Staff Working Paper/Document de travail du personnel—2024-18

Last updated: May 27, 2024

Financial Intermediation and Fire Sales with Liquidity Risk Pricing

by Yuteng Cheng[1] and Roberto Robatto[2]

[1] Banking and Payments Department
Bank of Canada
YCheng@bankofcanada.ca

[2] University of Wisconsin-Madison
robatto@wisc.edu

DOI: https://doi.org/10.34989/swp-2024-18 | **ISSN** 1701-9397

Acknowledgements

A previous version of this paper circulated under the title "Bank Runs and Endogenous Fire Sales with Consumption-Based Asset Pricing." We thank Pablo Kurlat, William Diamond, and Briana Chang for helpful conversations and suggestions. The views expressed herein are those of the authors and not necessarily those of the Bank of Canada.

Abstract

We provide a theory of fire sales in which potential buyers are subject to liquidity shocks and frictions that limit their ability to resell assets. The model predictions align with some stylized facts about the large sales of corporate bonds and Treasury securities during the COVID-19 economic crisis. The equilibrium is constrained efficient under weak conditions that apply if one interprets the key agents in the model as money market funds or mutual funds. Thus, as viewed through the lens of the model, the liquidity requirements proposed by the U. S. Securities and Exchange Commission for these intermediaries could hurt the economy.

Topics: Asset pricing; Financial markets; Financial system regulation and policies

JEL codes: G12, G23, G28

Résumé

Nous présentons une théorie des liquidations d'actifs dans laquelle les acheteurs potentiels sont assujettis à des chocs de liquidité et à des frictions qui limitent leur capacité à revendre les actifs. Les prévisions du modèle cadrent avec certains faits stylisés liés aux ventes massives d'obligations de sociétés et de titres du Trésor américain pendant la pandémie de COVID-19. L'équilibre est efficace en présence de conditions fragiles qui s'appliquent si l'on considère comme les agents les plus importants du modèle les fonds du marché monétaire ou les fonds communs de placement. Par conséquent, d'après le modèle, les exigences en matière de liquidité que propose la Securities and Exchange Commission des États-Unis (SEC) pour ces intermédiaires pourraient nuire à l'économie.

Sujets : Évaluation des actifs; Marchés financiers; Réglementation et politiques relatives au système financier
Codes JEL : G12, G23, G28

1 Introduction

Fire sales are common phenomena in periods of financial distress. These episodes are characterized by large sales of financial assets together with a reduction in their prices, despite little to no change in the fundamentals. These events arise when investors are forced to sell their assets for a variety of reasons. Examples of assets undergoing fire sales are abundant across markets and asset classes, ranging from assets held by banks in distress (Granja, Matvos, and Seru, 2017), asset-backed securities held by insurance companies (Merrill et al., 2021) and, more recently, safe assets such as highly rated corporate bonds and long-term Treasury securities (Falato, Goldstein, and Hortaçsu, 2021; Vissing-Jorgensen, 2021; Ma, Xiao, and Zeng, 2022), which were subject to fire sales at the peak of the COVID-19 crisis. Regulators have introduced various tools to limit fire sales, such as liquidity requirements on banks and money market mutual funds (MMMFs). Recent proposals by the U.S. Securities and Exchange Commission (SEC) call for tightening the liquidity requirements on MMMFs and also imposing such requirements on mutual funds.[1]

The literature provides several theories to explain fire sales. The main theories postulate that experts that are willing to pay a high price have limited wealth with which to purchase financial assets (Allen and Gale, 1998) and non-expert investors have a lower willingness to pay (Acharya and Yorulmazer, 2008; Dow and Han, 2018). The latter might be less willing to pay because they are only able to extract a lower cash flow relative to sellers and experts (Shleifer and Vishny, 1992; Kiyotaki and Moore, 1997; Lorenzoni, 2008; Dávila and Korinek, 2018) or because they have limited information about asset quality and are concerned about the adverse-selection problems (Chang, 2018; Dow and Han, 2018; Kurlat, 2016). If the selling pressure is limited, experts purchase all of the assets at high prices. But if the selling pressure is high, prices drop to fire-sale levels to encourage non-experts to step in.

The main fire-sale theories, however, might not fit some key stylized facts of the forced sales of high-quality corporate bonds and long-term Treasury securities that occurred at the peak of the COVID-19 crisis—and the associated drop in prices. These assets only require investors to collect cash flows; thus, theories based on investors' limited ability to manage these assets do not apply. In

[1]The proposal for MMMFs is available at the U.S. Securities and Exchange Commission, 17 CFR Parts 270 and 274 [Release No. IC-34441; File No. S7-22-21], RIN 3235-AM80, "Money Market Fund Reforms," https://www.sec.gov/rules/proposed/2021/ic-34441.pdf. For mutual funds, the proposal is available at the U.S. Securities and Exchange Commission, 17 CFR Parts 270 and 274 [Release Nos. 33-11130; IC-34746; File No. S7-26-22], RIN 3235-AM98, "Open-End Fund Liquidity Risk Management Programs and Swing Pricing; Form N-PORT Reporting," https://www.sec.gov/rules/proposed/2022/33-11130.pdf.

addition, these assets are typically not associated with asymmetric information; therefore, fire-sale theories based on adverse selection are unlikely to be relevant.

Our main contribution is to provide a novel theory of fire sales that applies to a large set of assets—including, but not limited to, those that require no particular skills for collecting cash flows and that are not subject to adverse-selection problems. Our theory combines resale frictions with endogenous variations in buyers' exposure to liquidity risk, which generate lower prices for long-term illiquid assets in response to increases in market-wide selling pressure.

In our framework, potential buyers are subject to future liquidity shocks and frictions that limit their ability to resell assets in the future. These frictions affect the price that potential buyers are willing to pay today to purchase a long-term asset. While it is well understood that frictions affecting the ability to resell an asset in the future can affect its price today (Amihud and Mendelson, 1986), our model has an important novelty. That is, the degree to which future reselling frictions can affect current asset prices is endogenous and varies with the amount of long-term assets that are sold. In normal times, when there are limited sales of long-term assets, buyers have little or no concern about future resale frictions and, thus, are willing to pay a high price. However, in crisis times—when selling pressure is higher—buyers are very concerned about future resale frictions, so they are only willing to pay a lower fire-sale price to purchase long-term securities. But why do resale frictions become (more) relevant in crisis times? In our model, investors have access to two assets—a short-term liquid asset and a long-term illiquid one—which are both in limited supply. During crises, sellers dump the long-term illiquid asset on the market and purchase the short-term liquid one, thereby draining liquidity from the market. Buyers are thus left with lower liquidity and when they are hit by a liquidity shock, they prefer to sell some of their holdings of the long-term security. But they might not be able to do so because of the reselling frictions. Anticipating this possibility, they are willing to pay a low fire-sale price when purchasing the long-term asset in the first place. Importantly, the fire-sale price is lower for the asset that is plagued by more resale frictions—a cross-sectional prediction that we use to validate our model. We label the channel that gives rise to a fire-sale price *liquidity risk pricing* to highlight the role played by the combination of two elements, namely, the concern about the future liquidity of the long-term asset and the risk that affects the buyers and subjects them to liquidity shocks and resale frictions.

A key element of our paper is our analysis of the efficiency properties of the equilibrium and the role of policy. We build on the approach used in the fire-sale literature (Dávila and Korinek, 2018; Kurlat, 2021) and we find that the equilibrium is generically inefficient because of the externalities that arise from market incompleteness. However, different from most of the fire-sale literature, we identify weak conditions under which the equilibrium is constrained efficient and, thus, under which policy interventions do not produce Pareto improvements. We argue that these conditions apply if the key agents in the model are interpreted as MMMFs or mutual funds—which were involved in fire sales at the peak of the COVID-19 crisis, as noted before. Hence, when viewed through the lens of our model, the SEC's proposed liquidity requirements for MMMFs and mutual funds would not produce welfare improvements and, if implemented, could harm the economy. Our policy implications complement those derived by other papers that highlight the unintended consequences of liquidity requirements, such as Malherbe (2014) .

The main logic behind the welfare results is that the typical elements that give rise to externalities in other fire-sale models are absent here. Specifically, our model does not include collateral constraints or sources of asymmetric information and all investors have the same ability to collect cash flows. In this sense, the model applies naturally to the analysis of MMMFs and mutual funds focused on high-quality assets such as highly rated corporate bonds or Treasury securities.

Our model is consistent with some stylized facts of fire sales observed in practice and we provide a brief comparison with the events that took place during the peak of the COVID-19 crisis. In March 2020, mutual funds and other investors sold a large amount of high-quality corporate bonds and long-term Treasury securities, resulting in large increases in yields unrelated to changes in the fundamentals (Falato, Goldstein, and Hortaçsu, 2021; Vissing-Jorgensen, 2021). For corporate bonds, most of the sales were absorbed by final customers—as dealers did not increase their balance sheet holdings—and the evidence in O'Hara and Zhou (2021) points to a strong reduction in buyers' willingness to pay, which is consistent with our model. For Treasury securities, dealers absorbed a large part of the sales and played a more important role (He, Nagel, and Song, 2022), but some stylized facts such as the increase in the on-the-run premium are consistent with the cross-sectional predictions of our model and hard to reconcile with explanations based solely on the role played by dealers.[2]

The baseline analysis uses a simple framework to convey the main elements and results of our fire-sale theory. The resell friction is extreme—no reselling is possible—and sellers can be interpreted as banks or MMMFs or mutual funds that are subject to withdrawal shocks that might force them to sell long-term assets to pay for such withdrawals. We then present alternative ap-

[2]The on-the-run premium is the spread between the yields on off-the-run and on-the-run Treasury securities. On-the-run securities are the most recently auctioned ones of any given tenor and off-the-run ones are all the other types of securities.

plications in which we combine the liquidity risk pricing with other channels that generate selling pressure. In particular, we show that selling pressure can arise from a tightening of the collateral constraint—such as in the case of insurance companies' fire sales of mortgage-backed securities in 2008 (Merrill et al., 2021)—or from bank runs in which depositors' withdrawals force banks to sell their long-term investments—such as the runs on the MMMFs that took place in 2008 and during the COVID-19 crisis (Schmidt, Timmermann, and Wermers, 2016; Li et al., 2021).

At a more general level, our theory of fire sales uses a pricing mechanism that is in line with recent advances in the asset pricing literature. While asset prices are in general equal to expected discounted cash flows, fire-sale prices in existing fire-sale theories are often the result of only the expectation part of the sale, with little or no role played by fluctuations in discount rates (i.e., investors' marginal utilities). Indeed, many of these theories can be derived in frameworks in which marginal investors have linear utility (Allen and Gale, 1998; Kurlat, 2021). Yet, this approach is in stark contrast with modern asset pricing, which emphasizes the importance of fluctuations in discount rates to explain movements in asset prices (Cochrane, 2011). In our model, combinations of liquidity shocks, resale frictions, and increased selling pressure in crisis times give rise to fluctuations in discount rates resulting in lower asset prices during fire sales despite there being no changes in the fundamentals.

1.1 Additional comparison with the literature

Classic theories of fire sales rely on second-best use (Shleifer and Vishny, 1992), limited cash in the market (Allen and Gale, 1998), and collateral constraints (Kiyotaki and Moore, 1997). More-recent approaches build on these insights, for instance, by providing a deeper understanding of why cash in the market might be limited (Acharya and Yorulmazer, 2008; Kurlat, 2016; Dow and Han, 2018). In a sense, our contribution is similar as buyers' exposure to liquidity and resale risk reduces their demand. Besides important economic insights about fire sales, our microfoundation implies different efficiency properties of the equilibrium (in comparison to, say, Allen and Gale, 2004; Dávila and Korinek, 2018; and Kurlat, 2021), leading to different policy prescriptions.[3]

There is a large empirical literature that tries to identify fire sales in various financial markets—in addition to the papers that study the fire sales of Treasury securities, discussed in the intro-

[3]There is also a literature that studies the efficiency of equilibria when firms are subject to frictions that give rise to externalities similar to those that arise in fire-sale models; see, for instance, He and Kondor (2016) and Lanteri and Rampini (2023).

duction. Coval and Stafford (2007) and Jotikasthira, Lundblad, and Ramadorai (2012) document equity securities fire sales that are driven by mutual funds outflows. Merrill et al. (2021) analyze forced sales of mortgage-backed securities by insurance companies. Other papers such as Ellul, Jotikasthira, and Lundblad (2011), Falato et al. (2021), Falato, Goldstein, and Hortaçsu (2021), and Manconi, Massa, and Yasuda (2012) provide evidence of fire sales in corporate bond markets both in normal times as well as during the 2007-08 financial crisis and the COVID-19 crisis, and Li and Schürhoff (2019) provide evidence of fire sales in municipal bond markets. Ambrose, Cai, and Helwege (2012) and Choi et al. (2020) challenge the evidence about corporate bond fire sales, but their analysis does not include the COVID-19 period.[4] The corporate bond fire sales that took place during the COVID-19 pandemic were unprecendented in size and, to our knowledge, there is no disagreement about the events of this period.

Finally, our work is also related to a large banking literature in which fire sales are combined with or produce financial fragility. This literature typically uses models in which fire sales are exogenous or arise from cash-in-the-market or second-best-use considerations (Caballero and Simsek, 2013; Diamond and Dybvig, 1983; Gertler and Kiyotaki, 2015; Goldstein et al., 2022). In this literature, Gale and Yorulmazer (2013) and Robatto (2019) consider agents' exposure to liquidity risk that generates a precautionary demand for liquid assets, similar to our framework. Both papers use fairly rich banking settings that give rise to inefficiencies and focus on central bank interventions. We instead show that the essential elements that give rise to fire sales under liquidity risk pricing do not necessarily generate inefficiencies, leading to very different welfare implications and policy prescriptions.

2 Baseline model

Consider an economy populated by two sets of investors, which we refer to as the buyers (b) and the sellers (s). The economy lasts three periods, $t = 0, 1, 2$. Time $t = 0$ denotes when a fire sale can happen—taking as given investors' initial time-0 portfolios. Time $t = 1$ is when resale frictions and liquidity shocks might affect the buyers. And time $t = 2$ is when any remaining payoff is realized. In Section 3, when conducting policy analysis, we will add a period $t = -1$ to study

[4] Choi et al. (2020) also challenge the SEC proposal to impose regulation on bond mutual funds, based on their result that fire sales are limited or absent. We also challenge the same policy proposals but based on a different logic—even if outflows from mutual funds generate fire sales, our theory suggests that policy interventions in the form of liquidity regulation do not produce Pareto improvements.

ex ante investment decisions, the efficiency properties of the equilibrium, and possible regulatory interventions.

2.1 Preferences, technology, and endowments

The sellers have linear utility from consumption at $t = 2$, c_2^s. The buyers consume at $t = 2$, but they might also derive utility from consumption at $t = 1$, depending on the realization of a preference shock:

$$U^b\left(c_1^b, c_2^b\right) = \begin{cases} u\left(c_1^b\right) + c_2^b & \text{w/prob } \theta \\ c_2^b & \text{w/prob } 1 - \theta \end{cases}$$

with $u(c) = \log c$. The preference shocks are in the spirit of Diamond and Dybvig (1983) and give rise to the need to finance some consumption expenditures at $t = 1$.

There are two assets: a short-term (liquid) asset, which can be interpreted as a storage technology, and a long-term asset. The liquid asset is standard—for each unit invested at time t, there is one unit available at $t + 1$. The long-term asset has a payoff R at $t = 2$. We assume $R > 1$. This assumption is important for the policy analysis of Section 3 but it does not affect the results of this section.

In terms of endowments, sellers' begin $t = 0$ with an amount k_{-1}^s of the long-term asset and we normalize their initial holdings of the liquid assets to zero. Sellers also have liabilities d_{-1}^s, which we assume are toward external agents, with d_{-1}^s representing the face value. As the notation suggests, one can think of these endowments as being determined in a period, $t = -1$, which is unmodeled at this point but which will be analyzed later in Section 3. The buyers' endowments are given by the amounts l_{-1}^b and k_{-1}^b of the liquid and long-term assets, respectively.

We note that at $t = 0$, the buyers and sellers are able to adjust their portfolio holdings of the liquid and long-term assets (see the next section, which describes markets). However, at the economy-wide level, it is not possible to change the overall supply of the two assets at $t = 0$—these are given by l_{-1}^b and $k_{-1}^s + k_{-1}^b$, respectively. The choices that lead to the economy-wide supply of these two assets will be analyzed in Section 3, when studying the ex ante investment decisions of investors.

2.2 Markets

At $t = 0$, there is a centralized market in which investors can trade the liquid and long-term assets. We denote q_0 as the price of the long-term asset and normalize the price of the liquid asset to one. We assume that short selling is not allowed, although the analysis can be extended without altering the logic of the results. For instance, we can allow for short selling subject to some costs or limits, or some primitives of the model such as preferences can be generalized in a way that agents choose endogenously to take only long positions.

At $t = 1$, there are frictions that limit investors' ability to trade. In particular, we consider the extreme scenario in which no trade is possible at $t = 1$ (as in, for instance, Caballero and Simsek, 2013). The result can be extended to allow for trading in markets with frictions, such as over-the-counter (OTC) markets or limited market participation.

