e} > 1,$$

and using market clearing

$$p^e = \frac{\delta}{R^e - 1} > \beta v'(0),$$

so the price is so high that the holders of crypto assets forgo the gains from trading with crypto assets by selling them all to the crypto bank. This is an equilibrium iff

$$R^m < \min\left\{\frac{\kappa R^e}{\mu\left[1-(1-\kappa)\beta R^e\right]}, R^e\right\} \equiv R_1^m.$$

Bottom line:

Since $v(.)$ is concave, crowding out crypto assets from the crypto bank by offering a higher rate of return CBDC will reduce the price of crypto assets p^e. We can see this from the equilibrium equation $p^e = v'\left((\delta + p^e)(E - e)\right)$ and using the implicit function theorem

$$dp^e = (E-e)v''(x)\, dp^e - (\delta + p^e)v''(x)\, de.$$

Hence

$$\frac{dp^e}{de} = \frac{-(\delta + p^e)v''(x)}{1-(E-e)v''(x)} > 0$$

and the more the crypto bank demands crypto assets, the higher their price p^e.

F. Supervision of crypto banks

The optimal choice of κ maximizes W subject to the constraint that (x_D, x_T) are equilibrium allocations of the economy with tokenized deposits. To facilitate the presentation of the analysis, we summarize the equilibrium allocations with tokenized deposits (x_D, x_T) in the following table:

	x_D	x_T
$\mathcal{A}_e$ or $R^m < R_1^m$	$x_D = \kappa R^e e \quad u'(x_D) = \frac{1-(1-\kappa)\beta R^e}{\kappa \beta R^e}$	$x_T = \rho B \quad u'(x_T) \geq \frac{1}{\beta R^m}$
$\mathcal{A}_{\tilde{m}}$ or $R_2^m \geq R^m \geq R_1^m$	$x_D = \mu R^m D \quad u'(x_D) = \frac{1}{\beta \mu R^m}$	$x_T = \rho B - R^m D \quad u'(x_T) = \frac{1}{\beta R^m}$
$\mathcal{A}_m$ or $R^m \geq R_2^m$	$x_D = \kappa R^m D \quad u'(x_D) = \frac{1-(1-\kappa)\beta R^m}{\kappa \beta R^m}$	$x_T = \rho B - R^m D \quad u'(x_T) = \frac{1}{\beta R^m}$

In region $\mathcal{A}_e$, there is no demand for tokenized deposits. As a consequence there is a dichotomy between the traditional and crypto sectors. In particular, the trade surplus in the traditional sector is independent of κ, while the surplus in the crypto sector is increasing in κ. Therefore the optimal policy in this region is to increase κ to 1, thus maximizing the pledgeability of the assets held by the crypto banks. Also, increasing κ will increase R_1^m so that this region applies for more parameters: It is more likely that the crypto will not rely on tokenized money, thus maintaining the (potentially) desirable separation between the crypto and the traditional sectors. But starting from an equilibrium in region $\mathcal{A}_e$ and increasing κ, the equilibrium remains in region $\mathcal{A}_e$.

In region $\mathcal{A}_{\tilde{m}}$, the crypto bank only acquires tokenized deposits from the traditional bank and passes it to its users who use them directly in the crypto space. Therefore κ does not affect the allocation in this region, because the crypto bank is a mere pass-through vehicle. However, κ affects the possibility of falling in this region: Indeed, increasing κ above some threshold value will imply that $R_1^m = R_2^m = R^e$ so that this region vanishes. Then the crypto bank will turn from simple pass-through to an intermediary. To the contrary, decreasing κ (e.g., by relaxing regulations) makes falling in $\mathcal{A}_{\tilde{m}}$ more likely, where crypto banks have no meaningful role. This is how limited regulation induces the crypto bank to "burn" in our model.

In region $\mathcal{A}_m$, the crypto bank acquires tokenized deposits and use them as collateral to secure its issuance of stablecoins. Intuitively, keeping x_D constant, it is clear that increasing κ implies a reduction in D, so that the consumption in the traditional sector (and surplus) increases. The reason is that a higher degree of pledgeability of the collateral in the crypto sector frees up resources in the traditional sector. However, x_D does not need to remain constant when κ changes. In particular, if the substitution effect is stronger than the income effect, then increasing κ may increase the demand for tokenized money so much that $R^m D$ increases. The proposition below says that if the coefficient of relative risk aversion ξ is high enough, then the substitution effect is sufficiently muted that increasing κ reduces the demand for tokenized deposits (cum interest, so $R^m D$) while still increasing the surplus for crypto users.

