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Risk and State-Dependent Financial Frictions

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Abstract

We augment a standard New Keynesian model with a financial accelerator mechanism and show that financial frictions generate large state-dependent amplification effects. We fit the model to US data and show that, when shocks drive the model far away from the steady state, the nonlinear model produces much stronger propagation of shocks than the linearized model. We document that these amplification effects are due to endogenous variation in financial conditions and not due to other nonlinearities in the model. Motivated by these findings, we propose a regime-switching dynamic stochastic general equilibrium framework where financial frictions endogenously fluctuate between moderate (low risk) and severe (high risk), depending on the state of the economy. This framework allows for efficient estimation with many state variables and improves fit with respect to the linear model.

Topics: Central bank research, Credit and credit aggregates, Financial stability, Monetary policy
JEL codes: E52, E58

1 Introduction

Since the Great Recession in 2007–2009, much research has focused on incorporating financial factors into macro models (Gertler and Gilchrist 2018, Christiano et al. 2018). In parallel, several empirical studies have shown that changing financial conditions alter how the financial sector affects the real economy (e.g., Adrian et al. 2019, Brunnermeier 2009, Hubrich and Tetlow 2015, Barnichon et al. 2018, Prieto et al. 2016). In particular, financial frictions tend to amplify the effects of macroeconomic shocks during periods of financial distress.

In this paper, we study the role of state-dependent financial frictions in a medium-sized New Keynesian model of the business cycle. New Keynesian models with financial frictions (NK-FF) have become a fundamental policy tool for central banks.[1] At the same time, they have been heavily criticized in the years after the Great Recession (Christiano et al. 2018). Two popular critiques are the following: (1) because NK-FF models take an overly simplified approach to modeling financial intermediaries, they fail to take into account the crucial role of financial factors for business cycle dynamics; and (2) because they are often linearized, they are unable to take into account the highly nonlinear dynamics of the financial sector.

Against this background, we address two questions in this paper. First, we ask whether financial frictions in a standard nonlinear NK-FF model generate a large amplification of shocks in macro and financial variables, as found in empirical studies. This relates directly to assessing the costs of linearizing these models for empirical analysis. And second, we investigate how to introduce these nonlinear dynamics into a framework that allows for efficient estimation.

Our contribution is twofold. First, we show that the cost of ignoring state-dependent effects of financial frictions is substantial. We rely on the NK-FF model by Christiano et al. (2014) and use a higher-order perturbation solution to investigate the extent to which nonlinear effects of financial frictions matter empirically. We document that the model generates large amplification effects in periods of financial distress or high risk and generates mild amplification effects in periods of financial tranquility or low risk. Second, we propose a regime-switching dynamic stochastic general equilibrium (DSGE) model in which the economy fluctuates endogenously between low-risk and high-risk states, allowing for state-dependent effects of financial frictions and for efficient estimation with many state variables.

We start by investigating the empirical relevance of the state-dependent financial frictions in the NK-FF model. We fit the model to US data and show that the nonlinear model produces much stronger propagation of shocks than its linearized version. We find

[1] See, e.g., Coenen et al. (2012) and Lindé et al. (2016) for a description and comparisons of the workhorse models used by central banks.

that output and investment drop by an additional 60% and 65% respectively during the Great Recession in the nonlinear model, while consumption drops by an additional 10% and the credit spread of the corporate bond jumps an additional 5.2 (annualized) percentage points. Importantly, we document that the bulk of these amplification effects are driven by variation in financial conditions, and not by other nonlinearities in the model. By contrast, amplification is almost absent throughout the 1980s and 1990s, which is consistent with the view that linear models provided a relatively good approximation of US business cycles during the Great Moderation. These two facts combined highlight the importance of allowing for state-dependent financial frictions in macroeconomic models.

We then propose a regime-switching DSGE framework where financial frictions fluctuate endogenously between moderate (low risk) and severe (high risk) depending on the state of the economy. We model the probability of switching from one state to the other as a function of the credit spread of the corporate bond. This allows us to discipline the nonlinear effects of financial frictions in the model with a measure of financial conditions in the data. We solve the regime-switching model using perturbation methods following Maih (2015), which gives us a key efficiency advantage with respect to projection methods. We then illustrate how this framework, combined with the filter proposed by Chang et al. (2018), can be used to efficiently estimate the New Keynesian model with state-dependent financial frictions. We demonstrate this using both simulated data and historical US data.

First, we generate data from the nonlinear NK-FF model and fit three models to these data: a linearized NK-FF model; a regime-switching NK-FF model with constant switching probabilities, where the switching follows an exogenous process; and a regime-switching NK-FF model with time-varying switching probabilities that are endogenous to financial conditions. Using model fit as a benchmark, we show that both regime-switching models greatly outperform the linear model and, most importantly, that the endogenous switching model outperforms the exogenous switching model. This is because, on average, high-risk states coincide with high spreads. By incorporating this information explicitly, the endogenous probability model produces better one-step-ahead forecasts when evaluating the likelihood function, which results in improved fit. Put differently, model fit improves as a result of the improved probabilistic assessment about when financial frictions matter most. Ultimately, both model fit and its determinants are of interest for policymakers.

We then fit the endogenous regime-switching model to US data and show that the transition to high-risk regimes is crucial to account for periods of financial stress, especially the Great Recession.

Alternatives to this approach include evaluating the nonlinear model, solved with either higher-order perturbation methods or fully nonlinear projection methods, using a particle filter. However, a particle filter is computationally more costly than our implementation of the regime-switching filter. Moreover, while both these solutions take the nonlinear nature of the financial contract into account, they do not exploit the extra flexibility implied by the time-varying nature of parameters and equilibria in our regime-switching framework.

We work with the New Keynesian model proposed by Christiano et al. (2014) (henceforth, CMR). The basic structure follows Christiano et al. (2005) and Smets and Wouters (2007), while financial frictions are introduced as in Bernanke et al. (1999) (henceforth, BGG). We choose this approach for two main reasons. First, several influential central banks have built their DSGE models on this structure.[2] Second, recent studies have highlighted the empirical relevance of this framework, in terms of both explaining the business cycle (CMR) and forecasting performance (Del Negro et al. 2015, Del Negro et al. 2016, Del Negro and Schorfheide 2013, Cai et al. 2018).

In the BGG model, an agency problem between financial intermediaries and productive firms gives rise to a premium for external finance. When firms' balance sheets weaken, the premium increases and real activity slows down, which has a further negative effect on borrowers' financial health, increasing the premium further, and so on. This is BGG's *financial accelerator* and its size is determined by the sensitivity of the premium to firm leverage. Crucially, this sensitivity is increasing in entrepreneurs' risk, defined as the variance of idiosyncratic productivity shocks faced by entrepreneurs.[3] Using this definition of risk, we exploit this relation to model state-dependent financial frictions.

Contribution to the literature Recent studies have provided empirical evidence of asymmetric effects of financial shocks and frictions on the real economy. Adrian et al. (2019) document a nonlinear relationship between financial conditions and the conditional distribution of GDP growth. They argue that DSGE models with financial frictions should therefore allow for nonlinear equilibrium relationships. Hubrich and Tetlow (2015) use a regime-switching vector autoregression model to show that the model that best explains the Great Recession features both changes in shock variances and in the parameters ruling the transmission of shocks. In a related study, Alessandri and Mumtaz (2017) indicate the presence of a regime change during the Great Recession. Barnichon et al. (2018) empirically document that financial shocks have asymmetric effects on the real economy, while Prieto et al. (2016) provide evidence of time-varying linkages between the financial sector and the macroeconomy. We build on this body of empirical evidence to develop a DSGE model that takes similar state-dependent dynamics into account.

[2]Policy institutions that use a New Keynesian model with financial frictions as in BGG for policy analysis include the IMF (GIMF model), the Federal Reserve Board (SIGMA model), the European Central Bank (New Area Wide Model), the Federal Reserve Bank of New York (FRBNY-DSGE model), and the Riksbank (Ramses II model), among others.

[3]The empirical relevance of this concept of risk for the business cycle goes back to Bloom (2009).

We also contribute to the literature that has analyzed developments in DSGE models before and after the 2008 financial crisis. Christiano et al. (2018) revise this literature and conclude that financial frictions in pre-crisis DSGE models seem to have only small quantitative effects, an observation that goes back to Kocherlakota (2000). Importantly, most studies discussed there consider linearized versions of NK-FF models, thereby neglecting the potential state-dependent effects of financial frictions over the business cycle. Other studies (e.g., Brunnermeier and Sannikov 2014, He and Krishnamurthy 2014) have shown that nonlinear models with financial frictions can generate large amplification effects. Our results provide additional evidence supporting the view that it is important to take nonlinear model dynamics into account for business cycle analysis, even in the pre-crisis generation of models.

Additionally, there is a growing literature studying the nonlinear effects of financial frictions in DSGE models. On the one hand, several papers have used New Keynesian models with occasionally binding constraints and regime switching to study the effects of different types of nonlinear financial constraints (Holden et al. 2018, Pietrunti 2017, Maria and Júlio 2018, Guerrieri and Iacoviello 2017, Bluwstein 2017, Lindé et al. 2016). Some of these papers take an empirical approach. For instance, Guerrieri and Iacoviello (2017) estimate a NK-FF model with an occasionally binding collateral constraint that captures the boom-bust dynamics observed in the US housing market in 2001–2009. Bluwstein (2017) documents that financial busts are more procyclical than booms and estimates a DSGE model with banks that face an occasionally binding borrowing constraint to explain this finding. More generally, the papers by Lindé et al. (2016), Del Negro et al. (2016), and Del Negro and Schorfheide (2013) have shown that allowing for time variation in the effects of financial frictions improves the forecasting performance of DSGE models.

On the other hand, various papers have used smaller nonlinear models that include important features of the financial sector, such as the endogenous buildup of financial risk and the asymmetric effects of financial constraints in normal times and in periods of financial distress (Adrian and Boyarchenko 2012, He and Krishnamurthy 2014, Brunnermeier and Sannikov 2014, Mendoza 2010) to show that financial frictions generate large amplification effects on macroeconomic variables. Many of these features are yet to be introduced to the models used by central banks. Importantly, due to the high computational burden involved in solving these models, they are typically much smaller than the standard NK-FF model. For the same reason, estimation of nonlinear DSGE models becomes computationally challenging (see, e.g., Gust et al. 2017). We contribute to this literature by combining elements of these two strands to develop a framework that features state-dependent financial frictions with time-varying risk in an otherwise standard NK-FF model and that allows for efficient estimation with many state variables.

Outline The paper is organized as follows. Section 2 describes the NK-FF model and discusses the nonlinear dynamics in the financial sector. Section 3 provides the details about the estimation and documents the quantitative effects of state-dependent financial frictions. Section 4 introduces the regime-switching DSGE model and discusses the estimation results. The last section concludes.

2 Model

We augment a standard New Keynesian model with the BGG financial accelerator following CMR. Absent financial frictions, the building blocks are similar to the well known models by Christiano et al. (2005) and Smets and Wouters (2007). In the following we present the main features of the model.

2.1 Goods Production

Final goods producers take intermediate goods, $Y_{jt},\ j \in [0,1]$, to produce a homogeneous good, Y_t, using the Dixit-Stiglitz technology:

$$Y_t = \left[\int_0^1 Y_{jt}^{\frac{1}{\lambda_{f,t}}} dj \right]^{\lambda_{f,t}}, \quad 1 \leq \lambda_{f,t} < \infty, \tag{1}$$

where $\lambda_{f,t}$ is a price-markup shock. The intermediate goods producer is a monopolist with technology

$$Y_{jt} = \begin{cases} \varepsilon_t K_{jt}^{\alpha}(z_t l_{jt})^{1-\alpha} - \Phi z_t^* & \text{if } K_{jt}^{\alpha}(z_t l_{jt})^{1-\alpha} > \Phi z_t^* \\ 0 & \text{otherwise} \end{cases}, \tag{2}$$

where $0 < \alpha < 1$ and ε_t is a transitory technology shock. z_t^* is a shock with a stationary growth rate with the property that Y_t/z_t^* converges to a constant in the non-stochastic steady state. Each firm supplies Y_{jt} at price P_{jt} and is subject to Calvo-style price rigidities, so that in each period only a random fraction $(1 - \xi_p)$ can re-optimize their price. The remaining fraction sets a price $P_{jt} = \tilde{\pi}_t P_{j,t-1}$, where

$$\tilde{\pi}_t = (\pi_t^{target})^{\iota}(\pi_{t-1})^{1-\iota}. \tag{3}$$

Here, $\pi_{t-1} \equiv P_{t-1}/P_{t-2}$ and π_t^{target} is the target inflation rate. Homogeneous goods can be converted to consumption goods, C_t, at a one-to-one rate. Alternatively, one homogeneous good can be converted to $\Upsilon^t \mu_{\Upsilon,t}$ investment goods, where $\Upsilon > 1$ and $\mu_{\Upsilon,t}$ is a shock. Perfect competition in these markets implies that the prices of consumption and investment goods are P_t and $P_t/(\Upsilon^t \mu_{\Upsilon,t})$, respectively. The trend rise in technology for producing investment goods is the second source of growth in the model, and $z_t^* = z_t \Upsilon^{\left(\frac{\alpha}{1-\alpha}\right)t}$.

2.2 Labor Market

As in the goods market, the labor market features a representative, competitive labor contractor that aggregates differentiated labor services, $h_{i,t}$, $i \in [0, 1]$, into homogeneous labor, l_t, using the Dixit-Stiglitz technology with production function

$$l_t = \left[\int_0^1 (h_{t,i})^{\frac{1}{\lambda_w}} di \right]^{\lambda_w}, \quad 1 \leq \lambda_w. \tag{4}$$

It then sells labor l_t to intermediate good producers at the nominal wage W_t. For each labor type, a monopoly union sets the wage rate $W_{i,t}$, subject to Calvo-style frictions. Hence, only a fraction $(1 - \xi_w)$ set their wage optimally while the remaining firms set their wage according to $W_{i,t} = (\mu_{z^*,t})^{\iota_\mu} (\mu_{z^*})^{1-\iota_\mu} \tilde{\pi}_{w,t} W_{i,t-1}$, where μ_{z^*} is the steady-state growth rate of z_t^* and

$$\tilde{\pi}_{w,t} \equiv (\pi_t^{target})^{\iota_w} (\pi_{t-1})^{1-\iota_w}, \quad 0 < \iota_w < 1. \tag{5}$$

2.3 Households

Each household contains every type of differentiated labor and a large number of entrepreneurs. Households also act as capital producers in the economy. Capital is produced according to the technology

$$\bar{K}_{t+1} = (1 - \delta)\bar{K}_t + (1 - S(\zeta_{I,t} I_t / I_{t-1})) I_t. \tag{6}$$

Households buy investment I_t to produce new capital subject to investment adjustment costs embodied in S, which is an increasing and concave function that we characterize below. $\zeta_{I,t}$ is a shock to the marginal efficiency of investment. In addition, households buy the existing stock of capital at price $Q_{\bar{K},t}$ and sell new capital at the same price.

Households' preferences are given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \zeta_{C,t} \left\{ \log(C_t - bC_{t-1}) - \psi_L \int_0^1 \frac{h_{i,t}^{1+\sigma_L}}{1 + \sigma_L} di \right\} \quad b, \sigma_L > 0, \tag{7}$$

where $\zeta_{C,t}$ is a preference shock and C_t represents per capita consumption of the household. The associated budget constraint reads

$$\begin{aligned} (1 + \tau^c) P_t C_t + B_{t+1} + \frac{P_t}{\Upsilon \mu_{\Upsilon,t}} I_t + Q_{\bar{K},t}(1 - \delta)\bar{K}_t \leq \\ (1 - \tau^l) \int_0^1 W_t^i h_{i,t} di + R_t B_t + Q_{\bar{K},t} \bar{K}_{t+1} + \Pi_t. \end{aligned} \tag{8}$$

Here Π_t stands for lump-sum payments including firm profits, transfers from entrepreneurs, and lump-sum transfers from the government. B_{t+1} is a one period bond that pays returns R_t, while τ^c and τ^l are exogenous tax rates. This budget constraint ensures that the sum of expenditures in consumption goods, new deposits, and purchases of investment goods and capital (left-hand side) does not exceed the household's income from labor, returns on deposits, revenues from selling capital, and lump-sump payments (right-hand side). In equilibrium, it holds with equality.