2.3 Aggregate shock at $t = 0$

At $t = 0$, sellers must repay a fraction, γ, of their debt, d^s_{-1}, while the remaining fraction, $1 - \gamma$, is due at $t = 1$. We assume γ is an aggregate shock that can take the value $\gamma \in \{0, \overline{\gamma}\}$ and is realized at the beginning of $t = 0$, that is, before the time-0 market opens. The realization of γ is common knowledge. In the state in which $\gamma = \overline{\gamma}$, the equilibrium will display sales of the long-term assets by sellers at a fire-sale price.

We interpret sellers as banks or MMMFs or mutual funds that experience withdrawals or outflows, with $\gamma = 0$ representing a low-withdrawal state and $\gamma = \overline{\gamma}$ a high-withdrawal state. Two remarks are in order. First, in Appendix A, we reformulate the model so that the withdrawals are paid at their net asset values (NAV), in line with the possibility of interpreting the sellers as mutual funds or certain MMMFs. Second, one possible interpretation of a high realization of γ is that of a run on the sellers' liabilities, d^s_{-1}, when such liabilities are held by external agents. Section 4.2 also studies runs on the sellers but under the assumption that their liabilities are held by buyers.

2.4 Buyers' choices

We now analyze the buyers' problem. They choose their holdings of liquid and long-term assets at $t = 0$ as well as their consumption at $t = 1$ if they are hit by the following preference shock:

$$\max_{l_0^b \geq 0,\, k_0^b \geq 0, c_1^b \geq 0} (1-\theta)\left[\underbrace{l_0^b + Rk_0^b}_{=c_2^b}\right] + \theta\left[u\left(c_1^b\right) + \underbrace{l_0^b - c_1^b + Rk_0^b}_{=c_2^b}\right] \tag{1}$$

subject to the time-0 budget constraint (2) and the time-1 liquidity constraint (3),

$$l_0^b + q_0 k_0^b \leq l_{-1}^b + q_0 k_{-1}^b, \tag{2}$$

$$c_1^b \leq l_0^b. \tag{3}$$

The liquidity constraint (3) implies that investors' consumption can only be financed using the liquid asset.

The first-order condition with respect to the time-1 consumption choice, c_1, implies

$$u'\left(c_1^b\right) \geq 1. \tag{4}$$

That is, absent the liquidity constraint (3), the agent would like to consume up to the point at which the time-1 marginal utility is equal to the time-2 marginal utility—given that these resources can be stored with a gross return of one. However, if the liquidity constraint is binding, the time-1 consumption will be lower, resulting in a time-1 marginal utility greater than one.

The time-0 decisions imply the standard asset pricing condition

$$q_0 = \frac{1}{(1-\theta)\times 1 + \theta\, u'\left(c_1^b\right)} \times R, \tag{5}$$

where the term $\frac{1}{(1-\theta)\times 1+\theta\, u'\left(c_1^b\right)}$ is the ratio of the marginal utility at $t = 2$ and the (average) marginal utility at $t = 1$. That is, the marginal utility at $t = 2$ is equal to one and the marginal utility at $t = 1$ is $(1-\theta) \times 1 + \theta\, u'\left(c_1^b\right)$.

The first-order conditions (4) and (5) imply

$$q_0 \leq R. \tag{6}$$

To preview some of the results, we note that if the buyers enter $t = 1$ with sufficiently large holdings of liquidity, their liquidity constraint (3) will not be binding, even if they are hit by a preference shock. In other words, their liquidity holdings will provide full insurance against preference shocks and the preference shocks will be irrelevant—on the margin—to determine the buyers' asset demand at $t = 0$. As a result, at $t = 0$, the buyers are willing to pay a price $q_0 = R$ to purchase long-term assets, that is, a price that depends solely on the cash flow such assets produce at $t = 2$.

If instead the buyers enter $t = 1$ with small holdings of liquidity, their liquidity constraint (3) will be binding if they are hit by a preference shock. This will make their time-1 marginal utility greater than one, that is, $u'\left(c_1^b\right) > 1$. As a result, when the buyers make their portfolio decisions at $t = 0$, they will value their liquid assets disproportionately more, as such assets relax the liquidity constraint (3). This will increase the buyers' demand for liquid assets and reduce their demand for long-term assets, resulting in a price $q_0 < R$ for long-term assets. That is, with only partial insurance against preference shocks, the buyers are willing to pay a lower price for the long-term asset because such an asset is illiquid at $t = 1$.

2.5 Sellers' choices

The sellers' problem is rather mechanical. They face exogenous withdrawals γd_{-1}^s, with $\gamma \in \{0, \overline{\gamma}\}$, and because they consume only at $t = 2$, they invest all of their resources in the long-term asset.

Formally, the sellers choose their non-negative holdings of liquid and long-term assets at $t = 0$, $l_0^s \geq 0$ and $k_0^s \geq 0$, to maximize consumption, c_2^s, subject to the time-0 budget constraint

$$l_0^s + q_0 k_0^s \leq q_0 k_{-1}^s - \gamma d_{-1}^s,$$

and where consumption, c_2^s, is given by the payoff of the time-0 investments net of the repayments, $(1 - \gamma)\, d_{-1}^s$, that are due to debt holders:

$$c_2^s = R k_0^s + l_0^s - (1 - \gamma)\, d_{-1}^s.$$

We restrict our attention to the relevant equilibrium cases in which $q_0 \leq R$. When $q_0 < R$, the long-term asset has a higher return than the liquid asset and, thus, the sellers invest all of their wealth in the long-term asset. That is, $l_0^s = 0$, and k_0^s is residually determined by the budget constraint:

$$k_0^s = \frac{q_0 k_{-1}^s - \gamma d_{-1}^s}{q_0}.$$

If $q_0 = R$, the liquid and long-term assets have the same returns, so the sellers are indifferent between the two. We show in the next section that the equilibrium is characterized by the sellers investing all of their wealth in the long-term asset, similar to the case in which $q_0 < R$.

2.6 Parameter restrictions

There is only one key parameter restriction that is necessary to derive the results, that is, that the buyers are endowed with enough liquidity l_{-1}^b at the beginning of $t = 0$ to pay for the withdrawals, $\overline{\gamma} d_{-1}^s$, in a high-withdrawal state. Formally, $l_{-1}^b > \bar{\gamma} d_{-1}^s$. To simplify the exposition, however, we include two additional normalizations. First, we assume that liquidity is at an intermediate level, that is,

$$1 \leq l_{-1}^b \leq 1 + \bar{\gamma} d_{-1}^s. \tag{7}$$

This implies that the fire-sale price, $q_0 < R$, arises only in the state with large withdrawals, $\gamma = \bar{\gamma}$. Relaxing this assumption would generate a price, $q_0 < R$, in both states, even though the price would be lower in the high-withdrawal state—in line with the results we derive. Second, we assume that the high-withdrawal shock is not too large:

$$\bar{\gamma} < \frac{R k_{-1}^s l_{-1}^b + d_{-1}^s l_{-1}^b (1-\theta) + d_{-1}^s \theta - l_{-1}^s \left[l_{-1}^b (1-\theta) + \theta \right]}{d_{-1}^s \left[d_{-1}^s (1-\theta) - l_{-1}^b - R k_{-1}^s \right]}. \tag{8}$$

This assumption guarantees that the sellers are always solvent in equilibrium. The model can be extended to allow for the sellers to default, but this would complicate the exposition without changing the nature of the results.

2.7 Equilibrium

The equilibrium definition is standard. Given a realization of the shock $\gamma \in \{0, \overline{\gamma}\}$, an equilibrium is a collection of the buyers' and sellers' portfolio choices at $t = 0$, the buyers' consumption choices at $t = 1$, buyers' and the sellers' consumption choices at $t = 2$, and a time-0 price q_0 for the long-term asset, such that the buyers and sellers maximize their utility and the time-0 market clears. Specifically, the market-clearing condition for liquidity at $t = 0$ is

$$l_0^b + \gamma d_{-1}^s = l_{-1}^b. \tag{9}$$

That is, the liquid asset available in the economy, l_{-1}^b, is allocated between the buyers' holdings l_0^b and the resources γd_{-1}^s that are used to repay the sellers' debt holders. (Recall from Section 2.1 that the sellers' initial endowment of liquidity is normalized to zero, so that all of the liquidity available at $t = 0$ is supplied by the buyers through their endowment, l_{-1}^b.) The other market-clearing condition—for the long-term asset—holds by Walras's law, but we also state it for completeness:

$$k_0^b + k_0^s = k_{-1}^b + k_{-1}^s. \tag{10}$$

We first solve for equilibrium, given the realization of the low-withdrawal state $\gamma = 0$. In this case, the sellers do not need to sell any debt and market clearing implies that the buyers hold all of the liquidity at the end of $t = 0$. Given the normalization imposed in Equation (7), the liquidity is high enough so that the buyers can fully self-insure against time-1 preference shocks. In a sense, time-1 trading frictions—even if very extreme—are not relevant because of the full insurance against the liquidity shock. As a result, no fire sales arise in this case, as summarized by the next proposition. All of the proofs are provided in Appendix B.

Proposition 2.1. ***(Equilibrium with low selling pressure: no fire sales.)*** *Given $\gamma = 0$, there exists an equilibrium in which the time-0 price of the long-term asset is $q_0 = R$, the buyers and sellers engage in no trades at $t = 0$ (i.e., $l_0^b = l_{-1}^b$ and $k_0^b = k_{-1}^b$ for the buyers, and $l_0^s = 0$, $k_0^s = k_{-1}^s$ for the sellers), and the time-1 consumption of the buyers that are hit by a preference shock is $c_1^b = 1$.*

Next, we solve for the equilibrium given the realization of the high-withdrawal shock, $\gamma = \bar{\gamma}$. To pay for the withdrawals, $\overline{\gamma} d_{-1}^s$, sellers sell some of their holdings of the long-term asset and drain some liquidity from the market. As a result, the buyers' holdings of the liquid asset are lower than in the scenario with $\gamma = 0$ and they are not able to fully self-insure against the preference shocks. This lack of liquidity introduces a wedge between the marginal utilities that affect the asset prices and, in particular, the time-0 price of the long-term asset. In other words, the time-1 trading frictions are now relevant for the time-0 decisions. At $t = 0$, the buyers fear that they might be hit by a time-1 preference shock and face a trading friction that would limit their ability to sell the long-term asset at $t = 1$. As a result, a fire sale arises; that is, the assets sold by the sellers to pay for their withdrawals are traded at a price that is lower than the expected payoff, R, as formalized by the next proposition.

Proposition 2.2. *(**Equilibrium with high selling pressure: fire sales.**) Given $\gamma = \overline{\gamma}$, there exists an equilibrium with fire sales; that is, the time-0 price of the long-term asset is*

$$q_0 = R \frac{\left(l_{-1}^b - \bar{\gamma} d_{-1}^s\right)}{\left(l_{-1}^b - \bar{\gamma} d_{-1}^s\right)(1-\theta) + \theta} < R,$$

the sellers sell part of their initial holdings of the long-term asset and hold no liquidity at $t = 0$ (i.e., $k_0^s < k_{-1}^s$ and $l_0^s = 0$), the buyers reduce their liquidity holdings and increase their long-term asset holdings at $t = 0$ (i.e, $l_0^b < 1$ and $k_0^b > k_{-1}^b$), and the time-1 consumption of the buyers that are hit by a liquidity shock is $c_1^b < 1$.

Before turning to the analysis of the ex ante investments at $t = -1$ and of possible policy interventions in Section 3, the next section derives additional predictions of the model and compares them with the fire sales observed during the COVID-19 crisis.

2.8 (Il)liquid long-term assets: cross-sectional model predictions and comparison with COVID-19 fire sales

We now provide some cross-sectional predictions that can be used to validate our model and we compare them with some empirical evidence. Specifically, we characterize the spread between two otherwise equivalent long-term assets that differ in their time-1 resale frictions. We show that this spread increases in times of fire sales and that this prediction is consistent with the fire sales of Treasury securities that took place during the COVID-19 crisis.

We keep working in the context of our simple model in which the long-term asset k is subject to an extreme reselling friction and, thus, cannot be traded at $t = 1$. We consider another long-term asset, which we denote by h, that has the same payoff R as the long-term asset k but can be traded at $t = 1$. The new long-term asset h is thus subject to fewer resale frictions at $t = 1$ and, in keeping with our approach of using stark assumptions to easily convey the results, we assume that time-1 trades of asset h are not subject to any frictions at all. We price asset h assuming that it is in zero net supply and, similar to the liquid asset, that short selling is not possible. We then obtain the spread between the yields of assets k and h and we analyze how such a spread can vary between normal times and times when there are fire sales.

Let h_0^b be the buyer's holdings of the liquid long-term asset and p_0 be the price at $t = 0$. The buyer's budget constraint, Equation (2), becomes

$$l_0^b + q_0 k_0^b + p_0 h_0^b \leq l_{-1}^b + q_0 k_{-1}^b. \tag{11}$$

Because the asset can be traded at $t = 1$, the buyer's time-1 constraint, Equation (3), becomes

$$c_1^b + l_1^b + p_1 h_1^b \leq l_0^b + p_1 h_0^b, \tag{12}$$

where l_1^b and h_1^b are the agent's holdings of the short- and long-term liquid assets after trading at $t = 1$ and p_1 is the time-1 price of the long-term liquid asset. Note that asset k is still assumed to be illiquid at $t = 1$ (i.e., it cannot be traded) and thus does not enter in Equation (12). The sellers' budget constraints can be amended similarly.

To price h, we note that such an asset is essentially a perfect substitute of the (short-term) liquid asset l. Hence, a no-arbitrage argument implies that its price depends only on the payoff R and is not affected by illiquidity considerations in fire-sale states. That is, $p_0 = p_1 = R$ both in the state with no fire sales (i.e., when $\gamma = 0$) and in the state with fire sales (i.e., when $\gamma = \bar{\gamma}$). The result is formalized by the next proposition.

Proposition 2.3. *Introducing the long-term liquid asset h in zero net supply does not change the equilibrium prices nor the allocations derived in Propositions 2.1 and 2.2. The price of this asset is $p_0 = R$ for both realizations of $\gamma \in \{0, \bar{\gamma}\}$ and all agents have zero holdings of such assets at all times.*

Next, we analyze the spread between the yields of the two long-term assets, given by ${}^1/_{q_0} - {}^1/_{p_0}$. This spread is zero when there are no fire sales and increases to strictly positive values in fire-sale times. The "zero" result follows from the stark assumptions we used to simplify the exposition. The analysis can be easily extended to produce a small positive spread, even in cases with no fire sales, by adding some small time-1 trading friction for the long-term liquid asset, and the result that the spread increases in fire-sales times would be unchanged.

The model predictions are consistent with some key empirical evidence about fire sales. First, Falato et al. (2021) show stronger fire-sale effects in less-liquid times. Second, the predictions align with some key stylized facts about the fire sales of corporate bonds and Treasury securities that took place at the peak of the COVID-19 crisis, which we discuss in detail below. Because the long-term assets in our model are not subject to adverse selection and do not require special skills to extract their cash flows, it seems natural to compare them with (high-quality) corporate bonds and Treasury securities.

We focus on the events of the first part of March 2020, that is, until the announcement of the main Federal Reserve interventions on March 23, 2020. Large sales of corporate bonds were associated with unprecedented outflows from corporate bond funds, exceeding $200 billion (Falato, Goldstein, and Hortaçsu, 2021; O'Hara and Zhou, 2021). Sales of Treasury securities by mutual funds and several other financial players were also unprecedented, exceeding $800 billion (Vissing-Jorgensen, 2021).

In the case of corporate bonds, O'Hara and Zhou (2021) and Kargar et al. (2021) document an increase in customer-to-customer trades, that is, those not involving dealers. Indeed, dealers played no major role in absorbing these sales, as their inventories remained essentially unchanged or even decreased a little.[5] Falato, Goldstein, and Hortaçsu (2021) provide evidence of fire sales in the corporate bond market. O'Hara and Zhou (2021) document that customer-to-customer sales (i.e., those not intermediated by dealers) took place at a substantial premium relative to intra-dealer trades, and this premium was much higher in comparison to the pre-COVID events. These facts are consistent with buyers stepping in and absorbing the increase in sales but with a lower willingness to pay than in normal times. As viewed through the lens of our model, this lower willingness to pay is justified by the buyers' exposure to the resell risk that is driven by the reduction in the liquidity available in the market.

In the case of Treasury securities, the yield on 10-year notes increased by about 60 basis points in the first part of March 2020. As discussed by He, Nagel, and Song (2022) and Vissing-Jorgensen (2021), concerns about default risk or an increase in inflation do not explain this increase, suggesting that forced sales and the market's inability to easily absorb them were responsible for the spike in yields. Different from corporate bonds, however, dealers played a more relevant role. Duffie (2020) and He, Nagel, and Song (2022) argue that a large part of Treasury sales were absorbed by dealers either directly or through an expansion of their repo financing to levered investors. Nonetheless, some facts are hard to explain by appealing solely to the role of dealers and are instead consistent with our model. In particular, the evidence points to larger distress in segments of the markets that were characterized by lower liquidity, such as longer-term and off-the-run securities

[5] Kargar et al. (2021) document an increase in so-called agency trades, in which dealers help buyers and sellers to find counterparties but do not take a position on the asset. The point we want to emphasize is that dealers did not absorb the increased supply of corporate bonds on the market.