More precisely, in region $\mathcal{A}_m$ the equilibrium is given by the following two equations in the two unknowns (R^m, D),

$$\begin{aligned} \kappa\beta R^m u'(\kappa R^m D) + (1-\kappa)\beta R^m &= 1 \\ \beta R^m u'(\rho B - R^m D) &= 1 \end{aligned}$$

Below, we show that if the (constant) coefficient of relative risk aversion ξ is greater than some threshold $\bar{\xi}$ defined in the proof, then a rise in κ increases the surplus in both the traditional and the crypto sectors.

Proposition 9. *There is $\bar{\xi}$ such that if the coefficient of relative risk aversion is $\xi > \bar{\xi}$, then $\kappa = 1$ is optimal.*

Proof.

$$\begin{aligned} \kappa\beta R^m u'(\kappa R^m D) + (1-\kappa)\beta R^m &= 1 \\ \beta R^m u'(\rho B - R^m D) &= 1 \end{aligned}$$

Define $x = R^m D$, then we have

$$\begin{aligned} \kappa x u'(\kappa x) + (1-\kappa)x &= D/\beta \\ x u'(\rho B - x) &= D/\beta \end{aligned}$$

Hence, total derivatives give

$$\begin{aligned} \left[x u'(\kappa x) + \kappa x^2 u''(\kappa x) - x\right] d\kappa + \left[\kappa u'(\kappa x) + \kappa^2 x u''(\kappa x) + (1-\kappa)\right] dx &= dD/\beta \\ \left[u'(\rho B - x) - x u''(\rho B - x)\right] dx &= dD/\beta \end{aligned}$$

From the last equation we have $dD/dx > 0/$. We use a constant coefficient of risk aversion ξ, so that

$$\begin{aligned} x\left[u'(\kappa x)\left(1+\frac{\kappa x u''(\kappa x)}{u'(\kappa x)}\right)-1\right]d\kappa+\kappa\left[u'(\kappa x)\left(1+\frac{\kappa x u''(\kappa x)}{u'(\kappa x)}\right)+\frac{(1-\kappa)}{\kappa}\right]dx &= dD/\beta \\ u'(\rho B-x)\left[1-x\frac{u''(\rho B-x)}{u'(\rho B-x)}\right]dx &= dD/\beta \end{aligned}$$

and

$$\begin{aligned} x\left[u'(\kappa x)(1-\xi)-1\right]d\kappa+\kappa\left[u'(\kappa x)(1-\xi)+\frac{(1-\kappa)}{\kappa}\right]dx &= dD/\beta \\ u'(\rho B-x)\left[1+\frac{x}{\rho B-x}\xi\right]dx &= dD/\beta \end{aligned}$$

Using the first-order condition,

$$\begin{aligned} u'(\kappa R^m D) &= \frac{1-(1-\kappa)\beta R^m}{\kappa\beta R^m} \\ u'(\rho B-R^m D) &= \frac{1}{\beta R^m} \end{aligned}$$

we obtain

$$\begin{aligned} x\left[\frac{1-(1-\kappa)\beta R^m}{\kappa\beta R^m}(1-\xi)-1\right]d\kappa+\kappa\left[\frac{1-(1-\kappa)\beta R^m}{\kappa\beta R^m}(1-\xi)+\frac{(1-\kappa)}{\kappa}\right]dx &= dD/\beta \\ \frac{1}{\beta R^m}\left[1+\frac{x}{\rho B-x}\xi\right]dx &= dD/\beta \end{aligned}$$