2.4 Financial Frictions

Financial frictions are added in the form of the standard BGG contract. As emphasized by Christiano et al. (2018), financial frictions can be broadly categorized in two groups: those arising inside financial institutions (theories of bank runs and rollover crises) and those arising between financial institutions and the people that borrow from them (theories of collateral-constrained borrowers). This model is of the latter type.

Following CMR, we index entrepreneurs by their level of net worth $N \geq 0$ and call each of them an N-type entrepreneur. If we denote the density of entrepreneurs with net worth N as $f_t(N)$, then the aggregate net worth in the economy is given by

$$N_{t+1} = \int_0^\infty N f_t(N)\, dN. \tag{9}$$

Each period, an N-type entrepreneur obtains a loan B^N_{t+1} at rate R^L_t and combines it with its own net worth N to buy raw capital $\bar{K}^N_{t+1}$ at the competitive price $Q_{\bar{K},t}$. Thus, her balance sheet is $B^N_{t+1} + N = Q_{\bar{K},t}\bar{K}^N_{t+1}$. After buying capital, she faces an idiosyncratic shock ω that converts $\bar{K}^N_{t+1}$ into $\omega\bar{K}^N_{t+1}$ units of effective capital. ω follows a log-normal distribution with a mean of one and a standard deviation given by σ_t, such that σ_t characterizes the cross-sectional dispersion in ω and, as in CMR, we interpret as a *risk shock*. After observing rates of return and prices, entrepreneurs decide what utilization rate u^N_{t+1} of effective capital units they supply to a competitive market at rate r^k_{t+1}. At the end of period $(t+1)$ each entrepreneur obtains a stochastic return ωR^k_{t+1}, regardless of her net worth, where

$$R^k_{t+1} = \frac{(1-\tau^k)[u_{t+1}r^k_{t+1} - a(u_{t+1})]\Upsilon^{-(t+1)}P_{t+1} + (1-\delta)Q_{\bar{K},t+1} + \tau^k\delta Q_{\bar{K},t}}{Q_{\bar{K},t}}. \tag{10}$$

Here, τ^k is an exogenous tax rate on capital income and a is an increasing and concave function that captures the costs of capital utilization.

The loan that each entrepreneur obtains in period t takes the form of a standard debt contract (R^L_t, L_t), where $L_t \equiv (N + B^N_{t+1})/N$ stands for leverage. Let $\bar{\omega}_t$ be the threshold value under which an entrepreneur cannot repay her loan, such that

$$\bar{\omega}_{t+1} = \frac{R^L_{t+1} B^N_{t+1}}{R^k_{t+1} Q_{\bar{K},t} \bar{K}^N_{t+1}}. \tag{11}$$

Entrepreneurs with $\omega < \bar{\omega}_{t+1}$ default on their loan, in which case financial intermediaries pay a monitoring cost equal to a fraction μ of the entrepreneur's assets and keep all that is left. Hence, the expected value of a loan for an entrepreneur can be written as

$$E_t \int_{\varpi_{t+1}}^{\infty} \left[R^k_{t+1} \omega Q_{\bar{K},t} \bar{K}^N_{t+1} - R^L_{t+1} B_{t+1}\right] dF(\omega, \sigma) = E_t[1 - \Gamma_t(\bar{\omega}_{t+1})] R^k_{t+1} L_t N, \tag{12}$$

with $\Gamma_t(\bar{\omega}_{t+1}) \equiv [1 - F(\bar{\omega}_{t+1})]\bar{\omega}_{t+1} + G_t(\bar{\omega}_{t+1})$ and $G_t \equiv \int_0^{\bar{\omega}_{t+1}} \omega dF_t(\omega)$.

In order to extend loans to entrepreneurs, financial intermediaries issue deposits to households at the competitive rate R_t. The fact that the relevant rate on these deposits is the risk-free rate reflects that the market for funds between households and financial intermediaries is frictionless. This rate is not contingent in $t + 1$ uncertainty, so in order for financial intermediaries to participate in the market, their expected return must be at least R_t:

$$[1 - F(\varpi_{t+1})] R^L_{t+1} B^N_{t+1} + (1 - \mu) \int_0^{\varpi_{t+1}} \omega dF_t(\omega) R^k_{t+1} Q_{\bar{K},t} \bar{K}^N_{t+1} \geq R_t B^N_{t+1}. \tag{13}$$

Free entry of financial intermediaries guarantees that they make zero profits in equilibrium, which implies that equation (13) effectively holds with equality in equilibrium. Combining equations equations (11) and (13) we can write

$$\frac{R^k_{t+1}}{R_t} = \frac{1}{\Gamma_t(\bar{\omega}_{t+1}) - \mu G_t(\bar{\omega}_{t+1})} \left(1 - \frac{1}{L_t}\right). \tag{14}$$

The $(\bar{\omega}_{t+1}, L_t)$ combinations that satisfy equation (14) determine a set of state $(t + 1)$-contingent standard debt contracts that are available for entrepreneurs. Entrepreneurs maximize their objective function (equation (12)) subject to this menu of contracts. Note that the $(\bar{\omega}_{t+1}, L_t)$ decision is independent of N. In fact, capital purchases of each entrepreneur are proportional to the entrepreneur's net worth, with a proportionality factor that is increasing in the expected discounted return to capital. We define the expected discounted return to capital $s_t \equiv E_t(R^k_{t+1}/R_t)$. Then, we can write

$$Q_{\bar{K},t} \bar{K}^N_{t+1} = \psi(s_t) N, \quad \text{with } \psi(1) = 1,\ \psi'(\cdot) > 0. \tag{15}$$

Since $Q_{\bar{K},t} \bar{K}^N_{t+1}$ are the entrepreneur's assets, it follows that $L_t = (Q_{\bar{K},t} \bar{K}^N_{t+1})/N$ or $L_t = \psi(s_t)$. This expression summarizes two important characteristics of the model. First, entrepreneurs will demand a positive amount of loans only when the expected return on capital is greater than the risk-free rate. And second, they will choose a higher leverage when the expected discounted return on capital s_t is higher. In equilibrium, s_t must be equal to the marginal cost of external finance or *external finance premium*. Hence, equation (15) can be reformulated as

$$s_t \equiv E_t \frac{R^k_{t+1}}{R_t} = s(L_t) \quad \text{with } s(1) = 1,\ s(\cdot)' > 0. \tag{16}$$

This expression is useful because it shows that the external finance premium is an increasing function of leverage.[4] We will come back to this relation when we discuss the equilibrium dynamics in the loan market. Finally, at the end of each period, a random fraction $(1 - \gamma_{t+1})$ of the entrepreneur's assets is transferred to the households, while the household makes a lump-sum transfer W^e_t to each entrepreneur.[5]

2.5 Aggregation

Aggregate raw capital is given by

$$\bar{K}_{t+1} = \int_0^\infty \bar{K}^N_{t+1} f_t(N) dN, \tag{17}$$

while aggregate capital rented to productive firms is $K_t = u_t \bar{K}_t$. Aggregate entrepreneurs' profits are given by $[1 - \Gamma_t(\bar{\omega}_{t+1})] R^k_t Q_{\bar{K},t-1} \bar{K}_t$, so that aggregate net worth evolves according to

$$N_{t+1} = \gamma_t [1 - \Gamma_{t-1}(\bar{\omega}_t)] R^k_t Q_{\bar{K},t-1} \bar{K}_t + W^e_t. \tag{18}$$

Aggregate debt is obtained as

$$B_{t+1} = \int_0^\infty B^N_{t+1} f_t(N) dN = Q_{\bar{K},t} \bar{K}_{t+1} - N_{t+1}, \tag{19}$$

and the loan rate is given by $R^L_{t+1} = R^k_{t+1} \bar{\omega}_{t+1} L_t$.

The aggregate resource constraint then reads as

$$Y_t = D_t + C_t + G_t + \frac{I_t}{\Upsilon \mu_{\Upsilon,t}} + a(u_t) \Upsilon^t \bar{K}_t, \tag{20}$$

where the last term stands for the capital utilization costs of entrepreneurs, D_t represents the total monitoring costs incurred by financial intermediaries

$$D_t = \mu G(\bar{\omega}_t)(1 + R^k_t) \frac{Q_{\bar{K},t-1} \bar{K}_t}{P_t},$$

and G_t is government spending, which follows an exogenous process.

[4] A detailed derivation of the function $s(\cdot)$ can be found in appendix A2.

[5] This is to ensure that entrepreneurs do not accumulate a level of net worth sufficient to operate with zero debt. These concepts are exogenous.

2.6 Monetary Policy, Adjustment Costs and Shocks

The central bank follows the Taylor rule

$$R_t - R = \rho_p(R_{t-1} - R) + (1 - \rho_p)\left[\alpha_\pi(\pi_{t+1} - \pi^*) + \alpha_{\Delta y}(g_{y,t} - \mu_z^*)\right] + \varepsilon_t^R, \qquad (21)$$

where R is the steady-state nominal risk-free rate, π^* is the central bank's inflation target, $g_{y,t}$ is the growth rate of GDP, and ε_t^R is a monetary policy shock.

Investment adjustment costs take the form

$$S(x_t) = \frac{1}{2}\{\exp[\sqrt{S''}(x_t - x)] + \exp[-\sqrt{S''}(x_t - x)] - 2\}, \qquad (22)$$

where $x_t = \zeta_{I,t} I_t / I_{t-1}$, x is the steady state value of x_t, $S(x) = S'(x) = 0$, and $S''(x) = S''$ is a model parameter.

Utilization adjustment costs follow

$$a(u) = r^k[\exp(\sigma_a(u - 1)) - 1]\frac{1}{\sigma_a}, \qquad (23)$$

where $\sigma_a > 0$. Note that utilization is one in the steady state, regardless of the value of σ_a.

The model dynamics are driven by 10 structural shocks: a transitory technology shock, a permanent technology shock, a price-markup shock, a consumption preference shock, a marginal efficiency of investment shock, a shock to the relative price of investment goods, a monetary policy shock, a fiscal shock, a shock to entrepreneurs' net worth, and the risk shock. In the model these are ε_t, $\mu_{z^*,t}$, $\lambda_{f,t}$, $\zeta_{C,t}$, $\zeta_{I,t}$, $\mu_{\Upsilon,t}$, ε_t^R, ε_t^G, γ_t, and σ_t, respectively. We impose an AR(1) structure for all shocks except the monetary policy shock, which is assumed to be i.i.d., and allow for an anticipated or *news* component for risk shocks.[6] We follow CMR and allow agents to anticipate information for up to eight quarters. Hence, the risk shock process reads as

$$\sigma_t = \rho_\sigma \sigma_{t-1} + \xi_{0,t} + \xi_{1,t-1} + \ldots + \xi_{8,t-8}. \qquad (24)$$

In this specification, the innovation to the σ_t process is the sum of i.i.d., mean zero random variables, consisting of an unanticipated component $\xi_{0,t}$ and the anticipated component summarized in $\xi_{1,t-1}$ to $\xi_{8,t-8}$. We impose CMR's correlation structure for the $\xi_{j,t}$s:

$$\rho_{\sigma,n}^{|i-j|} = \frac{E\xi_{i,t}\xi_{j,t}}{\sqrt{E\xi_{i,t}^2 E\xi_{j,t}^2}}, \quad i, j = 0, \ldots p, \qquad (25)$$

[6] CMR show that this anticipated component plays an important role in terms of model fit. They consider several alternative specifications and conclude that the *news* component matters most for the risk shock. We implement their preferred specification here.

where $E\xi_{0,t}^2 = \sigma_\sigma^2$ and $E\xi_{1,t}^2 = E\xi_{2,t}^2 = \ldots = E\xi_{8,t}^2 = \sigma_{\sigma,n}^2$. This means that the σ process is characterized by four free parameters: ρ_σ, $\rho_{\sigma,n}$, σ_σ^2, and $\sigma_{\sigma,n}^2$.

2.7 Equilibrium Dynamics in the Loan Market

In order to illustrate why financial frictions have state-dependent effects in the model, we start by characterizing equilibrium dynamics in the loan market. We calibrate the model as described in Table 1 and solve for the combinations of $(\bar{\omega}_{t+1}, L_t)$ that satisfy equation (14) for increasing values of the return on capital R_t^k conditional on different levels of risk σ.[7]

Table 1: Calibrated parameters

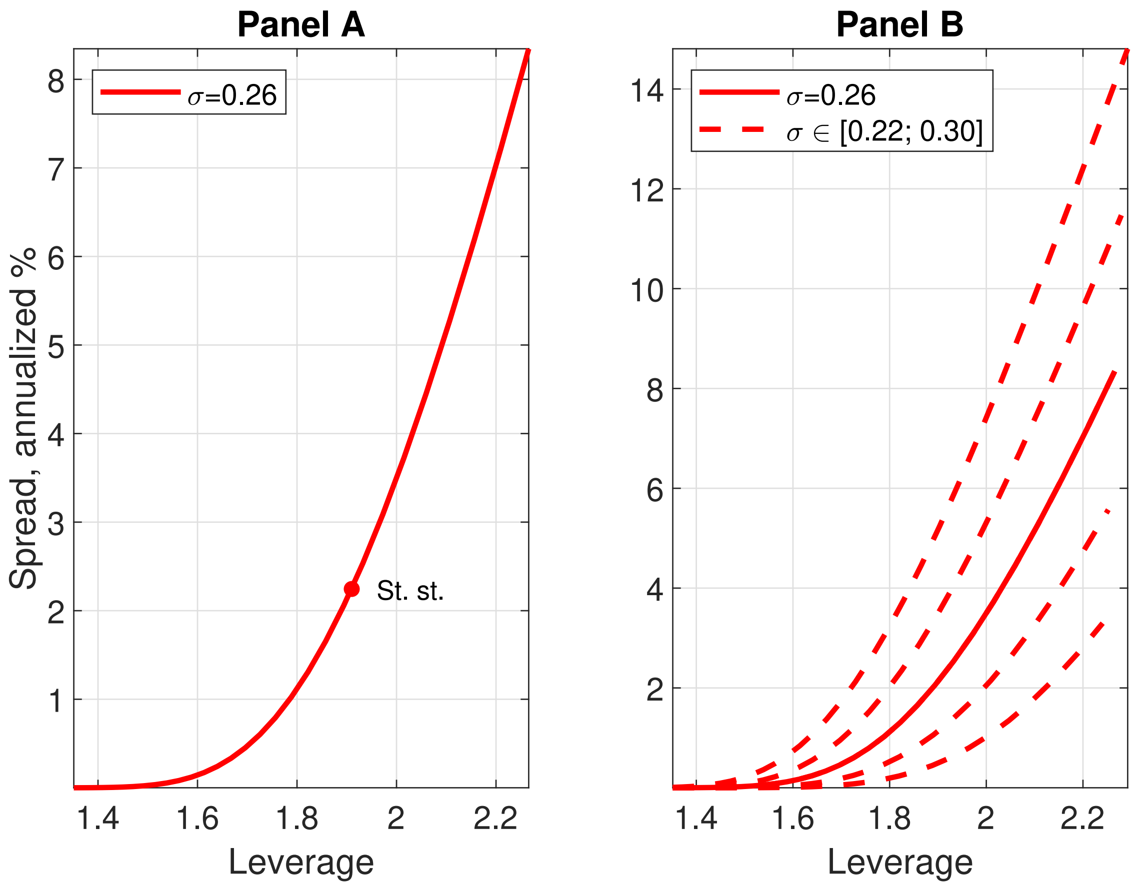
Parameter	Value	Description	Source/target
σ_L	1	Labor disutility	Christiano et al. (2014)
λ_f	1.2	Steady-state gross price markup	Christiano et al. (2014)
λ_w	1.05	Steady-state gross wage markup	Christiano et al. (2014)
α	0.4	Share of capital in production	Christiano et al. (2014)
δ	0.025	Depreciation rate of capital	Christiano et al. (2014)
τ^c	0.047	Tax rate consumption	Christiano et al. (2014)
τ^k	0.320	Tax rate capital	Christiano et al. (2014)
τ^l	0.241	Tax rate labor	Christiano et al. (2014)
μ	0.275	Monitoring cost	St.st. spread-leverage
γ	0.979	Survival rate of entrepreneurs	St.st. spread-leverage
W^e	0.134	Household-entrepreneur transfer	St.st. spread-leverage
σ	0.26	Steady-state risk shock	Christiano et al. (2014)
β	0.9985	Discount factor	Data
π	1.006	Steady-state inflation	Data
π^{target}	1.006	Central bank's inflation target	Data
μ_{z^*}	0.004	Steady-state economy growth rate	Data
Υ	0.004	Steady-state invest. specific growth rate	Data

Panel A in Figure 1 illustrates the equilibrium dynamics for our baseline calibration. The curvature of this schedule determines the responsiveness of the spread to fluctuations in the leverage position of entrepreneurs in equilibrium. For low returns on capital, entrepreneurs choose low leverage, which implies a low spread and a low sensitivity spread-leverage. As the return on capital increases, entrepreneurs take on more leverage and the spread increases.