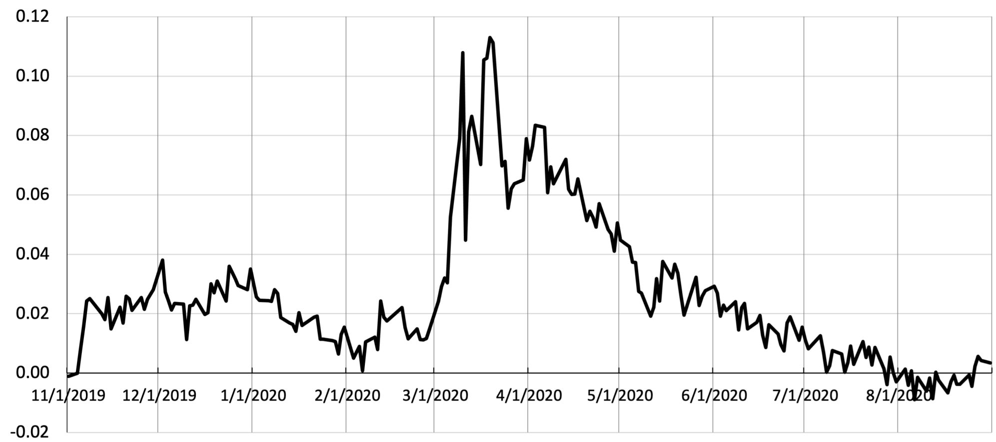

Figure 1: Evolution of the on-the-run spread

Spread between off- and on-the-run constant maturity 10-year Treasury yields (in percentages). For on-the-run yields, we use data from the par yield curve constructed by the Treasury department and based on the most recently auctioned securities. For off-the-run yields, we consider the corresponding par yield curve constructed by Gürkaynak, Sack, and Wright (2007), which is built using Treasury yield data other than on- and first off-the-run issues.

(He, Nagel, and Song, 2022; Fleming and Ruela, 2020; Ma, Xiao, and Zeng, 2022).[6]

Motivated by this evidence, Figure 1 plots the evolution of the on-the-run premium (i.e., the spread between yields on off- and on-the-run securities), focusing on 10-year notes.[7] The on-the-run premium is a well-known feature of the Treasury market (Krishnamurthy, 2002; Pancost, 2021), and on-the-run securities typically trade at lower yields (i.e., higher prices) than otherwise equivalent off-the-run securities. The lower yield of on-the-run securities is often explained by higher liquidity and the ability to obtain repo financing at better rates (Duffie, 1996; Vayanos and Weill, 2008). In the first half of March 2020, the on-the-run spread increased by about 10 basis points, as shown in Figure 1.

The increase in the on-the-run premium in March 2020 is consistent with the cross-sectional predictions of the model. In the context of the model, the on-the-run premium corresponds to the spread between long-term liquid and illiquid securities, which increases in times of fire sales, as discussed before. That is, yields on long-term off-the-run securities increased more than those of

[6]For any given maturity, on-the-run securities were the most recently auctioned, while all others were off-the-run.

[7]We construct the on-the-run premium by considering two otherwise equivalent on- and off-the-run securities. For on-the-run yields, we use data from the par yield curve constructed by the Treasury Department and based on the most recently auctioned securities. For off-the-run yields, we consider the corresponding par yield curve constructed by Gürkaynak, Sack, and Wright (2007), which is built using Treasury yield data other than on-the-run and first off-the-run issues. For details about the methodology used by the Treasury department, see https://home.treasury.gov/policy-issues/financing-the-government/interest-rate-statistics/treasury-yield-curve-methodology.

on-the-run securities, given the different reselling frictions at $t = 1$. Two observations reinforce the link between the model and the empirical evidence. First, explanations of the March 2020 Treasury selloffs based solely on dealers' balance sheet constraints (He, Nagel, and Song, 2022) cannot explain why prices dropped more for off-the-run securities.[8] Second, D'Amico and Pancost (2022) document a compression of the spread between repo rates with off-the-run versus on-the-run collateral. Absent other forces, this spread compression should have reduced the on-the-run premium as the gap in repo rates using on- versus off-the-run collateral became less important.[9]

3 Ex ante investments, efficiency, and policy

We now extend the model to include period $t = -1$, in which the buyers and sellers decide how to allocate their endowments between liquid and long-term assets. The main objective is to ask whether the choices the buyers and sellers make at $t = -1$ are efficient and whether regulatory interventions can improve the equilibrium outcome. Following Lorenzoni (2008), Dávila and Korinek (2018), Kurlat (2021), and the broad fire-sale literature, we focus on a regulator that can affect investors' portfolio choices at $t = -1$ but that takes as given asset trading and other decisions that take place in subsequent periods. In our model, the regulator takes as given trading and choices at $t = 0, 1, 2$. In this sense, the policy analysis is well-suited to study the SEC's proposal about the liquidity requirements on MMMFs and mutual funds. Our analysis is thus complementary to that of a large literature that focuses on ex post interventions, such as central bank liquidity injections during crisis times (Bianchi and Bigio, 2022; Gale and Yorulmazer, 2013; Robatto, 2023).

Our results differ substantially from those derived in other fire-sale models, in which equilibrium is typically inefficient and regulatory interventions are welfare improving. In our framework, the choices made by investors at time $t = -1$ in the decentralized equilibrium are the same as those made by the planner; that is, the equilibrium is constrained efficient under some conditions that we

[8]He, Nagel, and Song (2022) note that investors' exposure to liquidity shocks can interact with the limited balance sheet capacity of the dealers at the center of their model, but these authors do not formalize such considerations in their analysis.

[9]As noted by D'Amico and Pancost (2022), analyzing the dynamic of the repo spread is complicated by the seasonality induced by the auction cycle in the primary market: the spread spikes just before an auction and then drops dramatically. In March 2020, the seasonal dynamic of the spread followed this standard pattern around the 10-year Treasury note auction announced on March 5 and concluded on March 11. Yet the off-the-run premium remained elevated after this date, despite the subsequent compression of the repo spread to pre-auction levels. In fact, the highest value of the on-the-run premium in Figure 1 occurred on March 19, after the auction was concluded. We thank Aaron Pancost for his extensive discussions and clarifications about this point.

argue are likely to hold in practice if one maps the sellers to mutual funds or MMMFs. As a result, no regulatory intervention is required; that is, the model predicts that the liquidity requirements proposed by the SEC are welfare reducing for at least some agents.

Before formalizing the results in the next section, we briefly discuss what drives the difference in comparison to the other fire-sale models in the literature. In particular, we discuss the differences in comparison to models of fire sales with collateral constraints and models of fire sales with asymmetric information.

In models with collateral constraints, such as Lorenzoni (2008), price-dependent constraints give rise to what Dávila and Korinek (2018) refer to as collateral externalities. In contrast, the investors in our model are not subject to collateral constraints and, thus, these types of externalities do not arise here. There are of course situations, in practice, in which collateral constraints are relevant for fire sales. But for MMMFs and mutual funds—which correspond to the sellers in our model—these constraints do not seem relevant. We note, however, that even if our baseline model does not include collateral constraints, our liquidity risk pricing still applies if we extend the model to include such constraints, as we do in Section 4.

In fire-sale models with asymmetric information, such as Kurlat (2021), there might be low investment in long-term assets because of concerns of selling these assets to uninformed buyers that are willing to pay a low price due to adverse selection. Our model does not have information frictions, so this issue does not arise either. Along the lines of what we noted before, in practice there are situations wherein fire sales involve assets for which buyers might have limited information about future cash flows. But this is arguably not the case for highly rated corporate bonds and Treasury securities. These assets represent an important fraction of the MMMFs and mutual fund sector that the SEC wants to subject to liquidity regulation.

We now turn to our formal analysis and, after presenting the results, we discuss additional details and the intuition behind our results.

3.1 Extended model and choices at time $t = -1$

We extend the model by adding period $t = -1$ in which the buyers and sellers endogenously choose their portfolio holdings of liquidity and long-term assets. This section characterizes the buyers and sellers' choices in an unregulated equilibrium, and Section 3.2 analyzes the problem of a planner (or regulator). The main result, presented in Section 3.2, is that the unregulated equilibrium is constrained efficient, under conditions that we argue hold in practice.

Because our main result is that the unregulated equilibrium is efficient and no regulation is needed, we consider an extended version of the model presented in Section 2. The objective is to show that our message is not just the by-product of the simple structure employed in Section 2.

We extend the model along three dimensions: (i) we consider general buyers and sellers' utilities $U^b(c_0, c_1, c_2; \varepsilon)$ and $U^s(c_0, c_1, c_2)$, where ε parameterizes buyers' preference shocks; (ii) we allow a richer market structure at $t = 1$, although we maintain the assumption that there are some trading frictions that make resale difficult; and (iii) we allow for a general distribution of the withdrawal shock γ, captured by the cumulative density function (CDF) $F(\gamma)$. In principle, we could add additional sources of shocks, such as shocks to the return of the long-term asset, but they would not alter the results and, thus, we omit them. Under these assumptions, the outcomes at $t = 0, 1, 2$ are qualitatively identical to those of the baseline model and we provide them in Appendix C.[10] Here, we focus on the choices of investors at $t = -1$ and, in the next section, we focus on studying whether the planner wants to impose regulation at $t = -1$.

At $t = -1$, buyers and sellers have endowments e^b and e^s, respectively, and allocate them to liquid and long-term assets, subject to the budget constraints

$$l_{-1}^b + k_{-1}^b \leq e^b \tag{13}$$

and

$$l_{-1}^s + k_{-1}^s \leq e^s + d_{-1}^s, \tag{14}$$

respectively. We take as given d_{-1}^s and focus on the choices of $\left\{l_{-1}^b,\ k_{-1}^b,\ l_{-1}^s,\ k_{-1}^s\right\}$.[11]

We begin by analyzing the buyers' problem at $t = -1$. Let $V_0^b\left(l_{-1}^b, k_{-1}^b, d_{-1}^b\right)$ denote their indirect utility function at $t = 0$, that is,

$$V_0^b\left(l_{-1}^b, k_{-1}^b\right) = \mathbb{E}_\varepsilon\left[U^b\left(c_0^b, c_1^b, c_2^b; \varepsilon\right)\right] + \lambda_0^b\left[\left(l_{-1}^b + q_0 k_{-1}^b\right) - \left(l_0^b + q_0 k_0^b + c_0^b\right)\right], \tag{15}$$

where the first term on the right-hand side is the utility evaluated at the optimal consumption choices

[10] We assume that the utility functions are well behaved so that the equilibrium exists; see the discussion in Appendix C. We also assume that, in such an equilibrium, states with and without fire sales (in the sense of equilibrium prices $q_0 < R$ and $q_0 = R$) both have positive probabilities.

[11] Regarding d_{-1}^s, one can assume that there is a mass of external agents that may deposit their endowments with sellers. Assuming that the external agents have access only to the storage technology and are risk neutral and that sellers can make a take-it-or-leave-it offer, sellers will offer a zero return on deposits and d_{-1}^s will be equal to these sellers' total endowment.

and the second term is the product of the Lagrange multiplier of the time-0 budget constraint, λ_0^b, and the time-0 budget constraint itself (i.e., buyers use the liquidity l_{-1}^b and long-term asset k_{-1}^b purchased at $t = -1$ to invest in liquidity l_0^b, long-term asset k_0^b, and consumption c_0^b).

Buyers choose the $t = -1$ liquidity l_{-1}^b and the long-term asset holdings k_{-1}^b to maximize their expected indirect utility function

$$\max_{l_{-1}^b, k_{-1}^b} \mathbb{E}_{-1} \left\{ V_0^b \left(l_{-1}^b, k_{-1}^b \right) \right\}, \tag{16}$$

subject to the budget constraint (13). The problem in (16) is easy to analyze because we can exploit the envelope theorem to obtain the first-order condition

$$\mathbb{E}_{-1} \left\{ \lambda_0^b \left[q_0 - 1 \right] \right\} = 0. \tag{17}$$

This is a standard asset-pricing equation for the excess return, $q_0 - 1$, of investing in the long-term asset, as opposed to investing in the liquid asset, evaluated according to the marginal utility of wealth. The latter is given by the Lagrange multiplier of the time-0 budget constraint, λ_0^b.

Next, we analyse the sellers, whose choices are very similar to those of the buyers. The sellers' indirect utility function at $t = 0$ is

$$V_0^s \left(l_{-1}^b, k_{-1}^b, d_{-1}^s \right) = U^s \left(c_0^s, c_1^s, c_2^s \right) + \lambda_0^s \left[\left(l_{-1}^s + q_0 k_{-1}^s \right) - \left(l_0^s + q_0 k_0^s + c_0^s + \gamma \, d_{-1}^s \right) \right] + \mu_0^s l_0^s. \tag{18}$$

The first term on the right-hand side is the utility evaluated at the optimal consumption choices, the second term is the Lagrange multiplier of the time-0 budget constraint, λ_0^s, times the budget constraint itself, and the last term is the Lagrange multiplier of the non-negative constraint on liquidity holdings, μ_0^s, times such holdings, l_0^s. Similar to the buyers, the sellers use liquidity l_{-1}^s and long-term asset k_{-1}^s, which they purchased at $t = -1$, to invest in liquidity l_0^s, long-term asset k_0^s, and consumption c_0^s, but they also face withdrawals $\gamma \, d_{-1}^s$.

The sellers choose liquidity l_{-1}^s and long-term asset holdings k_{-1}^s to maximize their expected indirect utility function

$$\max_{l_{-1}^s, k_{-1}^s} \mathbb{E}_{-1} \left\{ V_0^s \left(l_{-1}^s, k_{-1}^s \right) \right\}, \tag{19}$$

subject to the budget constraint (14). The first-order condition is very similar to that obtained in

(17), that is,

$$\mathbb{E}_{-1}\left\{\lambda_0^s\left[q_0-1\right]\right\}=0, \tag{20}$$

and it has the same interpretation discussed for (17).

Without further specifying the utility functions and the CDF $F(\cdot)$ of the withdrawal shocks, γ, we cannot characterize in more detail the choices at $t=-1$. Appendix D characterizes such choices, using the functional forms employed in the baseline model of Section 2, and the next section continues with an analysis of the more general framework to study whether regulatory intervention can improve welfare.

3.2 Efficiency analysis: Do regulatory interventions improve welfare?

We now study whether the equilibrium is efficient, that is, if the equilibrium allocation corresponds to that chosen by a planner/regulator that internalizes the effects of the portfolio choices at $t=-1$ on the fire-sale price at $t=0$. Unlike several fire-sale models in the literature, the equilibrium here is efficient under weak conditions that we argue hold if one interprets the sellers as mutual funds or MMMFs. Hence, regulatory interventions are not needed and, if implemented, they hurt the economy.

We use a standard approach employed in the fire-sale literature. Various papers such Lorenzoni (2008), Dávila and Korinek (2018), and Kurlat (2021) consider a planner/regulator that makes the initial portfolio choices (in our model, the choices at $t=-1$) but has no influence on the trading and the choices that occur in the following time periods (in our model, from $t=0$ onward). Depending on the outcome, the literature identifies tools such as capital and liquidity requirements and taxes or subsidies that can be imposed on investors to implement the regulator's solution. By following the same approach, our results are easily comparable with the literature. In addition, this approach has a good fit with the analysis of actual policies such as liquidity requirements, including those proposed by the SEC. These interventions are imposed before the possible realization of fire sales (i.e., at time $t=-1$ in our model).

Before presenting the details, we want to clarify the type of analysis we are conducting. Our model—like those in several fire-sale papers in the literature—has two sets of agents (i.e., the buyers and the sellers). We thus use the concept of Pareto optimal regulatory interventions; that is, we say that a policy is beneficial if the planner can improve the welfare of one set of agents without hurting the welfare of the other. Consistent with this approach, we use the following definition of (constrained) efficiency.

Definition. *An equilibrium is constrained efficient if no regulatory intervention at $t = -1$ can improve the welfare of the buyers, the sellers, or both.*

Formally, we consider the problem of a planner/regulator (which we simply refer to as a 'regulator) that maximizes the welfare of both the buyers and the sellers, with a Pareto weight of ξ for the buyers and a Pareto weight normalized to one for the sellers. As noted before, we demonstrate that the equilibrium is constrained efficient under some conditions. We obtain this result by showing that there exists a Pareto weight such that the solution of the regulator's problem is the same as the allocation of the decentralized equilibrium. In other words, we show that the decentralized equilibrium with no interventions is on the Pareto frontier.