Combining both equations we get $dx/d\kappa$:

$$x\left[\frac{1-(1-\kappa)\beta R^m}{\kappa\beta R^m}(1-\xi)-1\right]d\kappa+\kappa\left[\frac{1-(1-\kappa)\beta R^m}{\kappa\beta R^m}(1-\xi)+\frac{(1-\kappa)}{\kappa}\right]dx = \frac{1}{\beta R^m}\left[1+\frac{x}{\rho B-x}\xi\right]dx$$

or

$$\frac{dx}{d\kappa}=\frac{x\left[1-\frac{1-(1-\kappa)\beta R^m}{\kappa\beta R^m}(1-\xi)\right]}{\kappa\left[\frac{1-(1-\kappa)\beta R^m}{\kappa\beta R^m}(1-\xi)+\frac{(1-\kappa)}{\kappa}\right]-\frac{1}{\beta R^m}\left[1+\frac{x}{\rho B-x}\xi\right]}$$

If $\xi \geq 1$, the numerator is positive and the denominator is negative whenever

$$\begin{aligned} \frac{1-(1-\kappa)\beta R^m}{\beta R^m}(1-\xi)+(1-\kappa)-\frac{1}{\beta R^m}\left[1+\frac{x}{\rho B-x}\xi\right] &< 0 \\ \left[1-(1-\kappa)\beta R^m\right](1-\xi)+\beta R^m(1-\kappa)-\left[1+\frac{x}{\rho B-x}\xi\right] &< 0 \\ (1-\xi)+(1-\kappa)\beta R^m\xi-\left[1+\frac{x}{\rho B-x}\xi\right] &< 0 \\ -1+(1-\kappa)\beta R^m-\frac{x}{\rho B-x} &< 0 \end{aligned}$$

which is always the case since $\beta R^m \leq 1$. Hence if $\xi \geq 1$ we have $dx/d\kappa \leq 0$. Therefore, $dD/d\kappa \leq 0$. Hence, as κ increases, consumption in the traditional sector increases.

Finally, we want to know the sign of $d(\kappa x)/d\kappa$, which is given by the sign of

$$\begin{aligned} x + \kappa\frac{dx}{d\kappa} &= x + \kappa\frac{x\left[1 - \frac{1-(1-\kappa)\beta R^m}{\kappa\beta R^m}(1-\xi)\right]}{\kappa\left[\frac{1-(1-\kappa)\beta R^m}{\kappa\beta R^m}(1-\xi) + \frac{(1-\kappa)}{\kappa}\right] - \frac{1}{\beta R^m}\left[1 + \frac{x}{\rho B - x}\xi\right]} \\ &= x\left[1 + \frac{1 - \frac{1-(1-\kappa)\beta R^m}{\kappa\beta R^m}(1-\xi)}{\frac{1-(1-\kappa)\beta R^m}{\kappa\beta R^m}(1-\xi) + \frac{(1-\kappa)}{\kappa} - \frac{1}{\beta R^m}\left[1 + \frac{x}{\rho B - x}\xi\right]}\right] \\ &= x\left[1 - \frac{\frac{1-(1-\kappa)\beta R^m}{\kappa\beta R^m}(1-\xi) - 1}{\frac{1-(1-\kappa)\beta R^m}{\kappa\beta R^m}(1-\xi) - 1 + \frac{1}{\kappa} - \frac{1}{\beta R^m}\left[1 + \frac{x}{\rho B - x}\xi\right]}\right] \end{aligned}$$

This is positive whenever

$$\frac{1}{\kappa} - \frac{1}{\beta R^m}\left[1 + \frac{x}{\rho B - x}\xi\right] < 0$$

so that the fraction in the expression in [.] is less than 1. Hence, we require

$$\begin{aligned} \frac{\beta R^m}{\kappa} &< 1 + \frac{R^m D}{\rho B - R^m D}\xi \\ \frac{\beta R^m}{\kappa} - 1 &< \frac{R^m D}{\rho B - R^m D}\xi \\ \left[\frac{\beta R^m}{\kappa} - 1\right]\left(\frac{\rho B}{R^m D} - 1\right) &< \xi \end{aligned}$$