Crucially, the spread increases more than proportionally as leverage goes up, reflecting the higher sensitivity spread-leverage for equilibria where firms are highly leveraged. This sensitivity determines the extent to which the financial health of entrepreneurs amplifies

[7] For this exercise we need to assign values to the following parameters of the financial contract: μ, γ, W^e, σ; and to β, π and μ_{z^*} in order to fix the nominal rate $R_t = (\pi\mu_{z^*})/\beta = 0.0115$. Note that these values are the same that we later fix when we estimate the model.

Figure 1: Equilibrium values for the spread, leverage, and risk



Notes: Panel A shows the equilibrium schedule for $R_t^k \in [0.0115, 0.0326]$. Panel B shows the equilibrium schedule fixing σ_t at 0.22, 0.24, 0.26 (baseline calibration, solid line), 0.28, and 0.30 (dashed lines) for $R_t^k \in [0.0115, 0.0489]$.

the propagation of shocks to the real economy. In the limit case where financial frictions are turned off, the spread and its sensitivity remain fixed at zero.

Linearizing the model requires selecting one point on this schedule and approximating the model dynamics around that steady state (the red dot in Figure 1, Panel A). The curvature of the schedule illustrates that this approximation can be quite poor when the model drifts away from the steady state. These nonlinear effects translate to the default probability of entrepreneurs, which is also more responsive for higher combinations spread-leverage.

Panel B of Figure 1 shows how the level of risk affects the spread-leverage dynamics. The solid line repeats the schedule from Panel A and the dashed lines depict the the leverage-spread schedule for increasing values of σ, from 0.22 to 0.3. An increase in σ shifts the entire schedule upward, implying a higher equilibrium for the spread, leverage, and the sensitivity spread-leverage. This panel provides a graphical illustration of the low-risk/high-risk setup that we present later in section 4, where the high-risk state is approximated around a higher value of σ, implying a larger propagation mechanism of the financial frictions to the real economy.

The economic intuition behind these two results is that financial frictions are amplified when financial intermediaries face higher expected losses from loan contracts. When leverage is high, entrepreneurs have relatively little skin in the game and the agency problem implies a higher scope for divergence of interests between borrowers and lenders. This is the traditional "financial accelerator" intuition, where endogenous dynamics in the credit market amplify macroeconomic shocks. In contrast, σ is a measure of the riskiness of entrepreneurs' returns, since a larger cross-sectional dispersion of idiosyncratic shocks yields a higher default probability in equilibrium. Hence, higher values of σ imply that financial intermediaries will charge a higher premium for each level of firm leverage.

We next document that these nonlinear dynamics result in quantitatively large asymmetries in key macroeconomic variables in the estimated New Keynesian model.

3 Estimation and State-Dependent Financial Frictions

We start by estimating the linearized model with full information Bayesian methods, the standard practice in most central banks. This provides us with a relevant benchmark for the values of deep parameters and variances of shocks that we can use to assess the role of state-dependent financial frictions.

3.1 Data

We use quarterly US data on 11 macro and financial time series covering the period 1985Q1–2010Q4 to estimate the model.[8] The first eight are macro variables in business cycle analysis: GDP, consumption, investment, and hours worked, all measured in real, per capita terms, plus the real wage, the relative price of investment goods, inflation and the federal funds rate. Note that in order to include the Great Recession in our analysis, our sample includes a number of quarters where the policy rate was at the effective lower bound. We deal with this by replacing the federal funds rate with the shadow rate by Wu and Xia (2016) for those quarters.

Additionally, we include three financial time series in the estimation: we measure the external finance premium with a BAA-rated corporate bond/10-year US treasury spread,[9]

[8]These are the same variables used by CMR, with the exception of the slope of the term structure. CMR include this concept, measured as the difference between the return on a 10-year Treasury yield and the federal funds rate, and add a long-term bond with measurement error to the model. Their goal is to diagnose whether the model dynamics are consistent with the observed slope of the term structure, and they show that the estimated model does well in this respect. The 10-year bond does not play a role for resource allocation in the model, but it involves the computation of expectations 40 quarters ahead, which slows down the solution and estimation. For this reason, we leave it out in our estimation.

[9]We obtain similar results when using the spread proposed by Gilchrist and Zakrajšek (2012).

entrepreneurs' net worth with the Dow Jones Wilshire 5000 index, converted into real, per capita terms, and firm credit as debt securities and loans of nonfinancial firms from the Flow of Funds tables, converted into real, per capita units. We assume that net worth is measured with error, so that our estimation of the model includes 11 shocks and 11 observables. Further details about data sources and transformations can be found in appendix A1.

3.2 Priors and Posteriors

A subset of parameters calibrated to match sample averages of the data is summarized in Table 1. We set π and π^{target} to the sample's average annual inflation rate of 2.3%. The households' discount factor β is fixed at 0.9985 to match the sample's average nominal interest rate of 4.6% and the average growth rate of the economy of 1.66%.

For the most part we stick to the parameterization of CMR, which is considered standard in the literature. We normalize ψ_L so that hours worked equals one in the steady state. The price and wage markups λ_f and λ_w are set to 1.2 and 1.05, respectively. The capital share in production α is set to 0.4 and the depreciation rate of capital δ to 0.025. Turning to the financial contract, we set the steady state productivity dispersion σ to 0.26 as in CMR. We fix the steady-state survival rate of entrepreneurs γ at 0.988, the transfer from households to entrepreneurs W^e at 0.13, and monitoring costs μ at 0.275, such that the steady state external finance premium matches the spread's sample average of 2.25%. This parameterization implies the following steady-state ratios: equity-to-debt ratio (firm leverage) of 1.91, consumption-to-output ratio of 0.54, investment-to-output ratio of 0.28, fiscal spending-to-output ratio of 0.18, and capital-to-output ratio of 8.4.

The priors and posterior estimates are presented in Table 2. The model does a good job in fitting not only the standard macro aggregates but also the spread and firm credit. This is despite the fact that the model does not include labor supply or wage markup shocks. Instead, as previously shown by CMR, the risk shock plays a key role for model fit, since it jointly explains a large share of both financial and non-financial variables.[10] With the estimated parameters and shocks at hand, in the following section we use a nonlinear solution of the model to document that financial frictions produce important state-dependent effects.

3.3 State-Dependent Financial Frictions

In this section we show that the NK-FF model with state-dependent financial frictions generates quantitatively large amplification of shocks in the real economy during periods of financial stress. We start by showing that the nonlinear spread-leverage equilibrium

[10] Figure A1 in appendix A3 shows the one-step-ahead forecasts of the model for all observables, while Figure A3 shows the data and the model dynamics when only feeding the risk shock for selected variables.

dynamics illustrated in Figure 1 imply large state-dependent effects in the estimated dynamic New Keynesian model. With the estimated parameters and shocks at hand, we use higher-order perturbation methods as in Dewachter and Wouters (2014) and Aruoba et al. (2017) to perform model simulations. Specifically, we fix the estimated parameters to their posterior mode and solve the model with a third-order Taylor approximation.[11] We simulate the model for 20,000 periods and do not allow for risk shocks in order to keep the level of risk fixed at its steady-state value.

[11]The ideal setting would be to have the fully nonlinear solution to estimate the model and run simulation experiments. However, because of the large number of state variables, the global solution of the New Keynesian model at hand involves a high computational burden, even for model simulations.

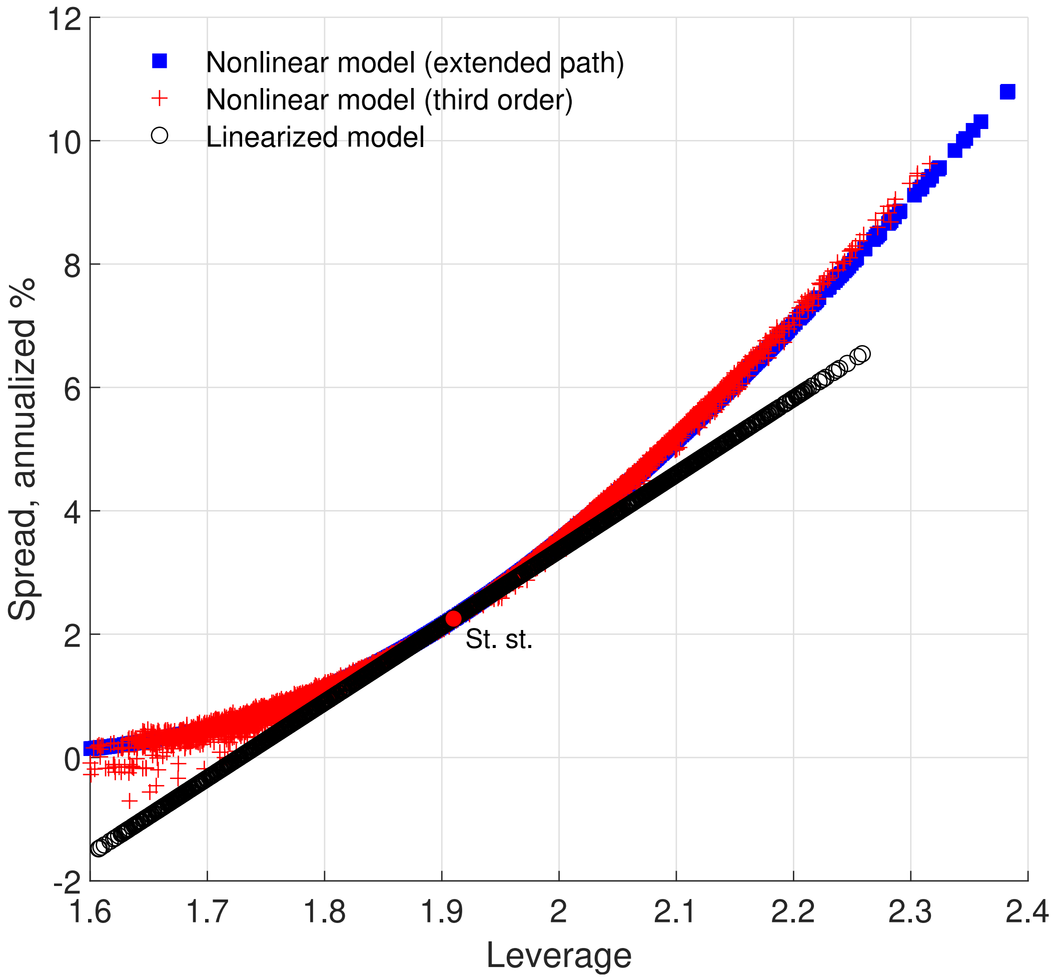
Table 2: Estimated parameters

	Description	Prior distribution	Posterior			
		Dist. mean[std.]	Mode	5%	Median	95%
ξ_w	Calvo wages	$\mathcal{B}$ 0.75 [0.1]	0.7555	0.6783	0.7422	0.8102
b	Habit in consumption	$\mathcal{B}$ 0.5 [0.1]	0.7406	0.6752	0.7463	0.8145
σ_a	Curvature capital util. cost	$\mathcal{IG}$ 1 [1]	2.6537	1.1619	2.8584	4.9703
S	Curvature invest. adj. cost	$\mathcal{N}$ 5 [3]	9.3535	4.5250	7.1253	9.9432
ξ_p	Calvo prices	$\mathcal{B}$ 0.5 [0.1]	0.8067	0.7649	0.8004	0.8352
α_π	Taylor rule: inflation	$\mathcal{N}$ 1.5 [0.25]	1.8732	1.6917	1.9070	2.1397
ρ_p	Taylor rule: smoothing	$\mathcal{B}$ 0.75 [0.1]	0.8458	0.8085	0.8389	0.8672
ι	Indexing: price inflation	$\mathcal{B}$ 0.5 [0.15]	0.8932	0.7968	0.8827	0.9566
ι_w	Indexing: wage inflation	$\mathcal{B}$ 0.5 [0.15]	0.4576	0.2173	0.4377	0.6703
ι_μ	Indexing: productivity	$\mathcal{B}$ 0.5 [0.15]	0.9444	0.8928	0.9371	0.9769
$\alpha_{\Delta y}$	Taylor rule: GDP	$\mathcal{N}$ 0.25 [0.1]	0.3415	0.2037	0.3643	0.5185
ρ_{λ_f}	AR price markup	$\mathcal{B}$ 0.5 [0.2]	0.9857	0.9522	0.9779	0.9966
ρ_ε	AR transitory technology	$\mathcal{B}$ 0.5 [0.2]	0.9725	0.9253	0.9655	0.9936
ρ_{ζ_I}	AR investment efficiency	$\mathcal{B}$ 0.5 [0.2]	0.4359	0.2843	0.4833	0.6639
ρ_{ζ_C}	AR intertemporal preference	$\mathcal{B}$ 0.5 [0.2]	0.9395	0.8767	0.9291	0.9748
ρ_μ	AR technology growth rate	$\mathcal{B}$ 0.5 [0.2]	0.0648	0.0164	0.0811	0.1548
ρ_σ	AR risk	$\mathcal{B}$ 0.5 [0.2]	0.9733	0.9511	0.9710	0.9899
ρ_{μ_Υ}	AR price of investment	$\mathcal{B}$ 0.5 [0.2]	0.9812	0.9627	0.9790	0.9943
ρ_g	AR fiscal	$\mathcal{B}$ 0.5 [0.2]	0.9417	0.8999	0.9383	0.9730
ρ_γ	AR equity	$\mathcal{B}$ 0.5 [0.2]	0.4323	0.2858	0.4036	0.5212
σ_ε	Std transitory technology	$\mathcal{IG}2$ 0.002 [0.0033]	0.0050	0.0045	0.0051	0.0058
σ_{λ_f}	Std price markup	$\mathcal{IG}2$ 0.002 [0.0033]	0.0093	0.0077	0.0097	0.0117
σ_{ζ_I}	Std investment efficiency	$\mathcal{IG}2$ 0.002 [0.0033]	0.0173	0.0153	0.0186	0.0221
σ_{ζ_C}	Std intertemporal preference	$\mathcal{IG}2$ 0.002 [0.0033]	0.0319	0.0224	0.0326	0.0457
σ_R	Std monetary policy	$\mathcal{IG}2$ 0.002 [0.0033]	0.0013	0.0012	0.0013	0.0015
σ_μ	Std technology growth rate	$\mathcal{IG}2$ 0.002 [0.0033]	0.0090	0.0081	0.0092	0.0104
σ_{σ_0}	Std unanticipated risk	$\mathcal{IG}2$ 0.002 [0.0033]	0.0016	0.0009	0.0073	0.0113
σ_{μ_Υ}	Std price of investment	$\mathcal{IG}2$ 0.002 [0.0033]	0.0028	0.0025	0.0028	0.0031
σ_N	Std net worth ME	$\mathcal{W}$ 0.01 [5]	0.0705	0.0628	0.0709	0.0796
σ_γ	Std equity	$\mathcal{IG}2$ 0.002 [0.0033]	0.0062	0.0057	0.0064	0.0072
σ_g	Std fiscal	$\mathcal{IG}2$ 0.002 [0.0033]	0.0218	0.0195	0.0221	0.0247
σ_{σ_n}	Std anticipated risk	$\mathcal{IG}2$ 0.0008 [0.0012]	0.0097	0.0050	0.0076	0.0106
$\rho_{\sigma,n}$	Corr. among signals	$\mathcal{N}$ 0 [0.5]	0.9468	0.8856	0.9570	0.9999

Notes: Table 2 shows estimation results for the linear NK-FF model. Prior distributions $\mathcal{B}$, $\mathcal{G}$, $\mathcal{N}$, $\mathcal{IG}2$, and $\mathcal{W}$ denote beta, gamma, normal, inverse gamma type 2, and weibull distributions, respectively. Posterior statistics are based on four chains of 250,000 MCMC replications, where the first 50.000 are discarded as burnin.