We are now ready to state the regulator's problem. At $t = -1$, the regulator chooses the investments in liquidity and long-term assets of the buyers, l^b_{-1} and k^b_{-1}, as well as those of the sellers, l^s_{-1} and k^s_{-1}, to solve the following problem

$$\max_{\substack{l^s_{-1}, k^s_{-1} \\ l^b_{-1}, k^b_{-1}}} \mathbb{E}_{-1} \left\{ V^s_0 \left(l^s_{-1}, k^s_{-1}, d^s_{-1}; q_0 \right) \right\} + \xi \mathbb{E}_{-1} \left\{ V^b_0 \left(l^b_{-1}, k^b_{-1}, d^b_{-1}; q_0 \right) \right\} \tag{21}$$

where V^s_0 and V^b_0 are the sellers and buyers' indirect utility functions, defined in (18) and (15), respectively, and ξ is the buyers' Pareto weight, as noted before. Maximization is subject to the budget constraint

$$\left(l^b_{-1} + k^b_{-1} \right) + \left(l^s_{-1} + k^s_{-1} \right) \leq \left(e^b + e^s \right) + \left(d^s_{-1} - d^b_{-1} \right). \tag{22}$$

The key difference in comparison to the buyers and sellers' individual problems is that the planner accounts for the effects its choices have on the time-0 price of the long-term asset, q_0. We return to this point in Proposition 3.1.

The first-order conditions relative to the buyers' choices of liquidity and long-term asset holdings, l^b_{-1} and k^b_{-1}, imply

$$\mathbb{E}_{-1} \left\{ \xi \lambda^b_0 \left(q_0 - 1 \right) + \left(\frac{\partial q_0}{\partial k^b_{-1}} - \frac{\partial q_0}{\partial l^b_{-1}} \right) \left(\xi \lambda^b_0 - \lambda^s_0 \right) \left(k^s_0 - k^s_{-1} \right) \right\} = 0. \tag{23}$$

Similarly, for the sellers' choice of holdings, we obtain

$$\mathbb{E}_{-1}\left\{\lambda_0^s\left(q_0-1\right)+\left(\frac{\partial q_0}{\partial k_{-1}^s}-\frac{\partial q_0}{\partial l_{-1}^s}\right)\left(\xi\lambda_0^b-\lambda_0^s\right)\left(k_0^s-k_{-1}^s\right)\right\}=0. \tag{24}$$

Note that we use the market clearing condition $k_0^b + k_0^s = k_{-1}^b + k_{-1}^s$ to derive and simplify the above conditions.

To discuss these optimality conditions and our main result about the efficiency of the equilibrium, we proceed in steps. First, we highlight that the regulator's problem differs from that of the private agents because the regulator internalizes the effects of its choices on the time-0 price q_0, as noted before; this is reflected by the second term in the expectation in each of the first-order conditions. Second, we discuss how the regulator's first-order conditions compare with those of the private agents, linking our results with Dávila and Korinek (2018). Third, we provide sufficient conditions under which the equilibrium is constrained efficient and we discuss such conditions when applying our model to the analysis of the liquidity requirements the SEC proposed for MMMFs and mutual funds.

We begin by formalizing that if the regulator disregards the effects of its choices on the time-0 price, q_0, then the regulator's choices are the same as those of the private agents. This is because the only channel through which the regulator's problem differs from that of the private investor is that where the regulator internalizes the effects of its choices on the time-0 price, q_0.

Proposition 3.1. ***(Effect of internalizing the impact of the regulator's choices on time-0 prices.)*** *If $\partial q_0/\partial l_{-1}^j = 0$ and $\partial q_0/\partial k_{-1}^j = 0$ for $j \in \{b, s\}$, the regulator's first-order conditions (23) and (24) are the same as the sellers and buyers' first-order conditions in the unregulated equilibrium, as shown in Equations (17) and (20).*

We now analyze the differences between the regulator's optimality conditions and those of the private agents. The fact that the regulator internalizes the effects of its choices on the time-0 price, q_0, can introduce a wedge between the regulator and the private agents' choices, which are captured by the second term in the expectations in Equations (23) and (24). This wedge depends on the product of the three elements that are described by the next proposition.

Proposition 3.2. ***(Regulator versus investors' choices.)*** *The difference between the regulator's choices and those of the unregulated equilibrium depends on the expectation of the product of three elements:*

- *The sensitivity of the time-0 price, q_0, with respect to the $t = -1$ liquidity choices l^b_{-1} and l^s_{-1} and with respect to the $t = -1$ choices of long-term asset holdings k^b_{-1} and k^s_{-1}, that is, $\partial q_0 / \partial l^b_{-1}$, $\partial q_0 / \partial k^b_{-1}$, $\partial q_0 / \partial l^s_{-1}$, $\partial q_0 / \partial k^s_{-1}$, ;*

- *The difference between the marginal utility of the wealth of the buyers, λ^b_0, and the sellers, λ^s_0, adjusted by the Pareto weight ξ: $\xi\lambda^b_0 - \lambda^s_0$; and*

- *The sellers' purchases of long-term assets $k^s_0 - k^s_{-1}$ (or sales, if negative).*

The three elements listed in Proposition 3.2—which affect the wedge between the unregulated choices and the planner's choices—are similar to those identified by Dávila and Korinek (2018) in regard to what they refer to as distributive externalities. We first explain the concept of distributive externalities, then we discuss how they can arise in our framework, and finally we provide sufficient conditions under which these externalities are actually absent, so that the equilibrium is efficient.

Distributive externalities arise in general when the marginal rate of substitution between the dates or the states differs across agents. To clarify the notion of distributive externalities, consider the following example. Say that the buyers and sellers have the same marginal utility of wealth in normal times but the sellers' marginal utility is much higher in fire-sale states. Say also that the regulator can tilt the portfolio decisions at $t = -1$ in a way that changes the time-0 price, q_0, during fire sales. If the price change increases the sellers' wealth, the intervention improves welfare by redistributing resources to those that value them the most.[12]

Because distributive externalities can arise in our framework, the equilibrium is generically not efficient. There are, however, important differences in comparison to the distributive externalities identified in the fire-sale framework of Dávila and Korinek (2018), and these differences lead to very different efficiency and policy implications. The model of Dávila and Korinek (2018) includes collateral constraints and assumes that the buyers can extract a lower cash flow from financial assets, relative to the sellers. In that setting, the distributive externalities can be due to binding collateral constraints or incomplete markets. Here, collateral constraints are absent and, thus, the only sources of distributive externalities are just the standard effects of incomplete markets (Geanakoplos and Polemarchakis, 1986). This difference is not only conceptual but it also allows us to easily derive conditions under which efficiency holds. We argue that such conditions are likely to hold in practice when applying our model to the analysis of MMMFs and mutual funds.

[12]If the general equilibrium effect reduces the buyers' welfare, the regulator can use transfers at $t = -1$ to offset this effect. When distributive externalities are present, it is typically possible to obtain a net increase in welfare for at least one set of agents while keeping the other agents' welfare unchanged through transfers.

We now provide sufficient conditions under which equilibrium is efficient—in the sense that a regulatory intervention at $t = -1$ does not improve welfare. We begin, in Proposition 3.3, with the benchmark in which markets between $t = -1$ and $t = 0$ are complete. This case is stated mainly for conceptual reasons; markets are typically incomplete in practice, so we turn to alternative sufficient conditions in Proposition 3.4 that are more likely to hold in practice.

Proposition 3.3. ***(Complete markets and efficiency.)*** *If the support of γ includes only two values, the unregulated equilibrium is constrained efficient.*

When the condition of Proposition 3.3 holds, the equilibrium is efficient because the number of states at $t = 0$ (i.e., two) is equal to the number of assets that can be traded at $t = -1$ (i.e., the liquid asset and the long-term asset). The result that market completeness guarantees efficiency also holds in Allen and Gale (2004) and Dávila and Korinek (2018). We are stating this result just for comparison and as a benchmark for the discussion, but the complete-market assumption likely does not hold in practice. Hence, the next proposition provides alternative conditions that guarantee the efficiency of the unregulated equilibrium without relying on complete markets. We then describe each condition in detail.

Proposition 3.4. ***(Other sufficient conditions for efficient equilibrium.)***

(i) Linear utility at $t = 2$: If the buyers and sellers have linear utility in time-2 consumption (i.e., if $\partial U^b/\partial c_2^b = \kappa^b$ and $\partial U^s/\partial c_2^s = \kappa^s$ for some $\kappa^b, \kappa^s > 0$), the unregulated equilibrium is constrained efficient.

(ii) Fire sales only in one state: If a fire-sale price $q_0 < R$ arises only in one state (i.e., only for one realization of γ), the equilibrium is constrained efficient.

(iii) No wealth effects on liquidity demand: Denoting $W_0^j = l_{-1}^j + q_0 k_{-1}^j$ to be the wealth of agent $j \in \{b, s\}$, if the demand for liquidity is not affected by wealth (i.e., if $\partial l_0^j/\partial W_0^j = 0$ for $j \in \{b, s\}$), the equilibrium is constrained efficient provided that, in fire-sale states, liquidity demand is downward sloping and sellers sell some of their long-term asset holdings at $t = 0$.

Item (i) of Proposition 3.4 states that the equilibrium is efficient if the buyers and sellers have linear utility at $t = 2$. To clarify this result, let us focus on the case in which both the buyers and the sellers have linear utility at $t = 2$, with a marginal utility of one. In this case, a dollar is valued equally by both—formally, the buyers and sellers have the same marginal utility of wealth.

Intuitively, the linear utility in time-2 consumption implies that any redistribution of wealth that is implicitly generated by fire sales is irrelevant because a dollar has the same utility value to both the buyers and the sellers. Similarly, policy interventions that transfer wealth from buyers to sellers (or vice versa) are also ineffective.

In practice, the assumption of the linear utility of consumption for buyers and sellers could be reasonable when applied to the MMMFs that experienced large redemptions in both 2008 and 2020—those that targeted institutional (wealthy) investors. The potential buyers of the assets sold by these funds were also likely to have been wealthy investors because the assets required the ability to trade in OTC markets. As a result, the assumption of linear utility could have applied in practice. Under this assumption, a dollar would have been worth the same to both buyers and sellers. This would likely have been the case because they were both sophisticated, wealthy, sophisticated investors.

Item (ii) of Proposition 3.4 states that if only one level of withdrawals, γ, leads to fire sales, the equilibrium is efficient, even if the withdrawal shock, γ, can take many values and the markets are incomplete. While this might be a strong assumption in general, it fits the case in which selling pressure is driven by runs, which tend to be all-or-nothing phenomena. In both 2008 and 2020, prime MMMFs were subject to runs, especially prime institutional funds (Duygan-Bump et al., 2013; Li et al., 2021), and the Federal Reserve stepped in by creating liquidity facilities. Ultimately, the withdrawals from these funds amounted to exactly 30% of their assets under management in *both* 2008 and 2020. It could be just a coincidence that we observed the same withdrawals from MMMFs in both 2008 and 2020. But the outcome could have been the result of structural features such as the speed at which prime institutional MMMFs holders reacted to news and runs, and the time it took for the central bank to set up a liquidity facility to stop a run, which might have been a similar case in 2008 and 2020. Formally, the shock, γ, is a reduced-form way to capture how these features translate into withdrawals from MMMFs. If these structural features do not experience substantial changes, a possible future run might end up producing similar withdrawals, and the assumption that there is only one withdrawal level that triggers fire sales might be a reasonable approximation.

The logic behind the result of item (ii) is related to the sensitivity of q_0 to the choices at $t = -1$. In fire-sale states, the time-0 price, q_0, is sensitive to the amount of liquidity carried by investors from $t = -1$. But in states with no fire sales, this sensitivity drops to zero. This is because the liquidity needs of the buyers are fully met when fire sales do not arise, so that their liquidity holdings are irrelevant on the margin. This feature makes the equilibrium outcomes essentially equivalent to the case in which there are only two states and the markets are complete (see Proposition 3.3), in the sense that one can compute the time-0 price by essentially collapsing the support of γ into two values—one for a state with no fire sales and one for a state with fire sales.

Item (iii) precludes the existence of wealth effects on buyers and sellers' liquidity demand. To understand the role of this assumption, note first that the regulatory interventions at $t = -1$ alter the time-0 relative supply of liquid assets and affect of investors' wealth at the beginning of $t = 0$. Without the wealth effects, however, the regulatory interventions have an impact only through changes in the supply of time-0 assets. Crucially, any given change can be obtained by targeting buyers or sellers; that is, the identity of the entity that brings, say, more liquidity to the time-0 market is irrelevant. In this sense, buyers and sellers are identical in the eyes of the regulator (up to a scaling factor given by the Pareto weight), and this symmetry removes the scope for Pareto-improving interventions.[13]

The condition in item (iii) is relevant in relation to a key difference in comparison to other models in the literature. In models with cash-in-the-market pricing, the buyers' demand is constrained by their wealth, so that an increase in the buyers' wealth has a first-order effect on prices. Relatedly, in models with second-best use and collateral constraints, an increase in the sellers' wealth typically leads to a bigger change in prices. The higher wealth relaxes the collateral constraints and, thus, limits fire sales and the sellers have a higher willingness to pay relative to the buyers because they can extract a higher cash flow. Because of these features, the assumption in item (iii) typically does not hold in other fire-sale models but it can hold in ours.

To sum up, the results of this section have important policy implications. The ex ante investment decisions of price-taking agents that face possible fire-sale prices are efficient—under conditions that we argue apply to MMMFs and mutual funds—and regulations such as liquidity requirements do not lead to Pareto improvement and might even reduce overall welfare.

4 Fire sales and liquidity risk pricing in other applications

In our baseline model of Section 2, we illustrated our theory of fire sales based on liquidity risk pricing in the context of a simple framework in which the sellers can be interpreted as banks,

[13] Note that item (iii) is conceptually different from (i). In our baseline model, quasi-linear utility implies that both (i) and (iii) hold. But in the model in Appendix A, the sellers have strictly concave utility—so that (i) does not hold—but (iii) holds anyway because the sellers' demand for liquidity is constrained at zero by a non-shorting condition.

MMMFs, or mutual funds that are subject to exogenous redemption shocks. The liquidity risk pricing at the core of our theory, however, is more general and can be used to study fire sales in several other applications with different sources of forced sales.

The main objective of this section is to show that the liquidity risk mechanism that gives rise to buyers' low willingness to pay in a fire sale can be combined with several channels that generate the selling pressure of the long-term asset. In the baseline model of Section 2, the selling pressure is triggered by exogenous withdrawal shocks that affect the buyers. In the application of Section 4.1, the sellers are subject to a collateral constraint and the selling pressure is triggered by an exogenous shock that tightens such a constraint. In the application of Section 4.2, we consider banks that can be subject to runs. In this case, the selling pressure is the result of a bank run that forces banks to liquidate their holdings of the long-term asset.

The applications we present here are relevant in practice. Fire sales in some empirical instances have been linked with binding collateral constraints, such as in the case of insurance companies in 2008 (Merrill et al., 2021), and bank runs are common in times of acute crisis; examples include the runs on MMMFs in September 2008 and March 2020, or those on Silicon Valley Bank and First Republic Bank in 2023 (Duygan-Bump et al., 2013; Jiang et al., 2023; Li et al., 2021; Schmidt, Timmermann, and Wermers, 2016).

Before turning to our analysis, we note that the source of the selling pressure might affect the efficiency of the equilibrium and the need for regulatory interventions even if it is irrelevant to generate fire sales with liquidity risk pricing. We briefly discuss this issue, but we leave to future research a full analysis of regulatory interventions for the extensions presented in this section. For instance, the version of the model with a collateral constraint (Section 4.1) is likely characterized by externalities related to such a constraint, which are instead absent in our baseline model. That said, there is an important difference in comparison to collateral externalities that arise in other fire-sale models in the literature. In models with second-best use, collateral constraints that affect sellers' behavior imply that long-term assets are sold to buyers with (exogenously) less ability to extract cash flow. Here, the collateral constraint that affects sellers might force such agents to sell assets in a fire sale, draining liquidity from the market and (endogenously) reducing buyers' ability to self-insure against liquidity/preference shocks. Ultimately, as noted by Kurlat (2021), efficiency analysis is heavily affected by the relation between forced sales and buyers' asset pricing conditions. This means that determining the microeconomic model that bests fits any given application is required to study efficiency and provide adequate policy implications.

4.1 Liquidity risk pricing and collateral constraints

This section modifies the baseline model along two dimensions: We include a collateral constraint that affects the sellers and we replace the withdrawal shocks with shocks to the terminal payoff, R, of the long-term asset. As noted in the previous section, the broad objective is to show that the liquidity risk pricing mechanism introduced in Section 2 can be coupled with other channels that give rise to high selling pressure.

A fire sale in this model arises when a negative shock reduces the payoff of the long-term asset to a sufficiently low level. The low payoff reduces the value of the long-term asset, tightening the sellers' collateral constraints and forcing them to sell some of their holdings of such assets, which in turn reduces the price of the long-term asset even more. When this high selling pressure materializes, the buyers' low willingness to pay arises from the same channel used in the baseline model of Section 2. That is, as the sellers sell the long-term asset and drain liquidity from the market, the buyers' holdings of liquidity drop, exposing them to a liquidity risk/preference shock that might hit them at $t = 1$.