Since $\rho B / R^m D > 1$ and we are in region $\mathcal{A}_m$, we know $1 \geq \beta R^m \geq \beta \max\left\{\frac{\mu-\kappa}{\beta\mu(1-\kappa)}, R^e\right\} = \beta R_2^m$ and

$$u'(\kappa R^m D) = \frac{1-(1-\kappa)\beta R^m}{\kappa\beta R^m}$$

If βR^m increases, $R^m D$ has to increase, so $R^m D \geq \frac{1}{\kappa}u'^{-1}\left(\frac{1-(1-\kappa)\beta R_2^m}{\kappa\beta R_2^m}\right)$ and $R^m D \leq \frac{1}{\kappa}u'^{-1}(1)$. Hence a sufficient condition for $d(\kappa x)/d\kappa > 0$ is

$$\xi > \left(\frac{1}{\kappa} - 1\right)\left(\frac{\kappa\rho B}{u'^{-1}\left(\frac{1-(1-\kappa)\beta R_2^m}{\kappa\beta R_2^m}\right)} - 1\right) \equiv \bar{\xi}$$

□

With log-utility ($\xi = 1$), it will be optimal to set $\kappa = 1$.

Proof. Suppose agents have log utility. We show that it is optimal to maximize κ. Note that $\frac{d}{d\kappa}R_1^m > 0$, and that $\frac{d}{d\kappa}R_2^m < 0$.

	$R^m M$	z	d
$\mathcal{A}_e$	0	$\frac{\kappa\beta R^e}{1-(1-\kappa)\beta R^e}$	ρB
$\mathcal{A}_{\tilde{m}}$	βR^m	$\beta\mu R^m$	βR^m
$\mathcal{A}_m$	$\frac{\beta R^m}{1-(1-\kappa)\beta R^m}$	$\frac{\kappa\beta R^m}{1-(1-\kappa)\beta R^m}$	βR^m

Replacing for the equilibrium values for R^m:

	$R^m M$	z	d
$\mathcal{A}_e$	0	$\frac{\kappa\beta R^e}{1-(1-\kappa)\beta R^e}$	ρB
$\mathcal{A}_{\tilde{m}}$	βR^m	$\mu\rho B/2$	$\rho B/2$
$\mathcal{A}_m$	$\frac{\beta R^m}{1-(1-\kappa)\beta R^m}$	$\frac{\kappa\beta R^m}{1-(1-\kappa)\beta R^m}$	βR^m

In the first region,

$$x_C = \frac{\kappa\beta R^e}{1-(1-\kappa)\beta R^e}$$

which is increasing in β. Hence, within this region, it is optimal to maximize κ too. In the second region, welfare is independent of κ. In the third region, traditional consumption is

$$x_T = \beta R^m = \frac{[2+\rho B(1-\kappa)] - \sqrt{4+((1-\kappa)\rho B)^2}}{2(1-\kappa)} = \frac{2+v-\sqrt{4+v^2}}{2v}\rho B$$

where $v = \rho B(1-\kappa)$. We know that x_T is increasing in κ as

$$\begin{aligned}\frac{d}{dv}\frac{2+v-\sqrt{4+(v)^2}}{v} &= \frac{v - v^2[4+(v)^2]^{-0.5} - 2 - v + \sqrt{4+(v)^2}}{v^2} \\ &= -\frac{1}{v^2}\left(\frac{v^2}{\sqrt{4+v^2}} + 2 - \sqrt{4+v^2}\right) \\ &= -\frac{1}{v^2\sqrt{4+v^2}}\left(v^2 + 2\sqrt{4+v^2} - 4 - v^2\right) \\ &= -\frac{2}{v^2\sqrt{4+v^2}}\left(\sqrt{4+v^2}-2\right) < 0.\end{aligned}$$

Hence

$$\frac{d}{d\kappa}x_T = \frac{d}{d\kappa}\beta R^m > 0.$$

Crypto consumption is

$$x_C = \frac{\kappa\beta R^m}{1-(1-\kappa)\beta R^m},$$

which is increasing in κ given R^m and increasing in R^m given κ. Hence $dx_C/d\kappa > 0$. Increasing κ reduces D and hence releases resources to the traditional sector. In this region, it is also optimal to set $\kappa = 1$. □

G. Stablecoins backed by government bonds

In the benchmark model, crypto banks cannot issue stablecoins backed by bonds. Here we present and solve for the general problem where the crypto bank can back the issuance of its stablecoins with crypto assets, tokenized assets, or government bonds.