Figure 2 shows the leverage-spread schedule implied by the model. When the model moves away from the steady state (red dot), the third-order approximation (red crosses) captures a substantial degree of the curvature depicted in Figure 1 that the linear model (black circles) misses. In order to assess the accuracy of the third-order approximation, we also compute the nonlinear solution of the model using the extended path method by Fair and Taylor (1983) (blue squares). This method preserves the fully nonlinear first-order conditions of the model but imposes certainty equivalence, just as linearizing does by definition. The fact that the stochastic third-order solution comes close to the fully nonlinear deterministic solution suggests that it is an accurate approximation to the nonlinear dynamics of the model.[12]

Figure 2: Leverage-spread schedule



As discussed above, for low (high) levels of leverage, the nonlinear solution implies a low (high) sensitivity spread-leverage. This state-dependent sensitivity, absent in the linear model, implies that the amplification effects of the financial accelerator fluctuate endogenously with the state of the economy. The reason is that when the sensitivity is low, shocks that reduce economic activity and entrepreneurs' net worth trigger only a small increase in spreads, which translates into a moderate increase in the financing costs

[12]We conduct the simulations by drawing 20,000 random shocks given the shocks' estimated standard deviation and then feeding them to each solution of the model. For the deterministic nonlinear solution, shocks hit the economy each period conditional on the state of the economy in that period and agents expect no further shocks thereafter. For the third-order solution we use pruning as implemented in Dynare 4.5.4. We also consider a second-order approximation for this exercise and get similar results (see Figure A4 in appendix A3). The third-order comes slightly closer to the nonlinear solution and computing time is only marginally higher than the second-order, so that we use third-order as our baseline solution.

for firms. By contrast, when the sensitivity is high, a contractionary shock triggers a large increase in firms' financing costs, with large effects on the real economy, as we document next.

How large are these amplification effects, as captured by the third-order approximation? To answer this question, we use the smoothed shocks obtained from the estimated linear model, feed them to the nonlinear model, and compare the dynamics with the linear solution.[13] These shocks generate the observed data in the linear model—in other words, they are the most likely shocks given the data and the model. If the simulated paths for the endogenous variables with the nonlinear solution are only marginally different, we would conclude that the cost of ignoring the state-dependent nature of financial frictions is negligible.

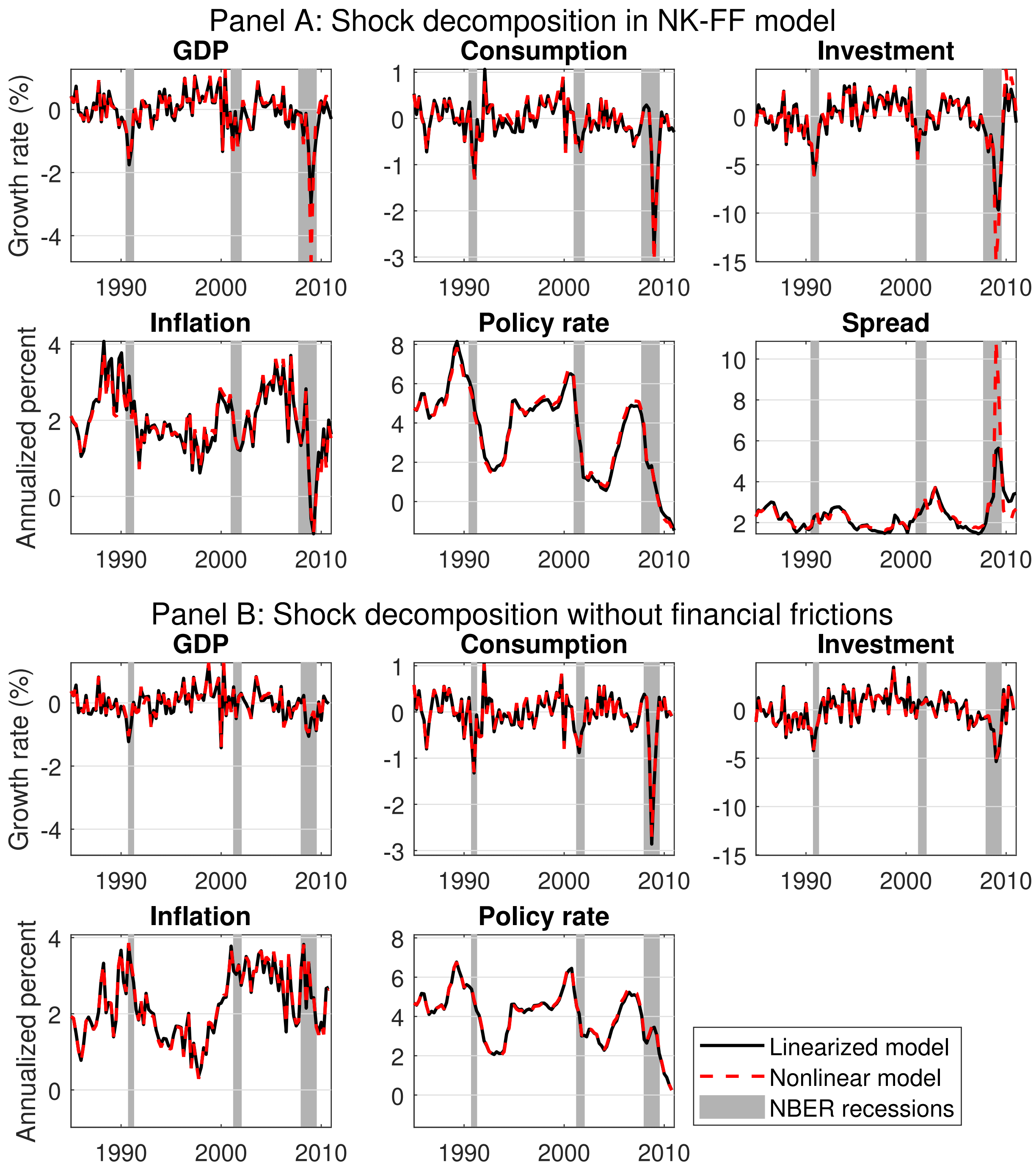
However, Figure 3 shows that amplification effects are quantitatively large. Panel A shows this exercise for the baseline estimated model, and it highlights the state-dependent nature of financial frictions. Amplification effects are large only in some states of the economy, when spreads are relatively high. Not surprisingly, the largest amplification occurs during the Great Recession, when financial conditions are worst. The nonlinear solution predicts that in 2008Q4 output and investment would have dropped by an additional 60% and 65% respectively, while consumption would have dropped by an additional 10%. The spike in the spread is 5.2 (annualized) percentage points higher. Altogether, the sample standard deviation is about 1.2 times larger for GDP and investment and 1.5 times larger for the spread in the nonlinear model.

It is noteworthy that amplification is small except during the Great Recession. This is consistent with the view that throughout the Great Moderation, linear models provided a good approximation to characterize business cycle dynamics. However, our results highlight that even if a linear model is a good approximation for most periods in our sample, it can be highly inaccurate in times of financial distress.

An alternative interpretation of these results is that a linear model estimates the wrong shocks, particularly in periods of high risk. Given that the propagation mechanisms are constant, the model needs much larger shocks than a model with time-varying propagation in those periods. Either way, the inference and predictions of the model will be misleading if these nonlinear dynamics are ignored.

[13]The shocks, including the news component of the risk shock, are shown in Figure A2 in appendix A3.

Figure 3: Amplification effects of financial frictions



Notes: NK-FF in Panel A stands for the New Keynesian model with financial frictions. Panel B shows the results for a model version without financial frictions.

To verify that the amplification effects described above are mainly due to financial frictions, Panel B of Figure 3 repeats the previous exercise in a model version where financial frictions are shut down. This exercise uncovers two interesting facts. First, the amplification in GDP, consumption, investment, and inflation is minimal, even during recessions, which confirms that the amplification effects of Panel A are due to state-dependent financial frictions and not other nonlinearities in the model. And second, financial shocks and frictions explain a large share of real variables, especially during recessions. The drop in GDP, investment, and inflation is much smaller in Panel B than in Panel A for the linear model as well, which reflects the important role played by the risk shock in explaining these variables.[14]

To have a closer look at the propagation mechanism behind the amplification effects shown in Figure 3, we compute state-dependent impulse responses conditional on financial conditions. Figure 4 shows the responses to risk shocks conditional on states of the business cycle when the spread is high (95$^{\text{th}}$ percentile) and low (5$^{\text{th}}$ percentile) before the shock hits.[15]

The average on-impact spread and net worth responses are more than twice as large in the high spread states as compared to the mean response, which explains the amplified response of investment and output in these states. Because firms' net worth falls sharply when financial conditions are worse, firms have to cut their investment more aggressively, which ultimately results in a larger drop in output. On the other hand, net worth and spreads barely respond to the shock in low spread states, which explains the muted responses of investment and GDP.

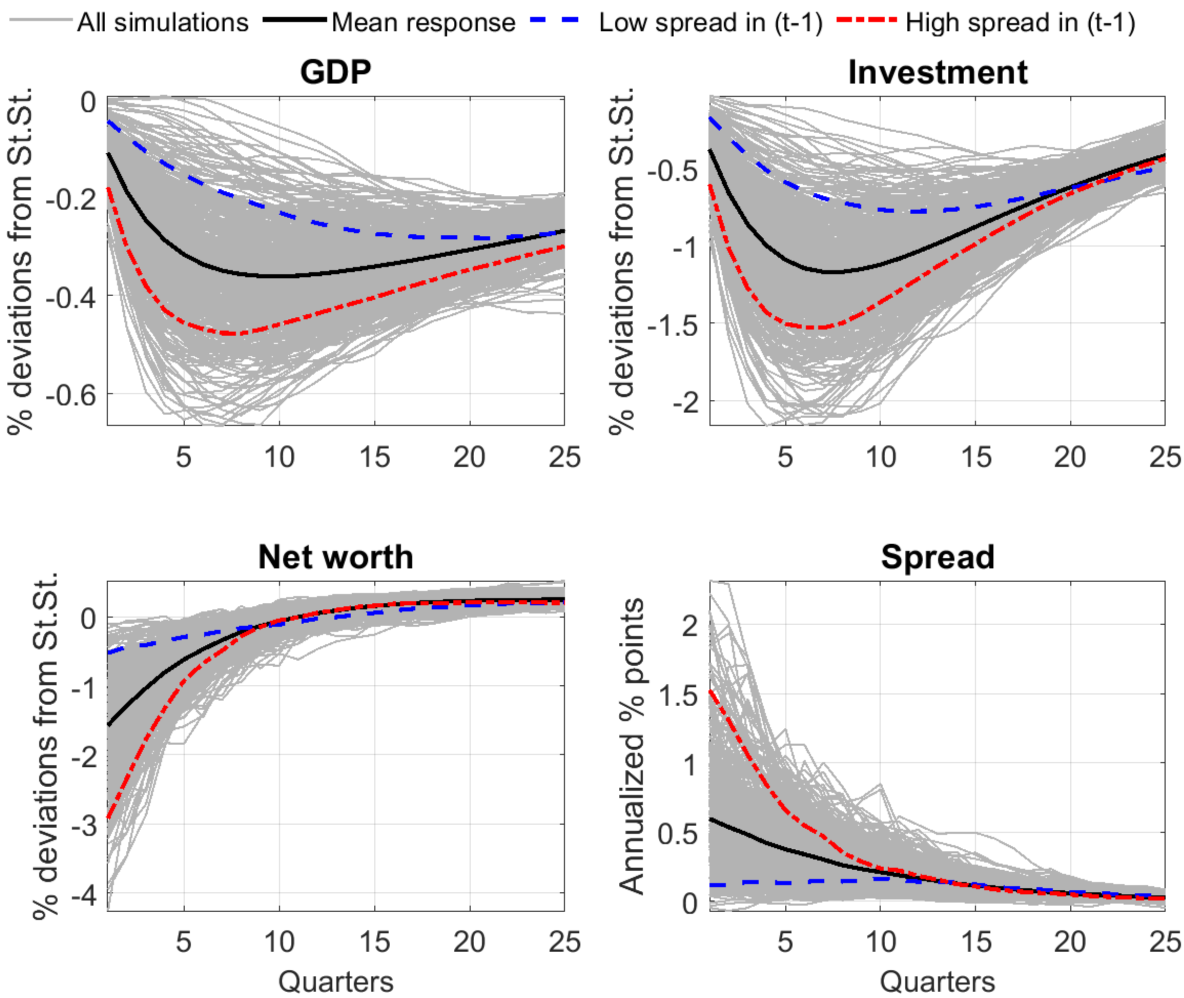
The thin grey lines show all the impulse response functions (IRFs) used to compute the average responses. They illustrate the asymmetric cyclical behavior of the model: there are a few (infrequent) cases where a single shock can trigger a big spike in the spread and a collapse of investment and output (on impact and at the response's trough) of more than twice the size of the average response over all the simulations, which is consistent with the amplification that the model generates during the Great Recession documented in Figure 3. These results are in line with the findings of Adrian et al. (2019), who document that the skewness of the distribution of GDP growth depends on financial conditions. In particular, the lower quantiles of the distribution vary as a function of current financial conditions, while the upper quantiles are relatively stable over time.

All in all, our results suggest an alternative interpretation for the conclusion of Christiano et al. (2018) that financial frictions in a pre-crisis NK model have small quantitative effects. Namely, that the effects of financial frictions are state-dependent. For example, the paper by Brzoza-Brzezina and Kolasa (2013) discussed there only allows for constant propagation mechanisms of financial frictions and argues that financial shocks explain only a small share of the real variables' volatility. We document not only that financial shocks play an important role for real variables (as already shown by CMR), but

[14]The relative standard deviations (third-order/linear simulations) in Panel A for GDP, consumption, investment, inflation, the FFR, and the spread are 1.1837, 1.0538, 1.1704, 0.9823, 1.0056, and 1.5206, respectively. Because the model features no financial frictions in Panel B, only the non-financial shocks are fed to the model. The relative standard deviations in this case are 0.9972, 0.9793, 0.9747, 1.0294, and 1.0079.

[15]Impulse response functions (IRFs) are computed by comparing two simulated paths for the endogenous variables which only differ in that one of them has an additional one-standard-deviation risk shock in period t.

Figure 4: State-dependent IRFs to a 1 σ_σ risk shock in the third-order model



Notes: Percentage deviations are computed with respect to the average counterfactual path without the 1 σ_σ risk shock. σ_σ is normalized to the value implied by assuming all variation in risk is unanticipated. The mean response is computed over 500 simulations. The impulse response functions (IRFs) for low and high spread states report the average response when the spread is below the 5^{th} and above the 95^{th} percentile before the shock hits, respectively.

that time-varying effects of financial frictions significantly increases the volatility in both financial and real variables. Against this background, in the next section we propose a regime-switching model that incorporates these time-varying effects and allows for efficient estimation.