Model. The timing and structure of the preferences, technology, endowments, and markets are the same as in the baseline model of Section 2, with a few differences. First, there are no withdrawal shocks (i.e., $\gamma = 0$ in all states). Second, the return R of the long-term asset is stochastic and is realized at the beginning, at $t = 0$, taking values R^H and R^L with probabilities $1 - \pi$ and π, respectively, with $R^H > 1 > R^L > 0$. Third, the sellers are subject to a time-0 collateral constraint of the form

$$\zeta \left(q_0 k_0^s\right) \leq q_0 k_{-1}^s + l_{-1}^s - d_{-1}^s. \tag{25}$$

This constraint has two possible interpretations. First, it can be viewed as a risk-weighted capital requirement, with a zero risk weight on liquid assets, a 100% risk weight on long-term assets, and a capital requirement ζ. Note that the right-hand side of (25) is the sellers' equity at $t = 0$. This is in line with the interpretation of sellers as insurance companies, which sold mortgage-backed securities (MBS) at fire-sale prices in 2008 (Merrill et al., 2021). Alternatively, the constraint can be derived by assuming limited commitment along the lines of Lorenzoni (2008), as discussed in Appendix E.

Parameter restrictions. We normalize the initial endowment of the sellers' liquidity to $l_{-1}^{s} = 0$ and the sellers' long-term asset to

$$k_{-1}^{s} = \frac{d_{-1}^{s}}{1 - \zeta}. \tag{26}$$

The value of the endowment of k_{-1}^{s} can be derived endogenously by adding a period $t = -1$ (along the lines of Section 3.1) and assuming that the sellers are subject to a collateral constraint of the same form as (25) at $t = -1$:

$$\zeta \left(k_{-1}^{s} \right) = k_{-1}^{s} + l_{-1}^{s} - d_{-1}^{s}. \tag{27}$$

This constraint can again be interpreted as a capital requirement or as arising from limited commitment (see Appendix E).

We normalize the buyers' initial holdings of liquidity to $l_{-1}^{b} = 1$. As in the baseline model, no restrictions are imposed on the buyers' initial holdings of the long-term asset, k_{-1}^{b}.

Finally, we assume that the collateral constraint is sufficiently tight—otherwise its effects would not be strong enough to obtain a fire sale. More precisely, we require that the parameter, ζ, satisfies

$$\zeta > 1 - \frac{R^{L} (1 - d)}{(1 - \theta d)}. \tag{28}$$

Buyers and sellers' choices. The analysis of the buyers is the same as that in the baseline model; see Section 2.4. In particular, the price, q_0, that they are willing to pay is given by Equation (5).

Regarding the sellers, their time-0 problem is different from that in the baseline model because of the collateral constraint (25). Thus, they solve the problem

$$\max_{l_0^s, k_0^s} \; R k_0^s + l_0^s - d_{-1}, \qquad R \in \{R^H, R^L\}$$

subject to the collateral constraint in Equation (25) and the budget constraint $q_0 k_0^s + l_0^s \leq q_0 k_{-1}^s$.

A seller's optimal choice depends on whether the collateral constraint is binding. When the constraint is not binding, the sellers' choices are the same as in the baseline model; that is, they hold no liquidity and invest everything in the long-term asset: $l_0^s = 0$ and $k_0^s = {}^{q_0 k_{-1}^s}/{}_{q_0}$. When the collateral constraint is binding, the seller's choices are

$$l_0^s = \frac{d_{-1}^{s} (1 - q_0)}{\zeta}, \quad k_0^s = \frac{d_{-1}^{s} \left[q_0 - (1 - \zeta) \right]}{q_0 (1 - \zeta) \zeta}. \tag{29}$$

Note that the collateral constraint is binding when the price of the long-term asset is $q_0 < 1$.[14]

Equilibrium. Given a realization of the return $R \in \{R^H, R^L\}$, an equilibrium is a collection of the buyers' choices, the sellers' choices, and the time-0 price, q_0, such that the time-0 market for liquidity clears, $l_0^b + l_0^s = l_{-1}^b$, and the time-0 market for the long-term asset clears, $k_0^b + k_0^s = k_{-1}^b + k_{-1}^s$.

We begin by solving the equilibrium when the realization of the long-term asset is $R^H > 1$. In this case, the equilibrium is essentially the same as that of the baseline with no fire sales (i.e., the case where $\gamma = 0$ in the baseline model). This is because the sellers' collateral constraint is not binding and they do not engage in any sale of their long-term asset holdings, as formalized by the next proposition.

Proposition 4.1. *(Equilibrium with collateral constraints and no fire sales) If $R = R^H > 1$, there exists an equilibrium in which the time-0 price of the long-term asset is $q_0 = R^H > 1$, the sellers' collateral constraint (25) is not binding, the buyers and sellers do not engage in trades at $t = 0$ (i.e., $l_0^b = 1$ and $k_0^b = k_{-1}^b$ for the buyers, and $l_0^s = 0$ and $k_0^s = k_{-1}^s$ for the sellers), and the time-1 consumption of the buyers that are hit by the preference shock is $c_1^b = 1$.*

Next, consider the case in which the return of the long-term asset is $R^L < 1$. The low return of the long-term asset depresses its price, tightening the sellers' collateral constraint (25). As a result, the sellers are forced to sell some of their holdings of the long-term asset, draining liquidity from the market. Similar to the baseline, the buyers end up with smaller liquidity holdings and, thus, are exposed to time-1 preference shocks. As a result, a fire sale arises: the liquidity risk pricing reduces the time-0 price of the long-term asset below its payoff, R^L, that is, $q_0 < R^L$. The next proposition formalizes this result.

Proposition 4.2. *(**Equilibrium with collateral constraints and fire sales.**) If $R = R^L < 1$, there exists an equilibrium with fire sales; that is, the time-0 price of the long-term asset is $q_0 < R^L$, the sellers' collateral constraint (25) is binding, they remain solvent (i.e., their net worth $q_0 k_{-1}^s + l_{-1}^s - d_{-1}^s$ is strictly positive), and they sell part of their endowment of the long-term asset and accumulate liquidity at $t = 0$ (i.e., $k_0^s < k_{-1}^s$ and $l_0^s > 0$); the buyers increase their holdings of the long-term asset and reduce their holdings of liquidity at $t = 0$ (i.e., $k_0^b > k_{-1}^b$ and $l_0^b < 1$), and their time-1 consumption that is hit by the preference shock is $c_1^b < 1$.*

[14]This can derived using Equation (25) evaluated with equality, the value of endowments k_{-1}^s in (26), and $l_{-1}^s = 0$.

To sum up, the results show that fire sales based on liquidity risk pricing can be coupled with different sources of forced sales—not only sellers' exogenous shocks, as in the baseline model, but also a tightening of sellers' collateral constraint, as in the model of this section. The next section presents yet another source of forced sales that can be combined with liquidity risk pricing, that is, a bank run.

4.2 Liquidity risk pricing and bank runs

We now present a model that embeds the liquidity risk pricing introduced in our baseline framework of Section 2 into an otherwise standard model of bank runs along the lines of Diamond and Dybvig (1983), hereafter, DD. The objective is to show that fire sales driven by liquidity risk pricing can be combined with yet another source of selling pressure, this one arising from banks' sales of their long-term asset holdings because of a run.

A key element of this application is that the depositors are similar to the buyers of Section 2 and, thus, have some ability to trade in the time-0 market. In particular, during a run, depositors not only withdraw from banks but they also purchase the long-term securities that are liquidated by banks. Fire-sale prices arise because, as banks become insolvent, depositors lose the insurance against future preference shocks that they would have had if banks had remained insolvent. Lacking this insurance, they are willing to pay low fire-sale prices to purchase long-term assets, making runs self-fulfilling events. An alternative approach to modeling runs would be to assume that the depositors are unable to trade and, in a run, the banks sell their long-term assets to buyers that are exposed to liquidity shocks. However, that approach is isomorphic to the model of Section 2, in which the withdrawal γ parameterizes the extent of the bank run.

This model has other important similarities and differences in comparison to DD and to the model of Section 2. Banks could be mapped into the sellers discussed in Section 2, although we follow the standard approach of DD and the banking literature of modeling banks as firm-like entities that compete with each other to attract depositors and earn zero profits in equilibrium. We also use the same types of assets as in Section 2 (i.e., a liquid asset and a long-term asset) and the same assumption about the lack of liquidation technology to transform long-term investments into liquid assets after initial investments have been made—a departure from DD. Finally, we use a market structure similar to that of the baseline model of Section 2 (i.e., a centralized market in which fire sales could take place, followed by a period in which there are no markets and some preference shocks are realized).

4.2.1 Model

We follow a timing similar to that of the baseline model of Section 2, but we also include the period in which ex ante investments are made, as in Section 3. Thus, there are four periods indexed by $t = -1, 0, 1, 2$. Period $t = 0$ is when a fire sale might arise—as in Section 2—in conjunction with a run. The technology is also the same as in the baseline model. That is, there is a liquid asset and a long-term asset.

We depart from the baseline model by enriching the structure of the preferences and markets. We do so to obtain that deposits are demandable at $t = 0$; we return to this point below.[15]

Preferences and endowments. The economy is populated by a continuum of agents, which we refer to as depositors (denoted by the superscript d). At $t = -1$, the depositors are identical and have endowment $e^d_{-1} = 1$. Similar to the buyers of Section 2, the depositors are subject to preference shocks but ones with a slightly different structure. At $t = 0$, some of the uncertainty related to the preference shocks is realized and the ex ante identical depositors are divided into two groups. A fraction, γ, of these depositors becomes *very impatient*, with a utility function, $u\left(c^d_0\right)$, that only depends on consumption at time $t = 0$. The remaining $1 - \gamma$ fraction faces additional preference shocks at $t = 1$, similar to DD and to the baseline model of Section 2. Specifically, they become *impatient* with probability θ and *patient* with probability $1 - \theta$, with utility function

$$U^d\left(c^d_1, c^d_2\right) = \begin{cases} u\left(c^d_1\right) + \beta c^d_2 & \text{with probability } \theta \\ \beta c^d_2 & \text{with probability } 1 - \theta, \end{cases}$$

where β is the discount factor. As in Section 2, we assume that $u(c) = \log c$. All of the preference shocks are independent and identically distributed across households and the law of large numbers holds. Therefore, the masses of very impatient, impatient, and patient agents are represented by γ, $(1 - \gamma)\theta$, and $(1 - \gamma)(1 - \theta)$, respectively.

The formulation of the preferences is slightly different from DD but the logic and implications are similar. That is, very impatient and impatient agents have urgent needs to consume at $t = 0$ and $t = 1$, respectively.

[15] Alternatively, one can impose the demandability at $t = 0$ exogenously and use the same structure of preferences and markets as in Section 2.

Banks. Banks (denoted by b) are established at $t = -1$ and liquidated at $t = 2$. Once established, each depositor has access to their bank at all times $t \in \{0, 1, 2\}$. In Appendix F, we show that the results are unchanged if new banks can be created at $t = 0$, to capture the idea that depositors might be able to transfer the resources they withdraw in a run to other parts of the financial sector.

Markets. The market structure is similar to that of the baseline model (i.e., a centralized market at $t = 0$ and a lack of markets at $t = 1$) but with a key difference: We assume that depositors have limited market access at $t = 0$. Specifically, some depositors are hit by a shock at $t = 0$, which makes them unable to trade in the centralized market—the realization is private information. To simplify the exposition, we assume that this shock is perfectly correlated to the preference shock that is realized at $t = 0$. In particular, the very impatient depositors that consume only at $t = 0$ are also hit by the same shock that prevents them from trading in the centralized market. Appendix F shows that the key results are unchanged if the time-0 preference and the market access shocks are uncorrelated.

The above structure implies that banks collect deposits at $t = -1$ and allow for withdrawals on demand at $t = 0, 1, 2$. Deposits are demandable at $t = 0$ and can offer liquidity to very impatient agents——with no market access—and deposits are also demandable at $t = 1$ and $t = 2$, as in DD and other banking models. If we assume that all of these depositors have access to the time-0 market, then very impatient depositors could simply trade at $t = 0$ to reach the first-best level of consumption (Jacklin, 1987), and banks could collect deposits at $t = 0$ from the remaining depositors that are subject to the time-1 preference shock. This would eliminate the equilibrium with bank runs in our setting.

Parameter restrictions We normalize the endowment of depositors to $e^d_{-1} = 1$ and the return, R, of the long-term asset to $1/\beta$, with $R > 1$ and $\beta < 1$.

4.2.2 Good equilibrium

We define an equilibrium as a contract that specifies banks' investments in liquid, l^b_{-1}, and long-term assets, $k^b_{-1} = 1 - l^b_{-1}$, as well as withdrawals, c^d_0, c^d_1, and c^d_2, for agents that report themselves as very impatient, impatient, or patient, respectively; depositors and banks' decisions regarding the quantity of the liquid and long-term assets to be traded at $t = 0$; and a price, q_0, that clears the market at $t = 0$.

The good equilibrium is standard. Banks offer a contract in which they collect endowments at $t=-1$ and allow agents to withdraw at $t=0$, $t=1$, or $t=2$. In equilibrium, very impatient agents withdraw at $t=0$, impatient agents withdraw at $t=1$, and patient ones wait until $t=2$ and receive an equal share of the funds that are available at that time. The next proposition shows that a Pareto optimal allocation is implemented in this equilibrium.

Proposition 4.3. (Good equilibrium) *There exists an equilibrium in which banks offer a contract that implements a Pareto optimal allocation, that is,* $l^b_{-1}=\gamma+(1-\gamma)\theta$, $k^b_{-1}=(1-\gamma)(1-\theta)$, $c^d_0=c^d_1=1$, *and* $c^d_2=R$. *In this equilibrium, the price* q_0 *of the long-term asset at* $t=0$ *is equal to the payoff,* R, *discounted by* β, $q_0=\beta R$ *(or,* $q_0=1$, *using the normalization* $\beta R=1$*).*

The good equilibrium is similar to the equilibrium of the baseline model of Section 2 under the realization of $\gamma=0$, in which the buyers are able to get full insurance against their time-1 liquidity risk. The proof (in Appendix B) is based on two steps. The first one is the standard argument that the depositors truthfully report the realization of their preference shock. The other step deals with trading in the time-0 market, which is novel in our environment. We elaborate more on this.

When banks solve for the contract they will offer to depositors, they solve for their portfolio holdings at $t=-1$ and for the withdrawals, c^d_0, c^d_1, and c^d_2, they offer to depositors at $t=0,1,2$, respectively. In addition, banks can adjust their holdings of liquidity and the long-term asset at $t=0$, l^b_0 and k^b_0. Thus, they solve the problem

$$\max_{l^b_{-1},k^b_{-1}}\left[\max_{l^b_0,k^b_0,c^d_0,c^d_1,c^d_2}\ \gamma\, u\left(c^d_0\right)+(1-\gamma)\,\theta\, u\left(c^d_1\right)+\beta\,(1-\gamma)\,(1-\theta)\,c^d_2\right] \tag{30}$$

subject to the budget constraints at $t=-1$ and $t=0$, given by $l^b_{-1}+k^d_{-1}\leq 1$ and $\gamma c^d_0+l^b_0+q_0k^b_0\leq l^b_{-1}+q_0k^d_{-1}$, the feasibility constraint that restricts time-1 repayments to the depositors of the liquidity, l^b_0, carried from $t=0$, $(1-\gamma)\,\theta c^d_1\leq l^b_0$, and the time-2 repayments from the return produced by the long-term asset, k^b_0, carried from $t=0$, $(1-\gamma)\,(1-\theta)\,c^d_2\leq Rk^b_0$.

The key and novel optimality condition of banks is a time-0 asset pricing condition similar to that derived for the buyers in the baseline model, that is, Equation (5):

$$q_0=\beta\frac{1}{u'\left(c^d_1\right)}\times R. \tag{31}$$

The banks' pricing kernel discounts the payoff, R, of the long-term asset, using the marginal utility of patient agents (i.e., one, discounted by β) in relation to that of the impatient agents (i.e., $u'\left(c^d_1\right)$).

When c_1^d is evaluated at the Pareto optimal level that is implemented by the good equilibrium, Equation (31) implies $q_0 = \beta R$ (or $q_0 = 1$, using the normalization $\beta R = 1$). Thus, the payoff, R, is discounted only according to the factor β and the preference shocks play no role in Equation (31) because the banks offer depositors full insurance.

4.2.3 Bad equilibrium

As in DD, there can be another equilibrium in which bank runs become self-fulfilling prophecies. In this equilibrium,the banks are subject to a run at $t = 0$ and they sell their holdings of long-term assets to replace the withdrawals. Crucially, the long-term asset is sold at a fire-sale price, $q_0 < 1$ (i.e., lower than the good equilibrium price). The fire-sale price arises because the banks are subject to runs and fail at $t = 0$, leaving the depositors without insurance against liquidity risk. Hence, at $t = 0$, the depositors increase their demand for liquidity and decrease their demand for the long-term illiquid asset, putting downward pressure on the price, q_0. This lower price, in turn, implies that the banks are insolvent at $t = 0$, making runs a self-fulfilling outcome.