Given μ, R^b, R^m, and R^e, a crypto bank maximizes its users' payoff by choosing the users' investment into the bank a, the quantity of tokenized money $\tilde{m}$, and stablecoins s directly held by its users, as well as the reserves of tokenized money m, bonds b, and crypto assets e that the crypto bank will hold to back its issuance of stablecoins:[11]

$$\max_{a,s,e,m,\tilde{m}} \left[-a - \tilde{m} + \beta u(s + \mu R^m \tilde{m})\right]$$

$$\begin{aligned} \text{subject to :} \qquad \underbrace{a - m - e - b + \beta \left[R^e e + R^b b + R^m m - s\right]}_{\text{net worth}} \quad &\geq \quad 0, \qquad (PC) \\ \kappa_e R^e e + \kappa_b R^b b + \kappa_m R^m m \quad &\geq \quad s \qquad (IC) \end{aligned}$$

where $\kappa_c \in (0, 1)$ denotes the pledgeability parameter of asset $c = b, e, m$ for the crypto bank. It is obvious that (PC) binds, so that the problem becomes

$$\begin{aligned} \max_{a,s,e,m,\tilde{m}} &\left[-b - e - m + \beta \left[R^b b + R^e e + R^m m - s\right] - \tilde{m} + \beta u(s + \mu R^m \tilde{m})\right] \\ &+ \beta\lambda \left[\kappa_e R^e e + \kappa_b R^b b + \kappa_m R^m m - s\right] \end{aligned}$$

where $\beta\lambda$ is the multiplier on the IC of the crypto bank. We can rewrite this problem as

$$\max_{a,s,e,m,\tilde{m}} \left[-b - e - m + \sum_{c=b,e,m} (1 + \lambda\kappa_c)\beta R^c c - \beta(1 + \lambda)s - \tilde{m} + \beta u(s + \mu R^m \tilde{m})\right].$$

Let $c = m, e, b$ denote the asset held by the crypto as collateral. If $\beta R^c < 1$, the crypto bank only holds asset $c = m, e, b$ if it relaxes IC, and it is indifferent when $\beta R^c = 1$. Also, the crypto bank holds the cheapest asset to satisfy its IC, that is, it will hold m whenever $(1 + \lambda\kappa_m)R^m > \max\left\{(1 + \lambda\kappa_e)R^e, (1 + \lambda\kappa_b)R^b\right\}$, e whenever $(1 + \lambda\kappa_e)R^e > \max\left\{(1 + \lambda\kappa_b)R^b, (1 + \lambda\kappa_m)R^m\right\}$, and b otherwise. It will be indifferent between two assets c_1, c_2 whenever $(1+\lambda\kappa_{c_1})R^{c_1} = (1+\lambda\kappa_{c_2})R^{c_2}$. With this understanding we can rewrite the problem of the crypto bank as

$$\max_{a,s,e,m,\tilde{m}} \left[-c + \beta \left[(1 + \lambda\kappa_c)R^c c - (1 + \lambda)s\right] - \tilde{m} + \beta u(s + \mu R^m \tilde{m})\right].$$

[11] This problem is equivalent to maximizing the bank's payoff subject to the user's participation constraint.

The first-order conditions are

$$\begin{aligned} s: \quad u'(s+\mu R^m \tilde{m}) &\leq 1+\lambda, \\ c: \quad (1+\lambda\kappa_c)\beta R^c &\leq 1, \\ \tilde{m}: \quad \mu\beta R^m u'(s+\mu R^m \tilde{m}) &\leq 1. \end{aligned}$$

1. First suppose there is c such that $\beta R^c = 1$. Then $\lambda = 0$, $u'(s+\mu R^m \tilde{m}) = 1$, and $\tilde{m} = 0$ if $\mu < 1$ since $\mu < 1$ implies $\mu\beta R^m u'(s+\mu R^m \tilde{m}) < 1$.