4 A Regime-Switching DSGE Model

In the previous section we have shown that financial frictions in the NK-FF model generate large state-dependent propagation effects on macro variables. However, estimating such a nonlinear DSGE model typically requires the use of computationally costly nonlinear filters, such as the particle filter. A good example is the work by Gust et al. (2017), who estimate a Smets-Wouters-type model that is subject to the effective lower bound on interest rates using projection methods to solve the model and a particle filter to evaluate the likelihood. Despite their impressive parallel implementation of the particle filter, each evaluation of the likelihood takes about eight seconds in a supercomputer with 300 cores, and 4.2 minutes in a standard two-core computer. This can be problematic when using Markov Chain Monte Carlo methods to estimate the model, as is standard in the literature, since the likelihood has to be computed many times. The model that we consider here is significantly larger (it has more state variables) and the models used by many central banks are larger still. Therefore, we pursue an alternative approach that allows for efficient estimation with many state variables.

We build on the work by Lindé et al. (2016), who augment the Smets-Wouters model with the BGG financial friction and a regime-switching (RS) framework with two states: one where financial frictions are mild (low spread-leverage sensitivity) and one where they are severe (high spread-leverage sensitivity). They show that the RS model improves model fit, especially during the Great Recession. In that framework, however, the switching probabilities are constant and the transition between states is exogenous: there is no mechanism in the structural model that translates information from the state of the economy to the switching probabilities.

By contrast, we consider time-varying switching probabilities and rely on the large explanatory power of the risk shock for the business cycle to model state-dependent financial frictions as a function of risk. Coming back to Figure 1, we model periods of low spread-leverage sensitivity as periods of low risk and high sensitivity as high risk. Crucially, we model the probability of switching from one state to the other as a function of the spread, which allows us to link the nonlinear effects of financial frictions in the model to a measure of financial conditions in the data.

We start with a simulation exercise to show that our Regime-Switching framework provides an accurate approximation to the state-dependent effects documented in section 3. We use the third-order solution of the model discussed there as the data-generating process (DGP) and then estimate three types of models using these data: an RS model where the switching process is a function of financial conditions, an RS model where the switching follows an exogenous process, and a linearized model. Not surprisingly, both RS models greatly outperform the linear model in terms of fit. But importantly, we show that the endogenous RS model outperforms the exogenous RS model. This is because model fit improves as a result of the improved probabilistic assessment of when financial frictions matter most in the endogenous model. We then move on to show that our endogenous RS model explains US business cycle data much better than a linearized benchmark does.

4.1 The Regime-Switching Framework

The model structure is as described in section 2. However, instead of solving the model by linearizing around the non-stochastic steady state, now we consider two steady states and assume that the agents know that with a certain probability the economy finds itself in one state or the other. Specifically, the problem can be formulated as solving for optimality conditions of the form

$$E_t \sum_{r_{t+1}=1}^{h} p_{r_t,r_{t+1}} f(x_{t+1}(r_{t+1}), x_t(r_t), x_{t-1}, \theta_{r_t}, \theta_{r_{t+1}}, \varepsilon_t) = 0, \tag{26}$$

where r_t stands for the regime in place in period t, $p_{r_t,r_{t+1}}$ is the probability of switching regimes from t to $t+1$, $x_t(r_t)$ is a vector of endogenous variables, θ_{r_t} is a vector of parameters, and ε_t is a vector of exogenous variables. In our application, the number of regimes h is equal to two. We solve the model using perturbation methods as described in Maih (2015) and take a linear approximation around the non-stochastic steady state in each regime, $\bar{x} = \{\bar{x}_{\sigma_l},\ \bar{x}_{\sigma_h}\}$, where σ_l and σ_h stand for the level of risk in the low-risk and high-risk states, respectively. This gives us two distinct policy functions that map the exogenous to the endogenous variables of the form $x_t(r_t) = \Gamma_{r_t}(\varepsilon_t, \theta_{r_t})$.

4.1.1 Transition Probabilities

The exogenous switching model follows the tradition of Markov switching models, where the switching is governed by a Markov chain that is independent of the model dynamics. Yet in many applications there are good reasons to think that the model dynamics or the state of the economy play an important role for the switching process: for instance, interest rate dynamics in a model where the states are in "constrained" or "unconstrained" by the effective lower bound, or financial conditions when the states are financial tranquility or distress. The endogenous switching model follows the formulation of Chang et al. (2018), where a threshold-type switching process uses information from the state of the economy to determine the switching probabilities.

The transition probabilities in the exogenous case are constant and can be written as[16]

$$P^{l,h} = \frac{1}{1+\exp(\alpha_{x,1})}; \qquad P^{h,l} = \frac{1}{1+\exp(\alpha_{x,2})}, \tag{27}$$

where $P^{l,h}$ is the probability of switching from the low-risk state to the high-risk state, $P^{h,l}$ is the probability of switching from the high-risk state to the low-risk state, and $\alpha_{x,1}$ and $\alpha_{x,2}$ are parameters.

[16] They could also just be written as $P^{l,h} = \alpha_{x,1}$ and $P^{h,l} = \alpha_{x,2}$, but the formulation above simplifies the comparison between the exogenous and endogenous frameworks.

By contrast, we postulate the following time-varying endogenous switching probabilities

$$P_t^{l,h} = \frac{1}{\exp(\alpha_{n,1}) + \exp\left(-\bar{\gamma}_1 \frac{s_t - \bar{s}_1}{s^{sd}}\right)}; \quad P_t^{h,l} = \frac{1}{\exp(\alpha_{n,2}) + \exp\left(\bar{\gamma}_2 \frac{s_t - \bar{s}_2}{s^{sd}}\right)}. \tag{28}$$

Note that this formulation allows for an exogenous component, captured by the parameters $\alpha_{n,1}$ and $\alpha_{n,2}$,[17] and an endogenous component where s_t is the spread at time t, s^{sd} is its standard deviation, $\bar{s}_1$ and $\bar{s}_2$ are threshold values, and $\bar{\gamma}_1$ and $\bar{\gamma}_2$ are parameters. This formulation implies that when the spread is relatively high, $P_t^{l,h}$ is relatively high and $P_t^{h,l}$ is relatively low, and vice versa when the spread is low. Note that when $\bar{\gamma}_1 = \bar{\gamma}_2 = 0$, $\alpha_{n,1} = \alpha_{x,1}$, and $\alpha_{n,2} = \alpha_{x,2}$, the endogenous switching model collapses into the exogenous switching model. In other words, we allow for but do not restrict the switching process to be driven by financial conditions.

4.2 Model Estimation and Fit with Simulated Data

Our goal is to compare the RS models in terms of their ability to account for the nonlinearities described in section 3. We use the nonlinear NK-FF from section 3 to generate artificial time series and focus on model fit as the relevant benchmark. Specifically, we generate a sample of 5,000 observations from the third-order NK-FF model and fit the RS models and a linear model to these data.[18]

Since our interest is in modeling state-dependent financial frictions, we keep most of the deep parameters fixed at the DGP values and estimate a small subset of parameters. For the RS models we estimate the level of risk in each regime, the standard deviations and correlation between the anticipated and unanticipated components of the risk shock, the parameters in the probability functions, and the discount factor in the low-risk regime. We allow for up to one year of anticipated effects (*news*) of the risk shock in this exercise, as the likelihood evaluation times for the RS models become exponentially large for two or more years. We estimate the discount factor in the low-risk regime to allow for a regime-specific risk-free interest rate. For the linearized model we estimate the level of risk and the standard deviations and correlation between the anticipated and unanticipated components of the risk shock. We estimate the RS models using the adaptation of the Kalman filter developed by Chang et al. (2018) (endogenous-switching Kalman filter) and the linearized model with the standard Kalman filter, both using the RISE toolbox

[17] We restrict these parameters to be non-negative to ensure that the probabilities are between zero and one. The endogenous component (the expression inside the second $\exp(\cdot)$) in each probability function is unrestricted.

[18] Adding more observations leaves the correlation structure of the variables and other moments essentially unchanged, so that 5,000 observations provide a good approximation to the DGP. Figure A5 in appendix A3 shows the simulated data used for this estimation exercise.

for Matlab (see `https://github.com/jmaih/RISE_toolbox`). We use full information Bayesian methods for both types of models.

Table 3: Likelihood evaluation time

	Linear	Regime-switching	Gust et al. (2017)
US data	$\simeq$ 0.1 seconds	$\simeq$ 0.5 seconds	8 seconds to 4 minutes
5,000 obs.	$\simeq$ 1 seconds	$\simeq$ 4 seconds	—
Filter	Kalman	Endogenous-switching Kalman	Particle

Notes: The computing times for the linear and regime-switching models correspond to a desktop computer with a processor Intel Core i7-6700 with 3.40 GHz and 16GB RAM. The variation in the times reported by Gust et al. (2017) corresponds to the difference between their parallel implementation of the filter in a 300-core supercomputer (8 seconds) and the implementation in a standard dual-core desktop (4 minutes).

Table 3 shows a comparison of the time required to evaluate the likelihood of the different model solutions that we consider, and we include the times reported by Gust et al. (2017) as a reference for the particle filter. Each evaluation of the likelihood with 5,000 observations takes 3.8 seconds for the RS models and about 1.1 second for the linearized model using a standard desktop computer. When we consider a sample of 104 observations (the sample size used in section 3), each evaluation takes about 0.5 seconds for the RS models and about 0.1 seconds for the linearized model. While the RS models are significantly slower to evaluate than the linearized model, they are orders of magnitude faster than the particle filter. At these speeds it is still feasible to estimate the RS models using a standard desktop computer.

Table 4 shows the priors and the estimation results. For the level of risk, we choose values that are reasonably close to the DGP of 0.26. The priors for $\alpha_{x,1}$ and $\alpha_{x,2}$ are chosen such that the steady-state probabilities $P^{l,h}$ and $P^{h,l}$ of the exogenous model are between 0.01–0.25 and 0.1–0.5, respectively, with a 95% probability. We use the same priors for $\alpha_{n,1}$ and $\alpha_{n,2}$ in order to facilitate the comparison between the two models, but, as we mentioned before, we restrict these parameters to be nonnegative. For $\bar{\gamma}_1$ and $\bar{\gamma}_2$ we choose a loose prior centered around one. Finally, we calibrate $\bar{s}_1$ and $\bar{s}_2$ to the median of the simulated spread used as observable.

Regarding the estimation results, the RS models identify a low-risk regime with σ_l around 0.24 and σ_h around 0.265, while the linearized model yields a value of $\sigma = 0.2637$, slightly higher than the DGP. These values imply different steady-state spread-leverage sensitivities, computed as $\frac{d \log s}{d \log L}$ and summarized in the lower part of the table.[19] The sensitivity for the RS models is close to 0.03 in the low-risk regime and 0.064 in the high-risk regime. A sensitivity of 0.03, for instance, means that if equilibrium leverage increases by 1%, then the equilibrium spread will increase by 3%. These elasticities imply

[19] The derivation for this expression in the steady state can be found in appendix A2.

Table 4: Estimated parameters: Regime-switching (RS) and linear models

	Description	Prior distribution Dist. mean[std.]	Posterior modes Linearized model	Exogenous RS model	Endogenous RS model
σ_{σ_0}	Std. unanticipated risk	$\mathcal{IG}$ 0.0100 [1]	0.0251	0.0369	0.0329
σ_{σ_n}	Std. anticipated risk	$\mathcal{IG}$ 0.0100 [1]	0.0191	0.0250	0.0232
$\rho_{\sigma,n}$	Corr. between signals	$\mathcal{N}$ 0 [0.5000]	0.5642	0.5328	0.5469
β_l	Discount factor (l)	$\mathcal{B}$ 0.9940 [0.0010]	—	0.9938	0.9940
σ_l	Risk level (l)	$\mathcal{B}$ 0.2450 [0.0050]	—	0.2388	0.2406
σ_h	Risk level (h)	$\mathcal{B}$ 0.2850 [0.0050]	0.2637	0.2653	0.2652
$\alpha_{x,1}$	Probability functions	$\mathcal{N}$ 2.8469 [0.8920]	—	3.8935	—
$\alpha_{x,2}$	Probability functions	$\mathcal{N}$ 1.0986 [0.5605]	—	3.6535	—
$\alpha_{n,1}$	Probability functions	$\mathcal{N}$ 2.8469 [0.8920]	—	—	3.5798
$\alpha_{n,2}$	Probability functions	$\mathcal{N}$ 1.0986 [0.5605]	—	—	1.7924
$\bar{\gamma}_1$	Probability functions	$\mathcal{G}$ 1 [0.5000]	—	—	1.0403
$\bar{\gamma}_2$	Probability functions	$\mathcal{G}$ 1 [0.5000]	—	—	1.6445
	Implied steady states and model fit		Linearized model	Exogenous RS model	Endogenous RS model
	St. st. probabilities [$P^{l,h}$; $P^{h,l}$]		—	[0.02; 0.025]	[0.027; 0.147]
	St. st. leverage [l; h]		1.91	[1.86; 1.91]	[1.86; 1.91]
	St. st. sensitivity spread-lev. [l; h]		0.0625	[0.0285; 0.0642]	[0.0301; 0.0641]
	ΔLog-marginal data density		—	Exo-linear: 1,708	Endo-exo: 130

Notes: Prior distributions $\mathcal{B}$, $\mathcal{N}$ $\mathcal{G}$, and and $\mathcal{IG}$ denote beta, normal, gamma, and inverse gamma distributions, respectively. The priors of the parameters $\alpha_{n,1}$ and $\alpha_{n,1}$ are truncated at zero. The subscripts $\{l, h\}$ stand for "low risk" and "high risk." respectively. For the linear model, the values refer to the unique regime we allow for. The lower block of the table shows the implied steady-state values for the probabilities, leverage, and the sensitivity spread-leverage, and model fit. For the RS models, the squared brackets indicate the steady-state values in each regime. In the last row, Exo-linear and Endo-exo stand for the difference between the MDD of the exogenous model with the linear model, and the endogenous model with the exogenous model, respectively.

that a risk shock will propagate about twice as strongly in the high-risk regime compared with the low-risk regime in the RS models. The elasticity in the linearized model is close to the RS models' high-risk regime, at 0.062. This means that the linearized model needs a relatively high sensitivity to account for periods of high financial turbulence in the simulated data, which will result in a poor fit during tranquil periods with low spreads.

How does this flexibility of the RS models translate into improved forecasting performance with respect to the linear model? In order to quantify these gains, the last row of Table 4 compares the marginal data densities (MDDs) of the three models. The exogenous RS model has an MDD that is 1,708 log points higher than the linear model, while the endogenous RS model outperforms the exogenous model by 130 log points.[20] These gains in MDD result from improved one-step-ahead forecasts of the RS models when evaluating the likelihood. Given that the effects of financial frictions get ampli-

[20]These differences are larger than typically found in empirical studies (as, for instance, in Lindé et al. 2016, Hubrich and Tetlow 2015) because our estimations are carried out in samples that are much larger than the typical macroeconomic time series.

fied when financial conditions deteriorate, the RS models—particularly the endogenous RS model—produce better forecasts in those periods by assigning a high probability to the high-risk regime, which features a stronger financial accelerator effect. This is also reflected in the smoothed regime probabilities and shocks. Figure A6 in the appendix shows that the smoothed regime probabilities are slightly more responsive to changes in the spread in the endogenous RS model. The linear model, in contrast, systematically filters larger positive (negative) risk shocks in periods of high (low) spreads, which translates into worse forecasts and fit. All told, the endogenous RS model provides an efficient alternative to model state-dependent financial frictions that endogenously evolve with the state of the economy.

4.3 Application to the Great Recession

In this section we fit the endogenous RS model to US data and show that the transition from a low-risk regime to a high-risk regime is key to account for the Great Recession. We use the same data set described in section 3 and estimate the same set of parameters as for the linearized model in that exercise, but we allow for some parameters to be regime-specific.