We consider the case of an unanticipated run, that is, in which banks' allocations of endowments at $t = -1$ to liquidity l_{-1}^b and long-term assets k_{-1}^b as well as the banking contracts offered to depositors are the same as in the good equilibrium.[16] That is, we solve for the post-deposit equilibrium, which is the equilibrium that includes only the prices and allocations for $t = 0, 1, 2$ that maximize the households' utility and clear the time-0 market, taking as given the choices made at $t = -1$.

At $t = 0$, all of the agents withdraw their funds from the bank. The amount each agent can withdraw depends on the equilibrium price, q_0, and we denote this as

$$w(q_0) = l_{-1}^b + k_{-1}^b q_0. \tag{32}$$

A very impatient depositor only enjoys utility at $t = 0$, so they immediately consume $w(q_0)$. Other agents need to decide how to invest their withdrawals, that is, how much to invest in liquidity, l_0^d, and long-term assets, k_0^d. Their problem is essentially identical to that of the buyers in Section 2—see the problem in (1)—with the only difference being that the time-2 consumption is now discounted

[16] This allocation in the good equilibrium is $l_{-1}^b = \gamma + (1-\gamma)\theta$ and $k_{-1}^b = (1-\gamma)(1-\theta)$.

by β. The optimal condition is thus an asset-pricing equation along the lines of Equation (5):

$$q_0 = \beta \frac{1}{(1-\theta) \times 1 + \theta u'\left(c_1^d\right)} \times R. \tag{33}$$

As a result, the equilibrium is similar to that of the baseline model of Section 2, with large sales of the long-term asset and a fire-sale price, q_0, as formalized by the next proposition. The difference is that the selling pressure here originates from a bank run rather than an exogenous shock to redemptions.

Proposition 4.4. *(Bank runs and fire sales) There exists a post-deposit equilibrium in which all of the depositors run and withdraw $w(q_0) < 1$ at $t = 0$. In this equilibrium, the long-term asset is traded at a fire-sale price, $q_0 < 1$.*

5 Conclusion

This paper provides a novel theory of fire sales in which a combination of liquidity risk and resale frictions generates a fire-sale price when (and only when) the selling pressure is high. The approach is quite general and can be combined with several sources of selling pressure, such as exogenous withdrawals from MMMFs or mutual funds, the tightening of collateral constraints, and panic-based runs.

The main application of the model is to the fire sales that took place at the peak of the COVID-19 crisis in March 2020. These involved high-quality assets such as Treasury securities and highly rated corporate bonds sold by mutual funds and MMMFs. The SEC responded to these events by proposing tight liquidity requirements. As viewed through the lens of our model, however, these policy interventions would reduce welfare, as we find the equilibrium to be constrained efficient under some weak conditions that we argue apply in practice. These results stand in contrast to the main fire-sale theories in the literature, in which the equilibrium is typically inefficient.

Our paper opens up several directions for future research. On the theoretical side, an open question is related to the welfare properties and possible regulatory interventions that might be required when our liquidity risk pricing is combined with alternative sources of forced sales that we have only briefly analyzed, such as the tightening of collateral constraints. On the empirical side, future research can analyze the extent to which our pricing mechanism contributes to fire sales.

References

Acharya, Viral V and Tanju Yorulmazer. 2008. "Cash-in-the-market pricing and optimal resolution of bank failures." *The Review of Financial Studies* 21 (6):2705–2742.

Allen, Franklin and Douglas Gale. 1998. "Optimal financial crises." *The Journal of Finance* 53 (4):1245–1284.

———. 2004. "Financial intermediaries and markets." *Econometrica* 72 (4):1023–1061.

Ambrose, Brent W, Kelly N Cai, and Jean Helwege. 2012. "Fallen angels and price pressure." *Journal of Fixed Income* 21 (3):74–86.

Amihud, Yakov and Haim Mendelson. 1986. "Asset pricing and the bid-ask spread." *Journal of financial Economics* 17 (2):223–249.

Bianchi, Javier and Saki Bigio. 2022. "Banks, liquidity management, and monetary policy." *Econometrica* 90 (1):391–454.

Caballero, Ricardo J and Alp Simsek. 2013. "Fire sales in a model of complexity." *The Journal of Finance* 68 (6):2549–2587.

Chang, Briana. 2018. "Adverse selection and liquidity distortion." *The Review of Economic Studies* 85 (1):275–306.

Choi, Jaewon, Saeid Hoseinzade, Sean Seunghun Shin, and Hassan Tehranian. 2020. "Corporate bond mutual funds and asset fire sales." *Journal of Financial Economics* 138 (2):432–457.

Cochrane, John H. 2011. "Presidential address: Discount rates." *The Journal of finance* 66 (4):1047–1108.

Coval, Joshua and Erik Stafford. 2007. "Asset fire sales (and purchases) in equity markets." *Journal of Financial Economics* 86 (2):479–512.

D'Amico, Stefania and N Aaron Pancost. 2022. "Special repo rates and the cross-section of bond prices: The role of the special collateral risk premium." *Review of Finance* 26 (1):117–162.

Dávila, Eduardo and Anton Korinek. 2018. "Pecuniary externalities in economies with financial frictions." *The Review of Economic Studies* 85 (1):352–395.

Diamond, Douglas W. and Philip H. Dybvig. 1983. "Bank Runs, Deposit Insurance, and Liquidity." *The Journal of Political Economy* 91 (3):401–419.

Dow, James and Jungsuk Han. 2018. "The paradox of financial fire sales: The role of arbitrage capital in determining liquidity." *The Journal of Finance* 73 (1):229–274.

Duffie, Darrell. 1996. "Special repo rates." *The Journal of Finance* 51 (2):493–526.

———. 2020. "Still the world's safe haven." *Hutchins Center Working Paper 62, Brookings Institution* .

Duygan-Bump, Burcu, Patrick Parkinson, Eric Rosengren, Gustavo A Suarez, and Paul Willen. 2013. "How Effective Were the Federal Reserve Emergency Liquidity Facilities? Evidence from the Asset-Backed Commercial Paper Money Market Mutual Fund Liquidity Facility." *The Journal of Finance* 68 (2):715–737.

Ellul, Andrew, Chotibhak Jotikasthira, and Christian T Lundblad. 2011. "Regulatory pressure and fire sales in the corporate bond market." *Journal of Financial Economics* 101 (3):596–620.

Falato, Antonio, Itay Goldstein, and Ali Hortaçsu. 2021. "Financial fragility in the COVID-19 crisis: The case of investment funds in corporate bond markets." *Journal of Monetary Economics* 123:35–52.

Falato, Antonio, Ali Hortacsu, Dan Li, and Chaehee Shin. 2021. "Fire-Sale Spillovers in Debt Markets." *The Journal of Finance* 76 (6):3055–3102.

Fleming, Michael and Francisco Ruela. 2020. "Treasury market liquidity during the COVID-19 crisis." Federal Reserve Bank of New York Staff Reports.

Gale, Douglas and Tanju Yorulmazer. 2013. "Liquidity hoarding." *Theoretical Economics* 8 (2):291–324.

Geanakoplos, John and Heraklis Polemarchakis. 1986. "Existence, regularity, and constrained suboptimality of competitive allocations when the asset market is incomplete." *Uncertainty, information and communication: essays in honor of KJ Arrow* 3:65–96.

Gertler, Mark and Nobuhiro Kiyotaki. 2015. "Banking, Liquidity and Bank Runs in an Infinite Horizon Economy." *American Economic Review* 105 (7):2011–43.

Goldstein, Itay, Alexandr Kopytov, Lin Shen, and Haotian Xiang. 2022. "Synchronicity and Fragility." Working Paper.

Granja, Joao, Gregor Matvos, and Amit Seru. 2017. "Selling failed banks." *The Journal of Finance* 72 (4):1723–1784.

Gürkaynak, Refet S, Brian Sack, and Jonathan H Wright. 2007. "The US Treasury yield curve: 1961 to the present." *Journal of monetary Economics* 54 (8):2291–2304.

He, Zhiguo and Péter Kondor. 2016. "Inefficient investment waves." *Econometrica* 84 (2):735–780.

He, Zhiguo, Stefan Nagel, and Zhaogang Song. 2022. "Treasury inconvenience yields during the COVID-19 crisis." *Journal of Financial Economics* 143 (1):57–79.

Jacklin, Charles J. 1987. "Demand deposits, trading restrictions, and risk sharing." *Contractual arrangements for intertemporal trade* 1.

Jiang, Erica Xuewei, Gregor Matvos, Tomasz Piskorski, and Amit Seru. 2023. "US Bank Fragility to Credit Risk in 2023: Monetary Tightening and Commercial Real Estate Distress." Working Paper.

Jotikasthira, Chotibhak, Christian Lundblad, and Tarun Ramadorai. 2012. "Asset fire sales and purchases and the international transmission of funding shocks." *The Journal of Finance* 67 (6):2015–2050.

Kargar, Mahyar, Benjamin Lester, David Lindsay, Shuo Liu, Pierre-Olivier Weill, and Diego Zúñiga. 2021. "Corporate bond liquidity during the COVID-19 crisis." *The Review of Financial Studies* 34 (11):5352–5401.

Kiyotaki, Nobuhiro and John Moore. 1997. "Credit cycles." *Journal of political economy* 105 (2):211–248.

Krishnamurthy, Arvind. 2002. "The bond/old-bond spread." *Journal of financial Economics* 66 (2-3):463–506.

Kurlat, Pablo. 2016. "Asset markets with heterogeneous information." *Econometrica* 84 (1):33–85.

———. 2021. "Investment externalities in models of fire sales." *Journal of Monetary Economics* 122:102–118.

Lanteri, Andrea and Adriano A Rampini. 2023. "Constrained-efficient capital reallocation." *American Economic Review* 113 (2):354–395.

Li, Dan and Norman Schürhoff. 2019. "Dealer networks." *The Journal of Finance* 74 (1):91–144.

Li, Lei, Yi Li, Marco Macchiavelli, and Xing Zhou. 2021. "Liquidity restrictions, runs, and central bank interventions: Evidence from money market funds." *The Review of Financial Studies* 34 (11):5402–5437.

Lorenzoni, Guido. 2008. “Inefficient credit booms.” *The Review of Economic Studies* 75 (3):809–833.

Ma, Yiming, Kairong Xiao, and Yao Zeng. 2022. “Mutual fund liquidity transformation and reverse flight to liquidity.” *The Review of Financial Studies* 35 (10):4674–4711.

Malherbe, Frédéric. 2014. “Self-Fulfilling Liquidity Dry-Ups.” *The Journal of Finance* 69 (2):947–970.

Manconi, Alberto, Massimo Massa, and Ayako Yasuda. 2012. “The role of institutional investors in propagating the crisis of 2007–2008.” *Journal of Financial Economics* 104 (3):491–518.

Merrill, Craig B, Taylor D Nadauld, René M Stulz, and Shane M Sherlun. 2021. “Were there fire sales in the RMBS market?” *Journal of Monetary Economics* 122:17–37.

O’Hara, Maureen and Xing Alex Zhou. 2021. “Anatomy of a liquidity crisis: Corporate bonds in the COVID-19 crisis.” *Journal of Financial Economics* 142 (1):46–68.

Pancost, Aaron N. 2021. “Zero-coupon yields and the cross-section of bond prices.” *The Review of Asset Pricing Studies* 11 (2):209–268.

Robatto, Roberto. 2019. “Systemic banking panics, liquidity risk, and monetary policy.” *Review of Economic Dynamics* 34:20–42.

———. 2023. “Liquidity Requirements and Central Bank Interventions During Banking Crises.” *Management Science* Available at https://doi.org/10.1287/mnsc.2023.4737.

Schmidt, Lawrence, Allan Timmermann, and Russ Wermers. 2016. “Runs on money market mutual funds.” *The American Economic Review* 106 (9):2625–2657.

Shleifer, Andrei and Robert W Vishny. 1992. “Liquidation values and debt capacity: A market equilibrium approach.” *The Journal of Finance* 47 (4):1343–1366.

Vayanos, Dimitri and Pierre-Olivier Weill. 2008. “A search-based theory of the on-the-run phenomenon.” *The Journal of Finance* 63 (3):1361–1398.

Vissing-Jorgensen, Annette. 2021. “The treasury market in spring 2020 and the response of the federal reserve.” *Journal of Monetary Economics* 124:19–47.

APPENDIX

A Application: Liquidity risk pricing and mutual funds

In this section, we present another example that showcases fire sales. The model is similar to that of Section 2, with the main difference being that the sellers are now a coalition of a unit mass of small investors, each with an endowment of k^s_{-1}, holding one share of the coalition. We demonstrate that this coalition can in fact be interpreted as a mutual fund. A fraction, $\gamma \in \{0, \bar{\gamma}\}$, of the coalition needs to consume at time 0, while the remainder will consume at time 2. We assume that the buyers' endowment of liquidity, l^b_{-1}, is sufficiently large so that there are no fire sales in the state in which no member of the coalition needs to consume (i.e., when $\gamma = 0$). We also assume that $\bar{\gamma} k^s_{-1} R + 1 > l^b_{-1}$.

Assuming log utility for consumption, the utility of the coalition at time 0 is given by

$$U^s = \gamma \log(c^s_0) + (1 - \gamma) \log(c^s_2).$$

On the other hand, the buyers are the same as in the baseline model.

At time 0, given the realization of γ, the seller coalition solves

$$\max_{l^s_0, k^s_0, c^s_0, c^s_2} \gamma \log\left(c^s_0\right) + (1 - \gamma) \log(c^s_2)$$

subject to

$$l^s_0 + q_0 k^s_0 \leq k^s_{-1} q_0$$
$$\gamma c^s_0 = l^s_0,$$
$$(1 - \gamma) c^s_2 = R k^s_0.$$

And it follows that

$$\gamma c^s_0 = \gamma q_0 k^s_{-1},$$

that is, the time-0 outflow of funds is only a fraction, γ, of the total asset value of the coalition. Hence, we can think of the coalition as a *mutual fund*, as shares are repaid at their net asset value (NAV).

The market clearing condition for liquidity is now

$$\gamma c_0^s + l_0^b = l_{-1}^b.$$

When $\gamma = 0$, the market clears at the price $q_0 = R$. When $\gamma = \bar{\gamma}$, the market clearing condition becomes

$$\bar{\gamma} q_0 k_{-1}^s + \frac{q_0(1-\theta)}{R - q_0\theta} = l_{-1}^b.$$

Recall that we have assumed $\bar{\gamma} k_{-1}^s R + 1 > l_{-1}^b$ and, thus, the market cannot clear at $q_0 = R$. Because the second term on the left-hand side of the market clearing condition is increasing in q_0, we conclude that q_0 has to be less than R.

B Proofs

Proof of Proposition 2.1 Let $\gamma = 0$ and $q_0 = R$. The sellers are indifferent between the two assets. For the buyers' problem, we formally write the Lagrangian function as

$$L_0^b = (1-\theta)(l_0^b + Rk_0^b) + \theta\left[u(c_1^b) + l_0^b - c_1^b + Rk_0^b\right] - \lambda_0(l_0^b + Rk_0^b - l_{-1}^b - Rk_{-1}^b) - \eta_0(c_1^b - l_0^b).$$

The necessary conditions for optimal c_1^b, l_0^b, k_0^b imply $\lambda_0 = 1$, $\eta_0 = 0$, and $c_1^b = 1$. Any $l_0^b \in [1, l_{-1}^b + Rk_{-1}^b]$ will be optimal. Hence, $l_0^b = l_{-1}^b$ and $k_0^b = k_{-1}^b$ for the buyers, and $l_0^s = 0$, $k_0^s = k_{-1}^s$ for the sellers is an equilibrium allocation, given $q_0 = R$. Because this allocation implies that no trading takes place at $t = 0$, market clearing holds. □

Proof of Proposition 2.2 When $\gamma = \bar{\gamma}$ and $q_0 < R$, the sellers will invest all of their wealth in the long-term asset after meeting the withdrawal needs, that is,

$$k_0^s = \frac{q_0 k_{-1}^s - \bar{\gamma} d_{-1}^s}{q_0}.$$

For the buyers' problem, the Lagrangian function becomes

$$L_0^b = (1-\theta)(l_0^b + Rk_0^b) + \theta\left[u(c_1^b) + l_0^b - c_1^b + Rk_0^b\right] - \lambda_0(l_0^b + q_0 k_0^b - l_{-1}^b - q_0 k_{-1}^b) - \eta_0(c_1^b - l_0^b).$$

The necessary conditions for optimal c_1^b, l_0^b, k_0^b are

$$\theta\left(\frac{1}{c_1^b}-1\right)=\eta_0,$$
$$1+\eta_0=\lambda_0,$$
$$R=q_0\lambda_0.$$

When $q_0 < R$, those conditions imply

$$c_1^b = l_0^b = \frac{1}{\frac{R}{\theta q_0}-\frac{1}{\theta}+1} < 1.$$

The equilibrium price q_0 is then derived from the market clearing condition (9), or

$$\frac{1}{\frac{R}{\theta q_0}-\frac{1}{\theta}+1}+\bar{\gamma}d_{-1}^s = l_{-1}^b.$$

Using our assumption that $1+\bar{\gamma}d_{-1}^s > l_{-1}^b$, the market cannot clear at $q_0 = R$; hence, such a condition implies that $q_0 < R$. $\square$

Proof of Proposition 2.3 First, consider a buyer is hit by a preference shock at $t = 1$. This buyer chooses holdings of the short- and long-term liquid assets l_1^b and h_1^b to maximize $u\left(c_1^b\right)+Rh_1+l_1^b$, subject to the budget constraint (12) and the non-negativity constraint $h_1^b \geq 0$ and $l_1^b \geq 0$. Denoting η_1^b to be the Lagrange multiplier of the constraint $h_1^b \geq 0$, the first-order condition with respect to h_1^b implies

$$u'\left(c_1^b\right)p_1 = R+\eta_1^b.$$

In the state with no fire sales, we have $u'\left(c_1^b\right) = 1$, and given $p_1 = R$, we obtain that $h_1^b = 0$ is optimal and the non-negativity constraint $h_1^b \geq 0$ is not binding. In the state with fire sales, we have $u'\left(c_1^b\right) > 1$, which implies $\eta_1^b > 0$.