2. Next suppose $\beta R^c < 1$ for all c.

   (a) If $s = 0$ then $\tilde{m} > 0$ and it solves $\mu\beta R^m u'(\mu R^m \tilde{m}) = 1$. This is the case iff for all c,

   $$u'(\mu R^m \tilde{m}) = \frac{1}{\mu\beta R^m} = 1+\lambda$$

   $$\left(1-\kappa_c+\frac{1}{\mu\beta R^m}\kappa_c\right)\beta R^c \leq 1.$$

   Hence, stablecoins are not issued whenever

   $$\beta R^m \leq \frac{\mu-\kappa_m}{\mu(1-\kappa_m)}$$

   $$\frac{\kappa_e \beta R^e}{\mu\left[1-(1-\kappa_e)\beta R^e\right]} \leq \beta R^m$$

   and

   $$\frac{\kappa_b \beta R^b}{\mu\left[1-(1-\kappa_b)\beta R^b\right]} \leq \beta R^m.$$

   (b) If $s > 0$ then

   $$u'(s+\mu R^m \tilde{m}) = 1+\lambda$$

   and for at least one c,

   $$\begin{aligned} (1+\lambda\kappa_c)\beta R^c &= 1. \\ \lambda &= \frac{1-\beta R^c}{\beta R^c \kappa_c}. \end{aligned}$$

   Hence for the one c,

   $$u'\left(\kappa_c R^c c+\mu R^m \tilde{m}\right) = \frac{1-\beta R^c(1-\kappa_c)}{\beta R^c \kappa_c} > 1,$$

with $\tilde{m}$ given by

$$\tilde{m} = \begin{cases} 0 & \text{if } \mu\beta R^m \frac{1-\beta R^c(1-\kappa_c)}{\beta R^c \kappa_c} < 1 \\ \geq 0 & \text{if } \mu\beta R^m \frac{1-\beta R^c(1-\kappa_c)}{\beta R^c \kappa_c} = 1 \end{cases}$$

i. If the **stablecoin is backed by crypto assets**, then

$$u'\left(\kappa_e R^e e + \mu R^m \tilde{m}\right) = \frac{1-\beta R^e(1-\kappa_e)}{\beta R^e \kappa_e},$$

and

$$\begin{aligned} (1+\lambda\kappa_e)\beta R^e &= 1 > \max\left\{(1+\lambda\kappa_m)\beta R^m, (1+\lambda\kappa_b)\beta R^b\right\} \\ & 1 > \max\left\{\left(1+\frac{1-\beta R^e}{\beta R^e \kappa_e}\kappa_m\right)\beta R^m, \left(1+\frac{1-\beta R^e}{\beta R^e \kappa_e}\kappa_b\right)\beta R^b\right\}. \end{aligned}$$

Hence,

$$\begin{aligned} \beta R^m &< \frac{\beta R^e \kappa_e}{1+\beta R^e(\kappa_e-\kappa_m)} \iff \frac{\beta R^m}{\kappa_e - \beta R^m(\kappa_e-\kappa_m)} < \beta R^e \\ \beta R^b &< \frac{\beta R^e \kappa_e}{1+\beta R^e(\kappa_e-\kappa_b)} \iff \frac{\beta R^b}{\kappa_e - \beta R^b(\kappa_e-\kappa_b)} < \beta R^e \end{aligned}$$

ii. If the **stablecoin is backed by bonds**, then

$$u'\left(\kappa_b R^b b + \mu R^m \tilde{m}\right) = \frac{1-\beta R^b(1-\kappa_b)}{\beta R^b \kappa_b},$$

and, symmetrically to the condition above,

$$\begin{aligned} \beta R^m &< \frac{\beta R^b \kappa_b}{1+\beta R^b(\kappa_b-\kappa_m)}, \\ \beta R^e &< \frac{\beta R^b \kappa_b}{1+\beta R^b(\kappa_b-\kappa_e)}. \end{aligned}$$

iii. If the **stablecoin is backed by tokenized money**, then

$$u'\left(\kappa_m R^m m + \mu R^m \tilde{m}\right) = \frac{1-\beta R^m(1-\kappa_m)}{\beta R^m \kappa_m},$$

and, symmetrically to the condition above,

$$\begin{aligned} \beta R^e &< \frac{\beta R^m \kappa_m}{1+\beta R^m(\kappa_m-\kappa_e)}, \\ \beta R^b &< \frac{\beta R^m \kappa_m}{1+\beta R^m(\kappa_m-\kappa_b)}. \end{aligned}$$

Notice that if $\kappa_m = \kappa_b$ and tokenized money has a liquidity premium relative to bonds ($\beta R^m \leq \beta R^b$), then this can't be the case.