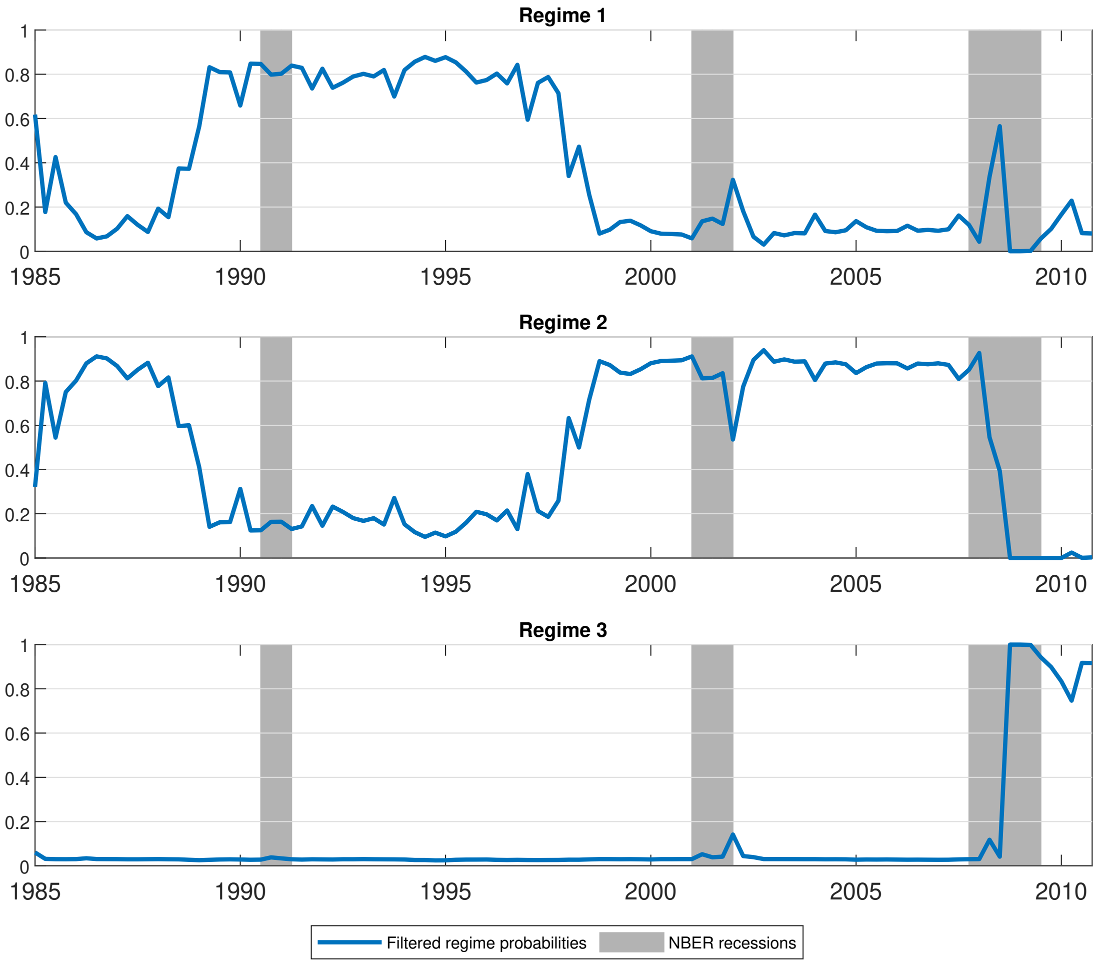
As in the estimation exercise from the previous section, we allow for regime-specific levels of risk, leverage, and discount factors. We extend our two-regime framework from the previous section and allow for three different regimes, which results in a better fit than a simpler two-regime (high-risk / low-risk) setting.[21] The full list of estimated parameters is reported in Table A1. Regime 1 is a low-risk regime with low spreads and a low spread-leverage sensitivity of 0.025. Regime 2 is an intermediate-risk regime associated with higher spreads and a slightly higher spread-leverage sensitivity of 0.028, while regime 3 is a financial crisis regime with high spread-leverage sensitivity of 0.08.

Figure 5 shows the model-implied filtered regime probabilities for US data. The model assigns a high probability to the low-risk regime from the late 1980s until 1998, when spreads were relatively low. The intermediate-risk regime gets a high probability in the early 1980s and from the late 1990s until the early stages of the Great Recession. To a large extent, this regime coincides with a period in which spreads are at an intermediate level and financial vulnerabilities are building up during the housing and financial boom that preceded the Great Recession. Finally, the high-risk regime starts showing meaningful positive probabilities only during the Great Recession, and it peaks at the height of that episode in 2008Q4.

The flexibility of the RS model to interpret changes in financial conditions as regimes with time-varying propagation effects of financial frictions, and the Great Recession as a

[21] We experimented with a two-regime setting and the result is that the entire pre-Great Recession period is interepreted as a low-risk regime.

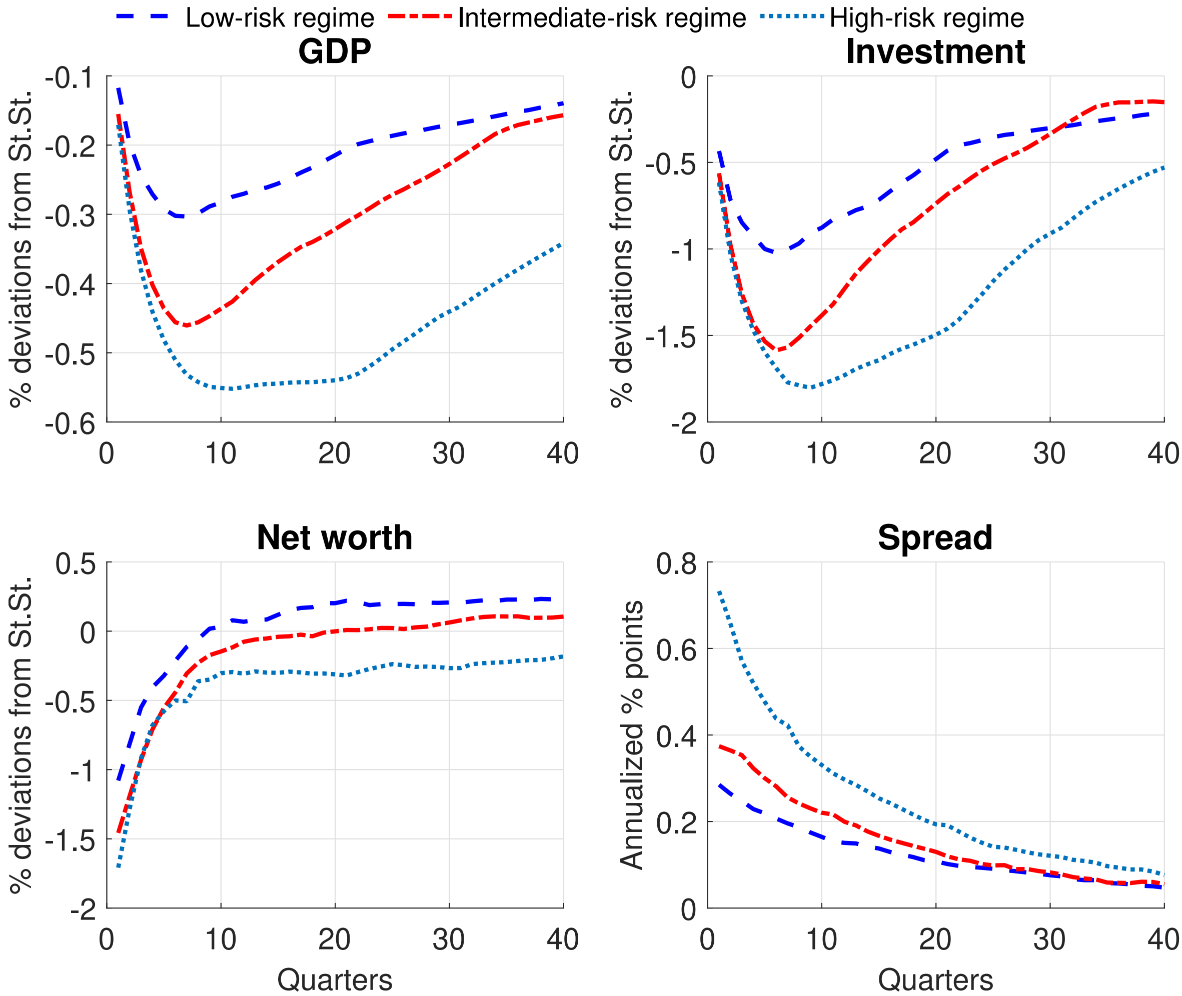
Figure 5: Filtered regime-switching model probabilities



high-risk regime in particular, results in a material improvement in model fit with respect to the linear model: a 250 log-points improvement in the marginal data density.

What explains these large gains in model fit? As mentioned above, the spread-leverage sensitivity in the high-risk regime is more than twice as large as that in the low-risk regime, which results in a large amplification of shocks in the high-risk regime, especially the risk shock. Figure 6 documents these state-dependent effects with generalized IRFs. These IRFs are computed with 20,000 simulations of the RS model based on the estimated standard deviations of the shocks, conditioning on whether the model is predicted to be in the low-, intermediate-, or high-risk regime before the shock hits. As expected, the propagation of risk shocks is much stronger in the high-risk regime. This gives the RS model a key advantage, as it does not rely as much on exogenous innovations as the linearized model does to account for periods of financial turmoil. Instead, the transition to regimes featuring a more prominent role of financial frictions results in better forecasts for the endogenous variables during those periods when evaluating the model.

Figure 6: State-dependent impulse response functions to a risk shock



5 Conclusion

The Great Recession exposed several open challenges for macro models used by central banks (Lindé et al. 2016). Christiano et al. (2018) discuss several paths forward to fine-tune pre-crisis New Keynesian models and improve their accuracy. They highlight that taking the nonlinear dynamics of these models into account is important to characterize business cycle dynamics.

In this paper, we contribute to this literature in two ways. First, we show that a pre-crisis New Keynesian model like the one used by many central banks generates large amplification of shocks in macro and financial variables during episodes of financial distress once state-dependent financial frictions are taken into account. These amplification effects are almost absent during the Great Moderation and become quantitatively large during the Great Recession. And second, we propose an endogenous regime-switching framework that incorporates these state-dependent effects, allowing for efficient estimation with many state variables and improving model fit. Allowing for these state-dependent effects is key to account for large shocks, such as the Great Recession.

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Technical Appendix

Risk and State-Dependent Financial Frictions

Martín Harding and Rafael Wouters

A1 Data Appendix

The following list describes the data and data transformations used to estimate the DSGE models in sections 3 and 4.

- Consumption: Personal consumption expenditures of services (FRED code: PCESV) and nondurable goods (FRED code: PCND), divided by the GDP deflator and by the total population, transformed into log-first differences minus the sample mean. Source: BEA.
- Firm credit: Nonfinancial business, debt securities and loans, liability, divided by the GDP deflator, divided by the total population, transformed into log-first differences minus the sample mean. Source: Financial Accounts of the United States - Z1/FA14
- GDP: Gross domestic product (FRED code: GDP), divided by the GDP deflator (FRED code: GDPDEF) and by the total population (FRED code: POPTOTUSA647NWDB), transformed into log-first differences minus the sample mean. Source: BEA.
- Hours worked: Nonfarm business sector hours of all persons (FRED code: HOANBS), divided by the total population, transformed to log-levels minus the sample mean. Source: US Bureau of Labor Statistics.
- Inflation: Implicit price deflator for GDP, percent change from preceding period, annualized percent divided by 400. (FRED code: A712RI1Q225SBEA). Source: BEA.
- Investment: Gross private domestic investment (FRED code: GPDI) and consumption expenditures of durable goods (FRED code: PCDG) divided by the GDP deflator and by the total population, transformed into log-first differences minus the sample mean. Source: BEA.
- Net worth: Wilshire 5000 Total Market Full Cap Index (FRED code: WILL5000INDFC), divided by the total population, transformed into log-first differences minus the sample mean. Source: Wilshire Associates.
- Nominal interest rate: Effective Federal Funds Rate (FRED code: FEDFUNDS), converted to quarterly by taking monthly averages, annualized percent divided by 400. Source: Board of Governors of the Federal Reserve System.
- Real wage: Nonfarm business sector compensation per hour (FRED code: COMPNFB), divided by the GDP deflator, transformed into log-first differences minus the sample mean. Source: US Bureau of Labor Statistics.

- Relative price of investment goods: Relative price of investment goods (FRED code: PIRIC), transformed into log-first differences minus the sample mean. Source: DiCecio (2009).

- Spread: Moody's seasoned Baa corporate bond yield relative to yield on 10-Year Treasury at constant maturity (FRED code: BAA10YM), converted to quarterly by taking monthly averages, annualized percent divided by 400 minus the sample mean. Source: Federal Reserve Bank of St. Louis.

A2 The Financial Contract and the Sensitivity Spread-Leverage

The derivations and properties of the functional forms under the log-normal distribution are discussed in detail in BGG and can be found in many other sources; here we focus on the derivation of the spread-leverage sensitivity, which plays an important role for our analysis. As stated in equation (12), the expected returns for entrepreneurs can be written as

$$E_t \int_{\varpi_{t+1}}^{\infty} \left[R^k_{t+1}\omega Q_{\bar{K},t}\bar{K}_{t+1} - R^L_{t+1}B_{t+1}\right] dF(\omega,\sigma) = E_t[1-\Gamma_t(\bar{\omega}_{t+1})]R^k_{t+1}L_tN, \tag{A.1}$$

with $\Gamma_t(\bar{\omega}_{t+1}) \equiv [1-F(\bar{\omega}_{t+1})]\bar{\omega}_{t+1} + G_t(\bar{\omega}_{t+1})$, $G_t \equiv \int_0^{\bar{\omega}_{t+1}} \omega dF_t(\omega)$. We can combine equations (11) and (13) to derive the participation constraint of banks and formulate the optimization problem of entrepreneurs. From equation (11), $R^L_{t+1}B^N_{t+1} = R^k_{t+1}Q_{\bar{K},t}\bar{K}_{t+1}\bar{\omega}_{t+1}$. Plugging this into equation (13) we obtain

$$[1-F(\varpi_{t+1})]\,R^k_{t+1}Q_{\bar{K},t}\bar{K}_{t+1}\bar{\omega}_{t+1} + (1-\mu)\int_0^{\varpi_{t+1}} \omega dF_t(\omega)R^k_{t+1}Q_{\bar{K},t}\bar{K}_{t+1} = R_tB^N_{t+1}, \tag{A.2}$$

where we have used the fact that this equation holds with equality in equilibrium. We can use the definitions of $\Gamma_t(\bar{\omega}_{t+1})$ and $G_t(\bar{\omega}_{t+1})$ to simplify this expression as follows:

$$\left([1-F(\varpi_{t+1})]\,\bar{\omega}_{t+1} + (1-\mu)G_t(\bar{\omega}_{t+1})\right)R^k_{t+1}Q_{\bar{K},t}\bar{K}_{t+1} = R_tB^N_{t+1} \tag{A.3}$$

$$[\Gamma_t(\bar{\omega}_{t+1}) - \mu G_t(\bar{\omega}_{t+1})]\,R^k_{t+1}Q_{\bar{K},t}\bar{K}_{t+1} = R_tB^N_{t+1} \tag{A.4}$$

and divide by N

$$[\Gamma_t(\bar{\omega}_{t+1}) - \mu G_t(\bar{\omega}_{t+1})]\,R^k_{t+1}L_t = R_t\frac{B^N_{t+1}}{N}. \tag{A.5}$$