Next, consider a buyer not hit by a preference shock. This agent maximizes $Rh_1^b + l_1^b$ subject to the budget constraint (12) and the non-negativity constraint $h_1^b \geq 0$ and $l_1^b \geq 0$. The first-order condition is $p_1 = R$, which is satisfied in all states for the holdings $h_1^b = 0$. For the sellers, the analysis is the same as for the buyers that were not hit by a preference shock.

Next, we turn to the buyers' problem at $t = 0$:

$$\max_{l_0^b, k_0^b, h_0^b} (1-\theta)\left[Rk_0^b + l_1^b + R\frac{l_0^b + p_1 h_0^b - l_1^b}{p_1}\right] + \theta\left[\log\left(l_0^b + p_1 h_0^b - p_1 h_1^b - l_1^b\right) + Rk_0^b + Rh_1^b\right]$$

subject to the budget constraint (11). The first-order conditions evaluated at $p_1 = R$ imply

$$(1-\theta) + \theta u'\left(c_1^b\right) = \frac{p_0}{q_0},$$

and together with (5), the first-order condition holds given the equilibrium price $p_0 = R$. Finally, because all agents' holdings of h are zero at all times, the equilibrium is unchanged and market clearing holds because the asset is in zero net supply. □

Proof of Proposition 3.1 The result follows from plugging ${\partial q_0}/{\partial l_{-1}^j} = 0$ and ${\partial q_0}/{\partial k_{-1}^j} = 0$ for $j \in \{b, s\}$ for $j \in \{b, s\}$ into the regulator's first-order conditions (23) and (24), and comparing them with (20) and (17). □

Proof of Proposition 3.2 Comparing the first-order conditions of the unregulated equilibrium, (17) and (20), with that of the regulator, (23) and (24), we observe that the difference is driven by the term

$$\mathbb{E}_{-1}\left\{\left(\frac{\partial q_0}{\partial k_{-1}^b} - \frac{\partial q_0}{\partial l_{-1}^b}\right)\left(\xi\lambda_0^b - \lambda_0^s\right)\left(k_0^s - k_{-1}^s\right)\right\}$$

for the buyers and

$$\mathbb{E}_{-1}\left\{\left(\frac{\partial q_0}{\partial k_{-1}^s} - \frac{\partial q_0}{\partial l_{-1}^s}\right)\left(\xi\lambda_0^b - \lambda_0^s\right)\left(k_0^s - k_{-1}^s\right)\right\}$$

for the sellers. □

Proof of Proposition 3.3 The result follows from the standard full spanning argument given by market completeness. Alternatively, the result follows as a corollary of item (ii) of Proposition 3.4. □

Proof of Proposition 3.4 Item (i): With linear utility in time-2 consumption, the marginal utilities of wealth at $t = 0$ are $\lambda_0^s = \lambda_0^b = R/q_0$. Hence, (23) and (24) collapse to (17) and (20), using $\xi = 1$.

Item (ii): We first note that in any state in which there are no fire sales, $q_0 = R$ and the time-1 liquidity constraint of the buyers in (3) is slack. These two features imply that ${\partial q_0}/{\partial l_{-1}^b} = 0$,

${}^{\partial q_0}\!/\!{}_{\partial k^b_{-1}} = 0$, ${}^{\partial q_0}\!/\!{}_{\partial l^s_{-1}} = 0$, and ${}^{\partial q_0}\!/\!{}_{\partial k^s_{-1}} = 0$ because of an arbitrage argument. Agents can obtain a gross return of one between $t = 0$ and $t = 2$ by investing in liquidity at $t = 0$ and carrying it until $t = 2$ and, thus, the same gross return must be achieved by investing in the long-term asset, thereby requiring that $q_0 = R$. This argument holds as long as the buyers are fully insured against liquidity shocks, so that any marginal variations in the agents' portfolio holdings at $t = -1$ do not affect q_0.

With this result, it is easy to see that the first-order conditions of the regulator simplify dramatically. Let us focus on Equation (23), although the discussion also applies to Equation (24). In Equation (23), the term $\left(\frac{\partial q_0}{\partial k^b_{-1}} - \frac{\partial q_0}{\partial l^b_{-1}}\right)\left(\xi\lambda^b_0 - \lambda^s_0\right)\left(k^s_{-1} - k^s_0\right)$ is zero in all states in which there are no fire sales because $\left(\frac{\partial q_0}{\partial k^b_{-1}} - \frac{\partial q_0}{\partial l^b_{-1}}\right) = 0$. Thus, the term $\left(\frac{\partial q_0}{\partial k^b_{-1}} - \frac{\partial q_0}{\partial l^b_{-1}}\right)\left(\xi\lambda^b_0 - \lambda^s_0\right)\left(k^s_{-1} - k^s_0\right)$ is, in general, not zero only in the state in which there is a fire sale. By assumption of the proposition, fire sales occur only in one state and, thus, one can pick the Pareto weight ξ such that $\xi\lambda^b_0 - \lambda^s_0 = 0$ in that state and the optimality condition of the regulator coincides with that of the private agents, (17).

Item (iii): Recall that q_0 is determined by the market clearing condition for liquidity

$$l^b_0\left(W^b_0; q_0\right) + l^s_0\left(W^s_0; q_0\right) + \gamma d^s_0 = l^b_{-1} + l^s_{-1}, \tag{34}$$

where $l^j_0\left(W^j_0; q_0\right)$ denotes the liquidity demand of the agent, and $j \in \{b, s\}$ is a function of their wealth, W^j_0, and the price, q_0. Because the wealth effects on l^s_0 and l^b_0 are assumed to be zero, the choices made at time $t = -1$ affect q_0 only through changes in $l^b_{-1} + l^s_{-1}$ on the right-hand side of (34). This implies that ${}^{\partial q_0}\!/\!{}_{\partial k^b_{-1}} = 0$ and ${}^{\partial q_0}\!/\!{}_{\partial k^s_{-1}} = 0$, so that the first-order conditions of the regulator (23) and (24) become

$$\mathbb{E}_{-1}\left\{\xi\lambda^b_0\left(q_0 - 1\right) - \left(\frac{\partial q_0}{\partial l^b_{-1}}\right)\left[\left(\xi\lambda^b_0 - \lambda^s_0\right)\left(k^s_0 - k^s_{-1}\right)\right]\right\} = 0 \tag{35}$$

and

$$\mathbb{E}_{-1}\left\{\lambda^s_0\left(q_0 - 1\right) - \left(\frac{\partial q_0}{\partial l^s_{-1}}\right)\left[\left(\xi\lambda^b_0 - \lambda^s_0\right)\left(k^s_0 - k^s_{-1}\right)\right]\right\} = 0. \tag{36}$$

In addition, (34) implies that a change in the liquidity holdings at time $t = -1$ produces the same effect on q_0, independently of whether such a change is due to the buyers or sellers' holdings of liquidity; that is, ${}^{\partial q_0}\!/\!{}_{\partial l^b_{-1}} = {}^{\partial q_0}\!/\!{}_{\partial l^s_{-1}}$. As a result, the second term in the expectation of (35) is the same as the second term in the expectation of (36). We rewrite this term but omit the superscript:

$$\mathbb{E}_{-1}\left\{-\left(\frac{\partial q_0}{\partial l_{-1}}\right)\left(\xi\lambda_0^b-\lambda_0^s\right)\left(k_0^s-k_{-1}^s\right)\right\}. \tag{37}$$

To complete the proof, we need to show that (37) is zero for some $\xi > 0$. First, as discussed in the proof of item (ii), note that ${}^{\partial q_0}/_{\partial l_{-1}^s} = 0$ in states in which there are no fire sales. Thus, we focus on fire-sale states. In such states, using the assumption of the proposition, we have ${}^{\partial q_0}/_{\partial l_{-1}^s} > 0$ because the liquidity demand is downward sloping in its price (i.e., downward sloping in $1/q_0$) and $k_0^s - k_{-1}^s < 0$ because the sellers sell some of their asset holdings. Then, because the marginal utilities of the buyers and sellers' wealth, λ_0^b and λ_0^s, are always strictly positive, we can consider the case in which $\xi \to 0$ and ξ is sufficiently large, to obtain

$$\mathbb{E}_{-1}\left\{-\left(\frac{\partial q_0}{\partial l_{-1}}\right)\left(-\lambda_0^s\right)\left(k_0^s-k_{-1}^s\right)\right\} < 0$$

and

$$\mathbb{E}_{-1}\left\{-\left(\frac{\partial q_0}{\partial l_{-1}}\right)\left(\xi\lambda_0^b-\lambda_0^s\right)\left(k_0^s-k_{-1}^s\right)\right\} > 0,$$

respectively. Hence, by continuity, there exists ξ so that (37) is zero.□

Proof of Proposition 4.1 The result can be established as in the proof of Proposition 2.1 and by noting that, in equilibrium, the collateral constraint in Equation (25) is not binding, using the assumptions (26) and $R^H > 1$. □

Proof of Proposition 4.2 We conjecture (and later verify) that the buyers' liquidity constraint, (3), is binding. Thus, the buyers' first-order condition (5) and the time-1 log utility assumption imply

$$l_0^b = \frac{q_0\left(1-\theta\right)}{R-q_0\theta}. \tag{38}$$

Next, consider the market clearing condition for liquidity $l_0^b + l_0^s = 1$. (Recall that the buyers' liquidity holdings l_{-1}^b are normalized to one and those l_{-1}^s of the sellers are normalized to zero.) Using (29) and (38), the market clearing condition becomes

$$\frac{q_0\left(1-\theta\right)}{R-q_0\theta}+\frac{d_{-1}^l\left(1-q_0\right)}{\zeta}-1=0.$$

We then evaluate this condition at two values of q_0: $q_0 = R^L$ and $q_0 = 1 - \zeta$. The former is the upper bound on the equilibrium price. Absent fire sales, the price cannot be higher than its expected payoff, otherwise, the buyers would obtain a higher return by investing all their wealth in liquidity. The latter is the price at which the sellers' net worth is zero, that is, $q_0 k^s_{-1} - d^s_{-1} = 0$, and is derived using the endowment of the long-term asset k^s_{-1} in (26). We have

$$\left[\frac{q_0(1-\theta)}{R^L - q_0\theta} + \frac{d^l_{-1}(1-q_0)}{\zeta} - 1\right]_{q_0=R^L} = \frac{d^l_{-1}\left(1-R^L\right)}{\zeta} > 0$$

and

$$\left[\frac{q_0(1-\theta)}{R^L - q_0\theta} + \frac{d^l_{-1}(1-q_0)}{\zeta} - 1\right]_{q_0=1-\zeta} = d^l_{-1} - \frac{R^L-(1-\zeta)}{R^L-(1-\zeta)\,\theta} < 0,$$

where the first inequality uses the assumption $R^L < 1$ and the latter uses the restriction on ζ in (28). Thus, because of the continuity, there exists one price $q_0 < R^L$ such that the liquidity market clears at $t = 0$ and the sellers have a strictly positive net worth. Finally, the conjecture that the buyers' liquidity constraint is binding is verified because $q_0 < R^L < 1$ implies $l^s_0 > 0$ and, by market clearing, $l^b_0 < 1$, so that $c^b_1 < 1$. $\square$

Proof of Proposition 4.3 We prove the result in three steps.

Step 1. We begin by deriving the Pareto optimal allocation by solving the problem of a planner that has full information of individuals' realized types at both $t = 0$ and $t = 1$. At time $t = -1$, the planner stores l^p_{-1} and invests k^p_{-1} in a long-term asset. Each very impatient agent consumes c^p_0 at time $t = 0$, each impatient agent consumes c^p_1 at time $t = 1$, and each patient agent consumes c^p_2 at time $t = 2$. To maximize social welfare, the planner solves the following optimization problem:

$$\max_{l^p_{-1},c^p_0,c^p_1,c^p_2} \gamma u(c^p_0) + (1-\gamma)\theta u(c^p_1) + (1-\gamma)(1-\theta)\beta c^p_2$$

subject to the resource constraints at $t = -1$, $t = 1$, and $t = 2$:

$$\begin{aligned} l^p_{-1} + k^p_{-1} &= 1, \\ (1-\gamma)\theta c^p_1 &\leq l^p_{-1} - \gamma c^p_0, \\ (1-\gamma)(1-\theta)c^p_2 &= k^p_{-1}R + l^p_{-1} - \gamma c^p_0 - (1-\gamma)\theta c^p_1. \end{aligned}$$

The solution to the above program satisfies $u'(c^p_0) = u'(c^p_1) = 1$. Under log utility, the Pareto optimal allocation is

$$l_{-1}^p = \gamma + (1-\gamma)\theta, \quad k_{-1}^p = (1-\gamma)(1-\theta), \quad c_0^p = 1, \quad c_1^p = 1, \quad c_2^p = R.$$

Step 2. We show that the Pareto optimal allocation solves the banks' problem, given the equilibrium price $q_0 = 1$ and that the time-0 market clears. Consider the Pareto optimal allocation derived previously and the banks' problem in (30). First, given $k^p = (1-\gamma)(1-\theta)$, each patient depositor can consume the Pareto optimal amount $c_2^p = R$. Second, it is feasible for each impatient depositor to consume at the Pareto optimal level c_1^p. Third, the time-0 optimality condition (31) evaluated to $q_0 = 1$ implies $c_1^d = c_1^p = 1$, using the normalization $\beta R = 1$; hence, the Pareto optimal allocations are implemented. Fourth, given the choices at $t = 0, 1, 2$, the first-order condition at $t = -1$ is just $1/q_0 = 1$, which holds, given the equilibrium price $q_0 = 1$. Finally, note that these choices imply that banks do not engage in any trade at $t = 0$ and because depositors do not engage in any trade either, the time-0 market clears. Regarding depositors' trades, very impatient depositors do not trade because they consume all of their withdrawals, since their utility depends only on time-0 consumption and they have no market access. Non-very-impatient depositors do not trade either because they do not withdraw at $t = 0$ and, thus, have no resources to trade.

Step 3. We check that depositors truthfully report their own type. Very impatient depositors will never misreport, since they only consume at time 0. A depositor not affected by the time-0 preference shock, by misreporting, collects $c_0^d = 1$ from the bank and is able to adjust their portfolio by entering the centralized market. Given $q_0 = 1$, let l_0' be the amount of liquidity they hold, which solves

$$\max_{l_0'} \; \theta \log(l_0') + \beta \Big[(1-\theta) l_0' + (1 - l_0') R \Big]$$
$$\text{s.t. } l_0' \leq 1.$$

That is, if they are impatient (which happens with probability θ), they consume all of their liquidity l_0' at $t = 1$ and the return on their long-term asset $(1 - l_0')R$ at $t = 2$; and if they are patient, they consume l_0' plus $(1 - l_0')R$. The solution is

$$l_0' = \frac{\theta}{1 - \beta(1-\theta)},$$

which is strictly less than 1 because $\beta < 1$. The expected utility from misreporting is

$$\theta \log \frac{\theta}{1-\beta(1-\theta)} + \beta \left[(1-\theta) \frac{\theta}{1-\beta(1-\theta)} + \left(1 - \frac{\theta}{1-\beta(1-\theta)}\right) R \right] < \theta \log 1 + (1-\theta)\, \beta R,$$

where the inequality uses $l_0' < 1$ and $(1-\theta)l_0' + (1-l_0')R < (1-\theta) \times 0 + (1-0) \times R$.