We rearrange and note that $\frac{B^N_{t+1}}{N} = L_t - 1$

$$\frac{R^k_{t+1}}{R_t} = \frac{1}{[\Gamma_t(\bar{\omega}_{t+1}) - \mu G_t(\bar{\omega}_{t+1})]}\left(1-\frac{1}{L_t}\right), \tag{A.6}$$

which is equation (14). Then, the problem of the entrepreneur is

$$\max_{L_t,\bar{\omega}_{t+1}} \quad E_t[1-\Gamma_t(\bar{\omega}_{t+1})]R^k_{t+1}L_tN \tag{A.7}$$

$$\text{s.t.} \quad [\Gamma_t(\bar{\omega}_{t+1}) - \mu G_t(\bar{\omega}_{t+1})]\,R^k_{t+1}L_tN = R_t(L_t-1)N, \tag{A.8}$$

where we have replaced $L_t = (Q_{\bar{K},t}\bar{K}_{t+1})/N$ and $B^N_{t+1} = (L_t - 1)N$ in equation (A.4). The first-order conditions associated to the problem are

$$L_t: \quad (1 - \Gamma_t(\bar{\omega}_{t+1}))s_t + \lambda_t \left[\Gamma_t(\bar{\omega}_{t+1}) - \mu G_t(\bar{\omega}_{t+1})\right] s_t - \lambda_t = 0 \tag{A.9}$$

$$\bar{\omega}_{t+1}: \quad -\Gamma_t'(\bar{\omega}_{t+1}) + \lambda_t \left[\Gamma_t'(\bar{\omega}_{t+1}) - \mu G_t'(\bar{\omega}_{t+1})\right] = 0 \tag{A.10}$$

$$\lambda_t: \quad \left[\Gamma_t(\bar{\omega}_{t+1}) - \mu G_t(\bar{\omega}_{t+1})\right] s_t L_t - (L_t - 1) = 0, \tag{A.11}$$

where we have replaced $s_t = R^k_{t+1}/R_t$. Now we can express λ_t, s_t and L_t as a function of $\bar{\omega}_{t+1}$. Specifically, from equations (A.10), (A.9), and (A.11) we get, respectively,

$$\lambda_t = \frac{\Gamma_t'(\bar{\omega}_{t+1})}{\Gamma_t'(\bar{\omega}_{t+1}) - \mu G_t'(\bar{\omega}_{t+1})} \tag{A.12}$$

$$s_t = \frac{\lambda_t}{1 - \Gamma_t(\bar{\omega}_{t+1}) + \lambda_t \left[\Gamma_t(\bar{\omega}_{t+1}) - \mu G_t(\bar{\omega}_{t+1})\right]} \tag{A.13}$$

$$L_t = \frac{1}{1 - \left[\Gamma_t(\bar{\omega}_{t+1}) - \mu G_t(\bar{\omega}_{t+1})\right] s_t}. \tag{A.14}$$

Equation (A.14) establishes the equilibrium relation between the leverage ratio and the spread that we have defined as $s(\cdot)$ in section 2. With these expressions at hand, we can now compute the steady-state sensitivity spread-leverage as a function of $\bar{\omega}$. We simply drop all the time indices, so that the value will be pinned down by the steady-state value of $\bar{\omega}$ along with the parameter values of the financial contract. We define this sensitivity as

$$\eta_{s,L} = \frac{d \log s(\bar{\omega})}{d \log L(\bar{\omega})}. \tag{A.15}$$

Define $\Psi(\bar{\omega}) \equiv 1 - \Gamma_t(\bar{\omega}_{t+1}) + \lambda_t \left[\Gamma_t(\bar{\omega}_{t+1}) - \mu G_t(\bar{\omega}_{t+1})\right]$, so that equation (A.13) can be rewritten as $s_t = \lambda(\bar{\omega})/\Psi(\bar{\omega})$. Plugging this into equation (A.14) we obtain $L(\bar{\omega}) = \Psi(\bar{\omega})/(1 - \Gamma(\bar{\omega}))$. And now we can compute the following derivatives:

$$\frac{d \log s(\bar{\omega})}{d\bar{\omega}} = \frac{\lambda'(\bar{\omega})}{\lambda(\bar{\omega})} - \frac{\Psi'(\bar{\omega})}{\Psi(\bar{\omega})} \tag{A.16}$$

$$\frac{d \log L(\bar{\omega})}{d\bar{\omega}} = \frac{\Psi'(\bar{\omega})}{\Psi(\bar{\omega})} + \frac{\Gamma'(\bar{\omega})}{1 - \Gamma(\bar{\omega})}, \tag{A.17}$$

where

$$\lambda'(\bar{\omega}) = \frac{\mu \left[\Gamma'(\bar{\omega})G''(\bar{\omega}) - \Gamma''(\bar{\omega})G'(\bar{\omega})\right]}{\left[\Gamma'(\bar{\omega}) - \mu G'(\bar{\omega})\right]^2} \tag{A.18}$$

$$\Psi'(\bar{\omega}) = \lambda'(\bar{\omega}) \left[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})\right] + \lambda(\bar{\omega}) \left[\Gamma'(\bar{\omega}) - \mu G'(\bar{\omega})\right] - \Gamma'(\bar{\omega}). \tag{A.19}$$

So that the expression for the steady-state spread-leverage sensitivity becomes

$$\eta_{s,L} = \frac{\frac{\lambda'(\bar{\omega})}{\lambda(\bar{\omega})} - \frac{\Psi'(\bar{\omega})}{\Psi(\bar{\omega})}}{\frac{\Psi'(\bar{\omega})}{\Psi(\bar{\omega})} + \frac{\Gamma'(\bar{\omega})}{1-\Gamma(\bar{\omega})}}. \tag{A.20}$$

A3 Linearized and Third-Order Model Additional Results

Figure A1: One-step-ahead model forecasts and data

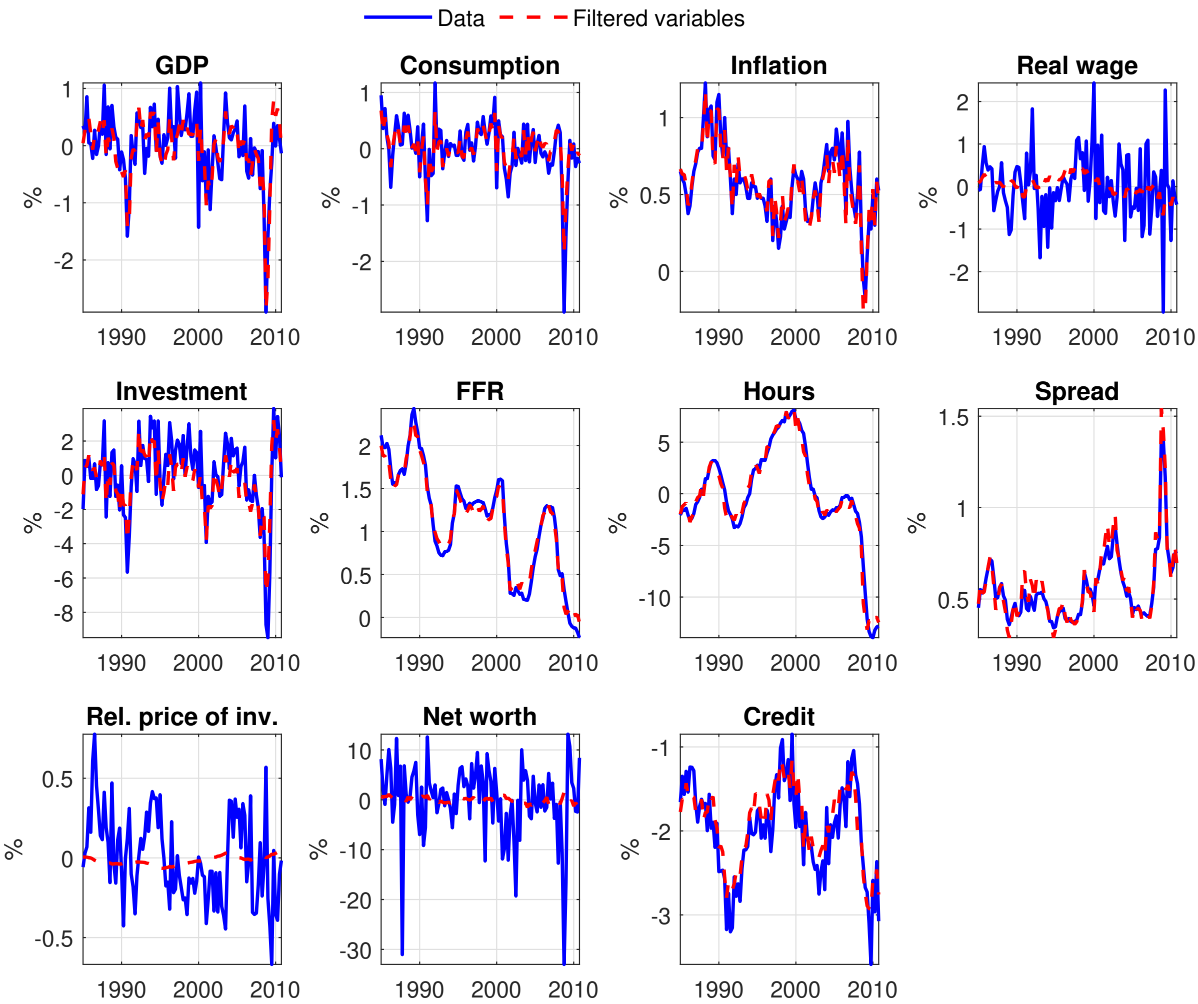
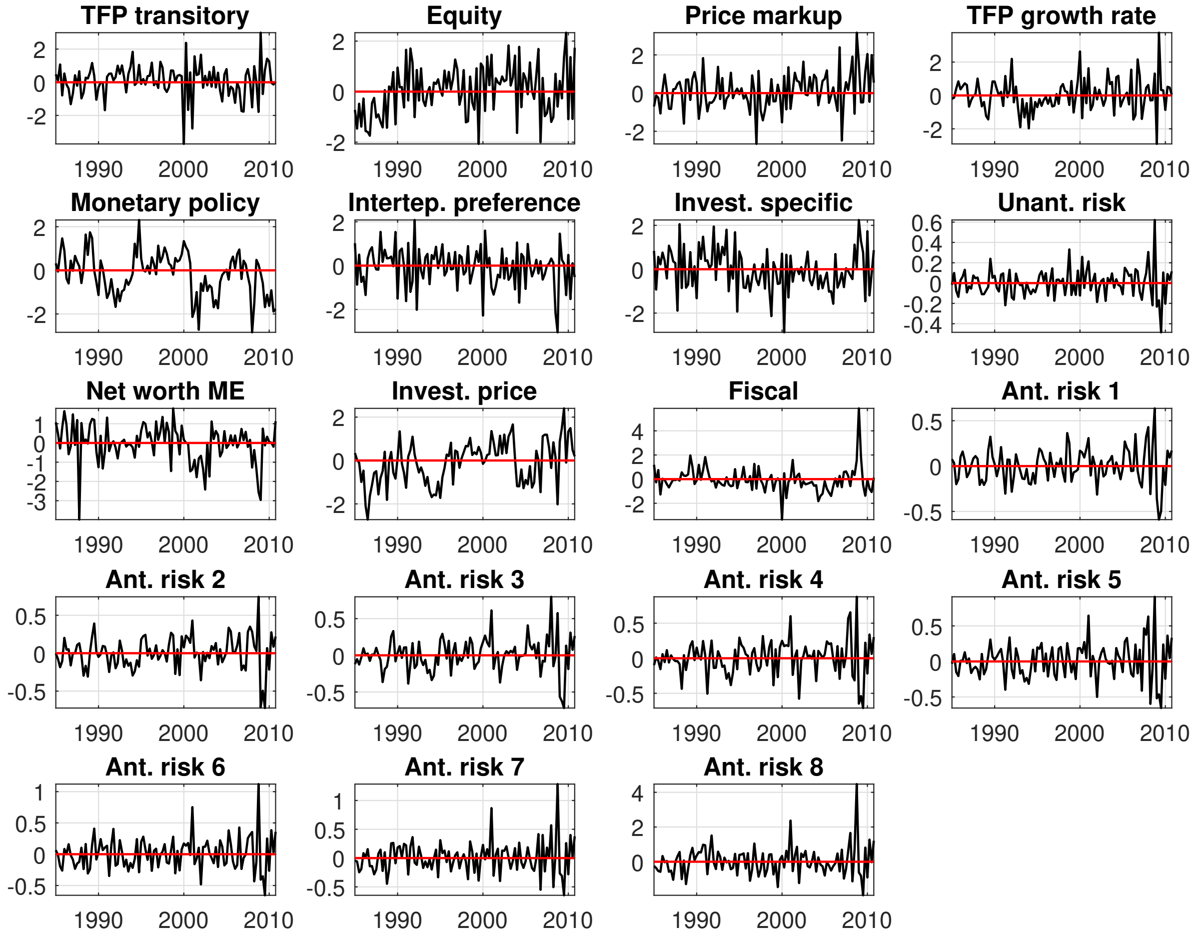
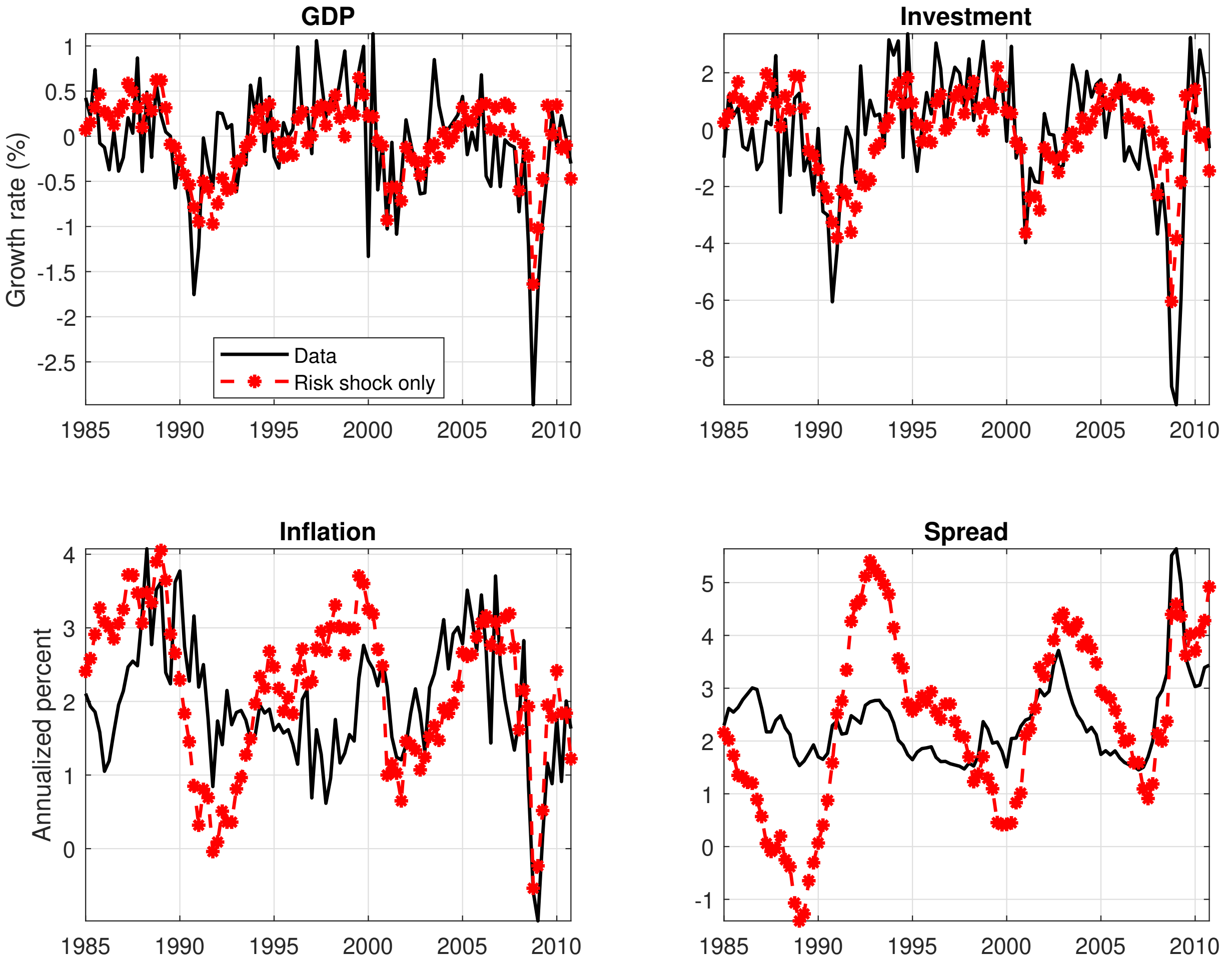