Finally, note that no misreporting will occur at $t = 1$ either. An impatient depositor will never misreport because, if they do, they will have no resources at $t = 1$ and an infinite marginal utility of consumption at that point. A patient agent will not misreport either. Misreporting implies that they will receive one unit at $t = 1$ and they can store it to consume at $t = 2$, whereas reporting truthfully implies that they consume $R > 1$.□

Proof of Proposition 4.4 We start with a guess that $q_0 < 1$ and verify this later. Let l_0' be the amount of liquidity held by a depositor not affected by the time-0 preference shock, which solves

$$\max_{l_0'} \; \theta \log(l_0') + \beta \left[(1-\theta) l_0' + (1-l_0') \frac{R}{q_0} \right]$$
$$\text{s.t. } l_0' \leq w(q_0).$$

The constraint $l_0' \leq w(q_0)$ cannot be binding; otherwise, the long-term asset market cannot clear. Hence l_0' is derived from the following first-order condition:

$$l_0' = \frac{\theta}{\frac{1}{q_0} - \beta(1-\theta)}.$$

The market clearing condition for the liquid asset market is denoted as

$$\gamma w(q_0) + (1-\gamma) l_0' = l_{-1}.$$

Suppose $q_0 \geq 1$, the left-hand side of the above condition is greater than $\gamma + 1 - \gamma$, which is greater than l_{-1}. Hence, the market cannot clear, which is a contradiction. As a result, the equilibrium price q_0 has to be strictly less than 1. □

C Baseline model with general utility and general shocks

In this section, we extend the baseline model in Section 2 along two dimensions: first, the buyers and the sellers have utilities in general form; second, the aggregate shock γ follows a CDF $F(\gamma)$. The sellers choose their non-negative holdings of liquid and long-term assets at $t = 0$, $l_0^s \geq 0$ and $k_0^s \geq 0$, as well as c_0^s, c_1^s, c_2^s, to maximize $U^s(c_0^s, c_1^s, c_2^s)$, subject to the time-0, time-1, and time-2 budget constraints:

$$\begin{aligned} c_0^s + l_0^s + q_0 k_0^s &\leq q_0 k_{-1}^s - \gamma d_{-1}^s, \\ c_1^s &\leq l_0^s, \\ c_2 &\leq l_0^s - c_1^s + R k_0^s - (1-\gamma) d_{-1}^s, \end{aligned}$$

all of which will be binding. The necessary conditions for optimality are

$$\begin{aligned} U_{c_0}^s(q_0 k_{-1}^s - \gamma d_{-1}^s - l_0^s - q_0 k_0^s, l_0^s, R k_0^s - (1-\gamma) d_{-1}^s) &= \lambda_0, \\ U_{c_1}^s(q_0 k_{-1}^s - \gamma d_{-1}^s - l_0^s - q_0 k_0^s, l_0^s, R k_0^s - (1-\gamma) d_{-1}^s) &= \lambda_0, \\ R U_{c_2}^s(q_0 k_{-1}^s - \gamma d_{-1}^s - l_0^s - q_0 k_0^s, l_0^s, R k_0^s - (1-\gamma) d_{-1}^s) &= \lambda_0 q_0. \end{aligned}$$

Let the solution be $c_0^s(\gamma, q_0), l_0^s(\gamma, q_0)$, and $k_0^s(\gamma, q_0)$. We assume U^s satisfies conditions such that $l_0^s(\gamma, q_0)$ is weakly decreasing in γ.[17]

The buyers have utility $U^b(c_0, c_1, c_2, \epsilon)$, where ϵ denotes the preference shock; that is, when $\epsilon = 0$, the buyers do not value c_1. As in Section 2, $\epsilon = 0$ with probability $1 - \theta$ and $\epsilon = 1$ with probability θ. The time-0 budget constraint becomes

$$c_0^b + l_0^b + q_0 k_0^b \leq l_{-1}^b + q_0 k_{-1}^b.$$

[17]To specify, one can solve $\frac{\partial l_0^s}{\partial \gamma}$ and $\frac{\partial k_0^s}{\partial \gamma}$ from the following three conditions:

$$\begin{aligned} -U_{00}\left(d_{-1}^s + \frac{\partial l_0^s}{\partial \gamma}\right) + q_0 \frac{\partial k_0^s}{\partial \gamma} + U_{01} \frac{\partial l_0^s}{\partial \gamma} + U_{02}\left(R \frac{\partial k_0^s}{\partial \gamma} + d_{-1}^s\right) &= \frac{\partial \lambda}{\partial \gamma}, \\ -U_{10}\left(d_{-1}^s + \frac{\partial l_0^s}{\partial \gamma}\right) + q_0 \frac{\partial k_0^s}{\partial \gamma} + U_{11} \frac{\partial l_0^s}{\partial \gamma} + U_{12}\left(R \frac{\partial k_0^s}{\partial \gamma} + d_{-1}^s\right) &= \frac{\partial \lambda}{\partial \gamma}, \\ -U_{20}\left(d_{-1}^s + \frac{\partial l_0^s}{\partial \gamma}\right) + q_0 \frac{\partial k_0^s}{\partial \gamma} + U_{21} \frac{\partial l_0^s}{\partial \gamma} + U_{22}\left(R \frac{\partial k_0^s}{\partial \gamma} + d_{-1}^s\right) &= \frac{\partial \lambda}{\partial \gamma}, \end{aligned}$$

where U_{ij} denotes for $U_{c_i c_j}$. We assume $\frac{\partial l_0^s}{\partial \gamma}$ as the solution to be non-positive for all γ.

When $q_0 = R$, as in the proof of Proposition 2.1, the necessary conditions for optimal $c_0^b, c_1^b, l_0^b, k_0^b$ in the buyers' problem are

$$(1-\theta)U^b_{c_0}(c_0^b, l_0^b - c_1^b + Rk_0^b|\epsilon = 0) + \theta U^b_{c_0}(c_0^b, c_1^b, l_0^b - c_1^b + Rk_0^b|\epsilon = 1) = \lambda_0$$
$$\theta\Big[U^b_{c_1}(c_0^b, c_1^b, l_0^b - c_1^b + Rk_0^b|\epsilon = 1) - U^b_{c_2}(c_0^b, c_1^b, l_0^b - c_1^b + Rk_0^b|\epsilon = 1)\Big] = \eta_0,$$
$$(1-\theta)U^b_{c_2}(c_0^b, l_0^b - c_1^b + Rk_0^b|\epsilon = 0) + \theta U^b_{c_2}(c_0^b, c_1^b, l_0^b - c_1^b + Rk_0^b|\epsilon = 1) + \eta_0 = \lambda_0,$$
$$R\Big[(1-\theta)U^b_{c_2}(c_0^b, l_0^b - c_1^b + Rk_0^b|\epsilon = 0) + \theta U^b_{c_2}(c_0^b, c_1^b, l_0^b - c_1^b + Rk_0^b|\epsilon = 1)\Big] = R\lambda_0.$$

The sellers choose $c_0^s(\gamma, R), l_0^s(\gamma, R)$, and $k_0^s(\gamma, R)$, and the buyers choose $l_0^b = l_{-1}^b - l_0^s(\gamma R) - \gamma d_{-1}^s$ and $k_0^b = k_{-1}^b + k_{-1}^s - k_0^s(\gamma, R)$ according to market clearing conditions and c_0^b and c_1^b, satisfying the intertemporal optimal conditions

$$U^b_{c_1}(c_0^b, c_1^b, l_0^b - c_1^b + Rk_0^b|\epsilon = 1) = U^b_{c_2}(c_0^b, c_1^b, l_0^b - c_1^b + Rk_0^b|\epsilon = 1),$$
$$(1-\theta)U^b_{c_0}(\epsilon = 0) + \theta U^b_{c_0}(\epsilon = 1) = (1-\theta)U^b_{c_1}(\epsilon = 0) + \theta U^b_{c_1}(\epsilon = 1).$$

Note that $l_0^b = l_{-1}^b - l_0^s(\gamma, R) - \gamma d_{-1}^s$ is decreasing in γ. Similar to the sellers, we assume that U^b satisfies conditions such that $c_1^b(\gamma)$ increases in γ. Let $\bar{\gamma}$ be such that $c_0^b(\bar{\gamma})$ and $c_1^b(\bar{\gamma})$ satisfy the above two conditions and $c_1^b(\bar{\gamma}) = l_0^b(\bar{\gamma})$; then the equilibrium with $q_0 = R$ exists when $\gamma \leq \bar{\gamma}$.

On the other hand, for each $\gamma > \bar{\gamma}$, we show that an equilibrium with $q_0 < R$ exists. When this is true, $\eta_0 > 0$, implying $c_1^b(\gamma, q_0) = l_0^b(\gamma, q_0)$. The price q_0 is then derived from the market clearing condition

$$l_0^s(\gamma, q_0) + \gamma d_{-1}^s + l_0^b(\gamma, q_0) = l_{-1}^b,$$

as in the proof of Proposition 2.1. We assume U^s and U^b are such that $l_0^s(\gamma, q_0)$ is weakly increasing in both γ and q_0, and $l_0^b(\gamma, q_0)$ is increasing in both γ and q_0, so that

$$l_0^s(\gamma, R) + \gamma d_{-1}^s + l_0^b(\gamma, R) > l_{-1}^b \text{ for all } \gamma,$$

and that

$$l_0^s(\gamma, 0) + \gamma d_{-1}^s + l_0^b(\gamma, 0) < l_{-1}^b \text{ for all } \gamma.$$

This guarantees that a unique solution, $q_0 \in (0, R)$, exists by continuity. In addition, the market clearing condition implies that

$$q_0'(\gamma) = -\frac{\frac{\partial l_0^s}{\partial \gamma} + \frac{\partial l_0^b}{\partial \gamma} + d_{-1}^s}{\frac{\partial l_0^s}{\partial q_0} + \frac{\partial l_0^b}{\partial q_0}} < 0.$$

That is, the higher the reselling pressure, the lower the fire-sale price becomes.

D Time -1 investments in the baseline model with two withdrawal shocks

In this appendix, we characterize in closed form the choice of the agents' investments in liquidity and long-term assets under the functional forms of the utility functions and the CDF $F(\cdot)$ that we used in the baseline model of Section 2. In particular, we assume that the shock γ can take two values, $\gamma \in \{0, \bar{\gamma}\}$, with probability $1-\pi$ and π, respectively.

In the model of Section 2, the time-0 marginal utilities of the buyers and sellers' wealth is the same and given by $\lambda_0^b = \lambda_0^s = R/q_0$. Thus, when $\gamma = 0$ and $q_0 = R$, we have $\lambda_0^b = \lambda_0^s = 1$, and when $\gamma = \bar{\gamma}$, we have $\lambda_0^b = \lambda_0^s = R/q(\bar{\gamma})$, where $q(\bar{\gamma})$ denotes the time-0 price in the state in which $\gamma = \bar{\gamma}$. Thus, the first-order condition (17) of the buyers—or equivalently the first-order condition (20) of the sellers—can be rewritten as

$$(1-\pi)(R-1) + \pi \frac{R}{q(\bar{\gamma})}(q(\bar{\gamma}) - 1) = 0$$

which implies

$$q(\bar{\gamma}) = \frac{\pi R}{R - (1-\pi)}.$$

Given this price, both the sellers and the buyers are indifferent regarding their holdings of liquidity and long-term assets in their portfolios at time $t = -1$. We can then normalize $l_{-1}^s = 0$ as we did in Section 2, and the buyer's liquidity choice l_{-1}^b is determined from the market clearing condition in the fire-sale state (i.e., when $\gamma = \bar{\gamma}$):

$$\begin{aligned} l_{-1}^b &= \frac{1}{\frac{R}{\theta q(\bar{\gamma})} - \frac{1}{\theta} + 1} + \bar{\gamma} d_{-1}^s \\ &= \frac{\theta \pi}{R - 1 + \theta \pi} + \bar{\gamma} d_{-1}^s. \end{aligned}$$

Finally, the buyers and sellers' investments in the long-term asset are determined by their budget constraints at $t = -1$, that is, (13) and (14), so that $k^b_{-1} = e^b - \frac{\theta\pi}{R-1+\theta\pi} - \bar{\gamma} d^s_{-1}$ and $k^s_{-1} = e^s + d^s_{-1}$.

E Collateral constraint and limited commitment

In this appendix, we derive the collateral constraint in Equation (25) by appealing to a limited commitment friction and following Lorenzoni (2008). Assume that a fraction α of debt d^s_{-1} is due at the end of time $t = 0$. If a seller fails to pay, it can make a take-it-or-leave-it offer to the debt holders. If the debt holder rejects the offer, liquidation takes place, which destroys a fraction ζ of the capital holdings. The rest of the capital (which is valued according to the price q_0) and the liquidity go to the debt holders. The resulting constraint is

$$d^s_{-1} \leq (1 - \zeta)\, q_0 k^s_0 + l^s_0.$$

Using the time-0 seller's budget constraint $q_0 k^s_0 + l^s_0 + \alpha d^s_{-1} \leq q_0 k^s_{-1} + l^s_{-1}$, taking the limit as $\alpha \to 0$ and rearranging, we recover the constraints in Equation (25).

Regarding the collateral constraint at time $t = -1$ in Equation (27), one can proceed along the lines of the time-0 constraint. That is, assume that a fraction α_{-1} of debt d^s_{-1} is due at the end of $t = -1$ (i.e., after the $t = -1$ investments have been made), that the seller can make a take-it-or-leave-it offer to all of the debt holders, and that if the debt holders reject the offer, liquidation takes place, which destroys a fraction ζ of the capital holdings. One can then take α_{-1} to be arbitrarily close to zero.

F Limited market access and creation of new banks at $t = 0$

Section 4.2 derived a run equilibrium with fire sales driven by our novel liquidity risk pricing. Two assumptions are imposed to simplify the exposition, but they are arguably somewhat restrictive. First, the time-0 preference shock is perfectly correlated with the market access shock, and agents cannot trade at $t = 0$ if and only if they are very impatient. Second, banks can only be established at $t = -1$ and, thus, new banks cannot be established at $t = 0$ even though agents can gather in a centralized market at that time.

We now consider an extension with less-restrictive assumptions.

(i) The time-0 preference shock is uncorrelated with the market access shock. Specifically, only a fraction $p < 1$ of depositors has access to the centralized market. The remaining $1 - p$ can either store the amount withdrawn from banks in the form of the liquid asset or consume it.

(ii) New banks can be created at $t = 0$.

With respect to (ii), we note that the ability to create banks at $t = 0$ captures the idea that some depositors may be able to transfer their resources to other parts of the financial sector during a run, thereby obtaining full or near-full insurance against their residual liquidity risk. This possibility would alter depositors' pricing kernel and limit the fire-sale mechanism that we analyzed.[18]

Despite the weaker assumptions, the results of the good and bad equilibria are qualitatively unchanged, as formalized by the next proposition.

Proposition F.1. *Assume (i) and (ii) hold. Two equilibria exist: a good equilibrium that implements the first best and a post-deposit run equilibrium in which all depositors withdraw $w(q) < 1$ at $t = 0$. In the run equilibrium, the long-term asset is traded at a fire-sale price $q_0 < 1$.*

Proof. For the good equilibrium, the argument about $q_0 = 1$ still follows. We only need to check the incentive compatibility of the depositors not hit by the preference shock at time 0. If the depositor has access to the centralized market, the IC is the same as in the good equilibrium in the main part of the paper. If the depositor has no market access, the one unit of liquidity withdrawn delivers an expected utility given by

$$\theta \log 1 + (1 - \theta)\beta,$$

which is less than the expected utility the depositor receives by not withdrawing, which is $\theta \log 1 + (1 - \theta)\beta R$.

For the bank run equilibrium, in the case of autarchy, agents that are not affected by the preference shock at time 0 withdraw $w(q_0)$ from the old bank and receive an expected utility of

$$\theta \log w(q_0) + \beta w(q_0).$$

These agents that successfully access the centralized market can create a new bank that provides

[18] If all of the depositors can join new banks at $t = 0$, the bad equilibrium does not exist.

insurance against liquidity risk. The new bank collects $p(1-\gamma)w(q_0)$ from these agents and solves

$$\max_{\hat{l}_0,\hat{k}_0,\hat{c}_1,\hat{c}_2} \; p(1-\gamma)\theta\log(\hat{c}_1) + p(1-\gamma)(1-\theta)\beta\hat{c}_2$$

subject to

$$\begin{aligned} \hat{l}_0 + q_0\hat{k}_0 &\leq p(1-\gamma)w(q_0) \\ p(1-\gamma)\theta\hat{c}_1 &= \hat{l}_0, \\ p\,(1-\gamma)\,(1-\theta)\hat{c}_2 &= R\,\hat{k}_0. \end{aligned}$$

The solution for the liquidity holdings is

$$\hat{l}_0 = p(1-\gamma)\theta q_0.$$

Given this result, the market clearing condition for this liquidity is now denoted as

$$\gamma w(q_0) + (1-p)(1-\gamma)w(q_0) + p(1-\gamma)\theta q_0 = \gamma + (1-\gamma)\theta. \tag{39}$$

The first term in (39) is the aggregate demand from the very impatient buyer types. The second term is the demand from the remaining agents, which are in autarchy. The third term is the demand from the newly established bank. Similar to the case with full participation, it can be shown that $q_0 < 1$ in this equilibrium. If $q_0 \geq 1$, the left-hand side of (39) is greater than the right-hand side because

$$\begin{aligned} &\gamma w(q_0) + (1-p)(1-\gamma)w(q_0) + p(1-\gamma)\theta q_0 \\ \geq&\gamma + (1-p)(1-\gamma) + p(1-\gamma) \\ >&\gamma + (1-\gamma)\theta, \end{aligned}$$

and the market cannot clear. Thus, $q_0 < 1$. $\square$