Figure A2: Smoothed shocks



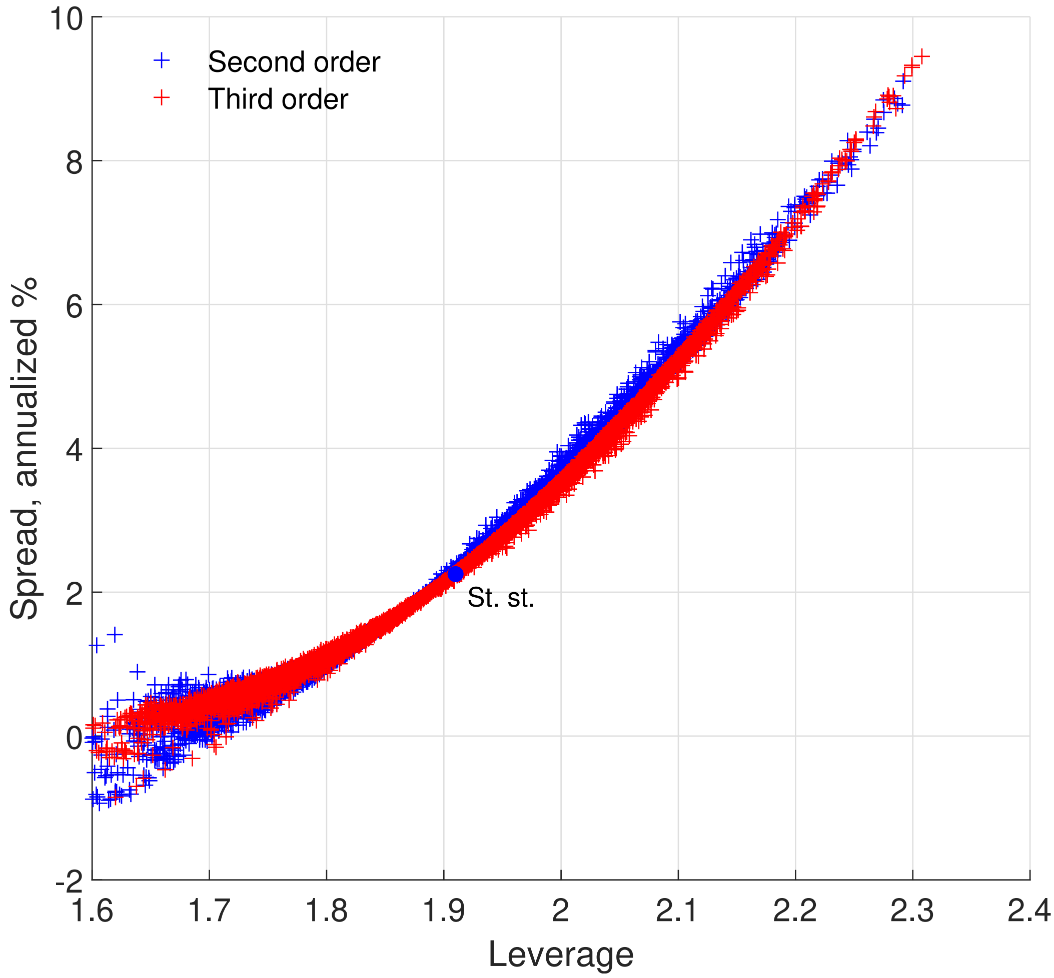
Notes: Figure A2 shows the smoothed shocks of the estimated DSGE model, normalized such that the units of the y-axis is standard deviations of each shock. The panels of anticipated risk 1–8 show the anticipated component of the risk shock from 1 to 8 quarters ahead.

Figure A3: Data and the risk shock



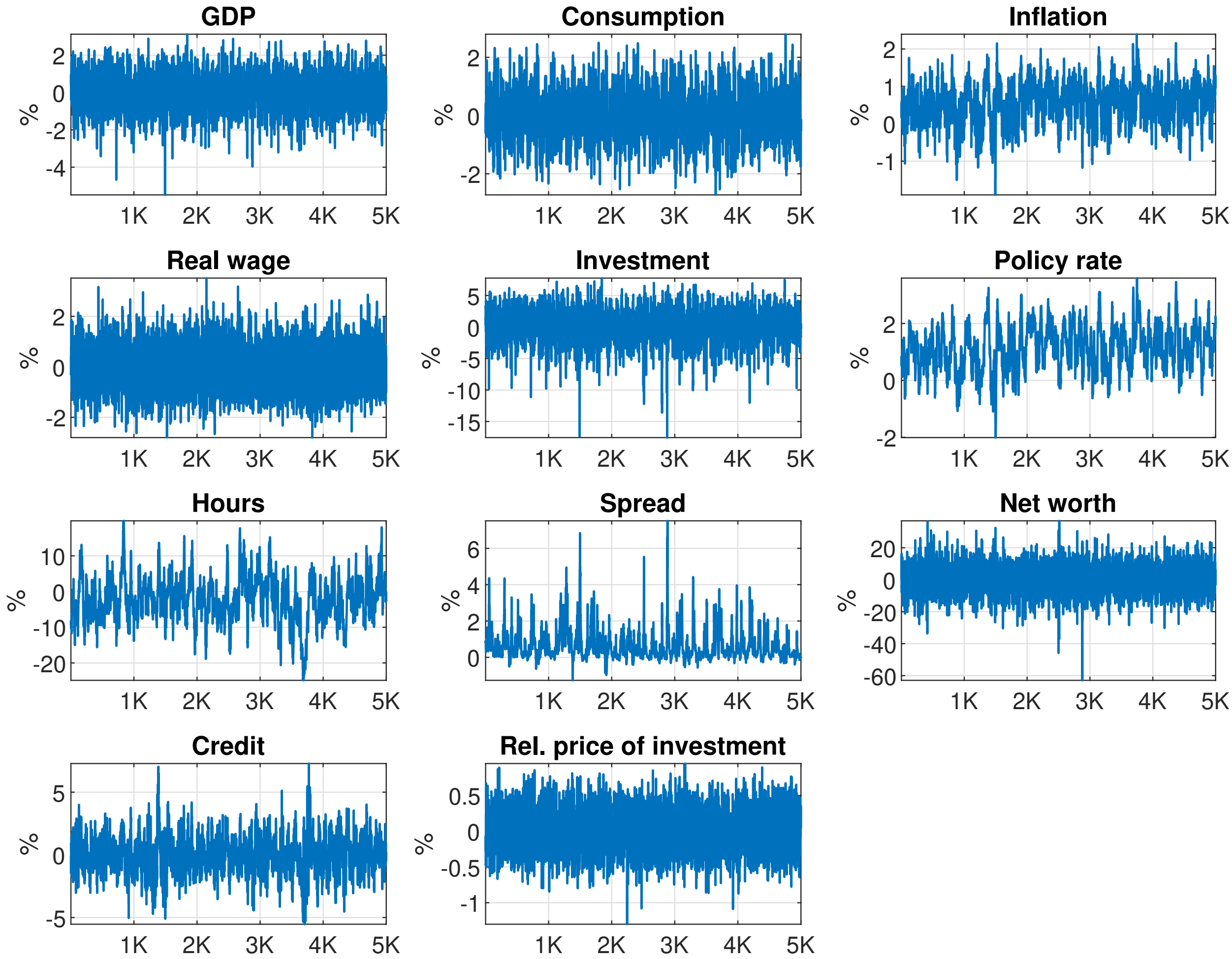
Notes: The black (solid) line shows the data and the red (starred) line shows the smoothed series for each variable when only feeding the anticipated and unanticipated components of the risk shock to the model.

Figure A4: Leverage-spread schedule



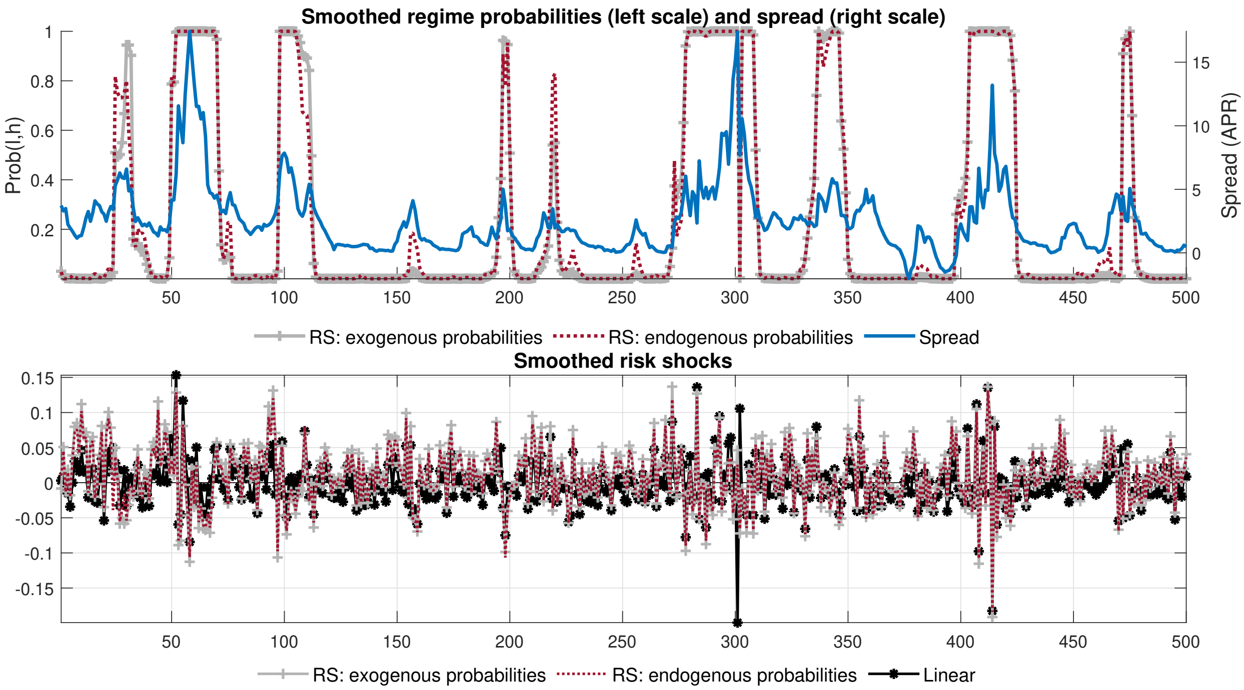
Notes: Figure A4 shows the simulated leverage-spread in the nonlinear model: second- and third-order approximations.

Figure A5: Simulated data from the third-order model



A4 Regime-Switching Model Additional Results

Figure A6: Smoothed probabilities and shocks: Regime-switching (RS) and linear models



Notes: The charts show the first 500 observations from the total of 5,000 used to estimate the models.

Table A1: Estimated parameters for regime-switching model

	Description	Dist. mean[std.]	mode
ξ_w	Calvo wages	$\mathcal{B}$ 0.7500 [0.1000]	0.6764
b	Habit in consumption	$\mathcal{B}$ 0.5000 [0.1000]	0.7240
σ_a	Curvature capital util. cost	$\mathcal{G}$ 1.0000 [1.0000]	1.2581
S	Curvature invest. adj. cost	$\mathcal{N}$ 5.0000 [3.0000]	5.7084
ξ_p	Calvo prices	$\mathcal{B}$ 0.5000 [0.1000]	0.8195
α_π	Taylor rule: inflation	$\mathcal{N}$ 1.5000 [0.2500]	1.7839
ρ_p	Taylor rule: smoothing	$\mathcal{B}$ 0.7500 [0.1000]	0.8306
ι	Indexing price inflation	$\mathcal{B}$ 0.5000 [0.1500]	0.9026
ι_w	Indexing wage inflation	$\mathcal{B}$ 0.5000 [0.1500]	0.4435
ι_μ	Indexing productivity	$\mathcal{B}$ 0.5000 [0.1500]	0.9432
$\alpha_{\Delta y}$	Taylor rule: GDP	$\mathcal{N}$ 0.2500 [0.1000]	0.4050
ρ_{λ_f}	AR technology growth	$\mathcal{B}$ 0.5000 [0.2000]	0.0772
ρ_ε	AR price markup	$\mathcal{B}$ 0.5000 [0.2000]	0.9336
ρ_{ζ_I}	AR transitory technology	$\mathcal{B}$ 0.5000 [0.2000]	0.9842
ρ_{ζ_C}	AR investment efficiency	$\mathcal{B}$ 0.5000 [0.2000]	0.5572
ρ_μ	AR intertemporal pref.	$\mathcal{B}$ 0.5000 [0.2000]	0.9482
ρ_σ	AR equity	$\mathcal{B}$ 0.5000 [0.2000]	0.4271
ρ_{μ_Υ}	AR risk	$\mathcal{B}$ 0.5000 [0.2000]	0.9827
ρ_g	AR fiscal	$\mathcal{B}$ 0.5000 [0.2000]	0.8576
ρ_γ	AR price of investment	$\mathcal{B}$ 0.5000 [0.2000]	0.9848
σ_ε	Std. technology growth	$\mathcal{IG}$ 0.0100 [1.0000]	0.0093
σ_{λ_f}	Std. price markup	$\mathcal{IG}$ 0.0100 [1.0000]	0.0096
σ_{ζ_I}	Std. transitory technology	$\mathcal{IG}$ 0.0100 [1.0000]	0.0053
σ_{ζ_C}	Std. investment efficiency	$\mathcal{IG}$ 0.0100 [1.0000]	0.0202
σ_R	Std. intertemporal pref.	$\mathcal{IG}$ 0.0100 [1.0000]	0.0327
σ_μ	Std. monetary	$\mathcal{IG}$ 0.0100 [1.0000]	0.0013
σ_{σ_0}	Std. risk	$\mathcal{IG}$ 0.0100 [1.0000]	0.0090
σ_{μ_Υ}	Std. price of investment	$\mathcal{IG}$ 0.0100 [1.0000]	0.0028
σ_N	Std. equity	$\mathcal{IG}$ 0.0100 [1.0000]	0.0064
σ_γ	Std. fiscal	$\mathcal{IG}$ 0.0100 [1.0000]	0.0187
σ_g	Std. net worth ME	$\mathcal{W}$ 0.0100 [5.0000]	0.0729
σ_{σ_n}	Std. anticipated risk	$\mathcal{IG}$ 0.0100 [1.0000]	0.0126
$\rho_{\sigma,n}$	Corr. between signals	$\mathcal{N}$ 0.0000 [0.5000]	0.7269
$\alpha_{1,2}$	Probability functions	$\mathcal{G}$ 1.2877 [0.4061]	1.6991
$\alpha_{2,1}$	Probability functions	$\mathcal{G}$ 1.6509 [1.1660]	2.3478
$\alpha_{1,3}$	Probability functions	$\mathcal{G}$ 1.9109 [0.4734]	3.4908
$\alpha_{3,1}$	Probability functions	$\mathcal{G}$ 1.6509 [1.1660]	0.8969
$\bar{\gamma}_1$	Probability functions	$\mathcal{G}$ 2.0000 [0.5000]	2.2431
$\bar{\gamma}_2$	Probability functions	$\mathcal{G}$ 2.0000 [0.5000]	1.8940
β_1	Discount factor regime 1	$\mathcal{B}$ 0.9960 [0.0010]	0.9941
β_2	Discount factor regime 2	$\mathcal{B}$ 0.9960 [0.0010]	0.9971
σ_l	Risk level regime 1	$\mathcal{B}$ 0.2500 [0.0050]	0.2460
σ_2	Risk level regime 2	$\mathcal{B}$ 0.2600 [0.0050]	0.2492
σ_3	Risk level regime 3	$\mathcal{B}$ 0.2750 [0.0050]	0.2789
c_r	Interest rate constant	$\mathcal{N}$ 0.0150 [0.0050]	0.0151