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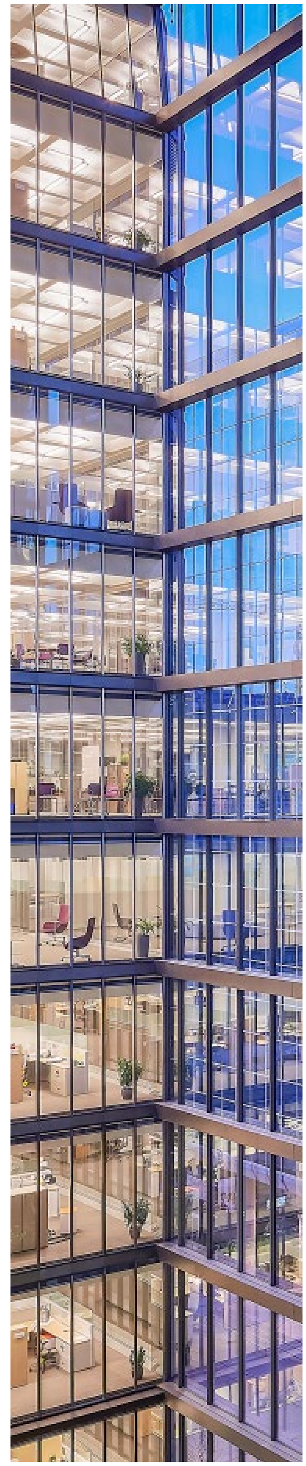

Central Bank Liquidity Facilities and Market Making

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Abstract

In the onset of the COVID-19 crisis, central banks purchased large volumes of assets in an effort to keep markets operational. We model one such central bank, which purchases assets from dealers to alleviate balance sheet constraints. Asset purchases can prevent market breakdown, improve price efficiency and reduce dealer risk positions. A central bank that purchases assets at their expected value is able to achieve market outcomes as if dealers were unconstrained. Absent other concerns, central banks can maximize welfare by purchasing assets at a premium, though they may create market distortions. Alternatively, central banks who bear costs associated with large interventions may only be willing to purchase assets at a discount. In the absence of leverage constraints, lending programs are as effective as asset purchases; when leverage constraints are present, lending programs lose effectiveness.

Topics: Coronavirus disease (COVID-19), Economic models, Financial institutions, Financial markets, Market structure and pricing

JEL codes: G10, G20, L10

1 Introduction

In March 2020, the spread of COVID-19 led to global market turmoil. Behind this turmoil was a need to convert assets to cash by large asset managers to meet redemptions and margin calls. As a result, large volumes of fixed-income securities were sold to dealers who had limited balance sheet space and risk tolerance. Many previously liquid markets either became illiquid or seized altogether. To calm markets, central banks intervened on a historically large scale and scope using several measures, including asset purchase programs.[1] Asset purchase programs restore market liquidity in specific assets by acting on the balance sheets of intermediaries, since most fixed-income trading by clients is intermediated by dealers in OTC markets.

Asset purchase programs target fixed-income securities that investors sell to meet liquidity needs in a crisis. Many of these investors are non-bank financial intermediaries (NBFI), such as asset managers, mutual funds and hedge funds. These NBFIs channel savings and investment outside the traditional banking sector, often through leveraged securities holdings and securities financing markets. The capacity of bank-owned intermediaries has not kept pace with growth in the NBFI sector, increasing demands on the balance sheets of bank-owned dealers. [2] We study how central bank asset purchase programs should be structured to overcome frictions created by the dealer balance sheet channel.

We model these interventions through two key frictions. First, we model clients who are unable to trade with each other directly; instead, trades are intermediated by a set of risk-averse, Cournot-oligopolistic dealers. This friction represents the reality in many fixed-income markets, where dealers play a key role in intermediating trades. Second, we allow the central bank to engage in interventions with dealers rather than all market participants. In practice, many central banks have a network of bank-owned dealers with whom they

[1] The Bank of England, the US Federal Reserve and the Bank of Canada each launched purchase programs for a wide range of fixed-income securities in addition to their traditional bank-focused lending programs.

[2] For example, in the UK, EU and the US, the size of the sovereign bond market has recently exceeded the assets of bank-owned dealers who intermediate these markets, suggesting capacity limitations for intermediaries (Hauser 2021; Duffie 2020).

engage regularly in market operations, including issuance of government debt and open market operations for monetary policy. Operational constraints and other considerations may prevent central banks from trading directly with non-dealers.

In a baseline version of the model, a financial crisis, characterized by high demand for liquidity by sellers, leads to price deviations from common value and a reliance on agency trading by dealers. Agency trading occurs when dealers perform only a matching function at large price deviations rather than taking assets onto their balance sheets. These outcomes reflect what was observed in March 2020 in the US and Canadian corporate bond markets (Fontaine et al. 2021; Kargar et al. 2020). In our model, high balance sheet costs can lead to market breakdown where dealers are unwilling to purchase any additional assets, which is consistent with the severe breakdown in intermediation that occurred in March 2020 (Fontaine and Walton 2020; Fleming and Ruela 2020).

We show that central bank purchases increase dealers' capacity to purchase assets from investors. A central bank purchasing assets from dealers in a crisis can prevent market breakdown, mitigate mispricing, reduce dealer risk positions and improve welfare at the cost of a larger central bank balance sheet. Preventing market breakdown prevents potential default for investors relying on market liquidity to raise cash. This is consistent with the empirical work by Carlson and Macchiavelli (2020), who find that the Federal Reserve's facilities allowed dealers to provide better bond market liquidity. Mispricing of assets is relevant for monetary policy if the targeted assets directly affect the sovereign yield curve or if monetary policy pass-through is impaired in the targeted asset. Low dealer risk positions are desirable since bank-owned dealers also face funding challenges in a crisis, for example, from customers' draws on committed lines of credit.

The impact of asset purchases depends on the price set by the central bank. When the central bank conducts purchases at the asset's common value, dealers trade with market participants as if they had no balance sheet costs of inventory. In return, these dealers sell their entire outstanding inventory to the central bank. Alternatively, if the central bank pursues a welfare-maximizing policy, it purchases assets at a premium, inducing dealer short positions. In this case, the premium paid and the corresponding size of the central bank's asset purchases increase in dealers' balance sheet costs, asset riskiness and selling demand. However, while the total size of the intervention increases in the number of dealers, the premium paid decreases as the number of dealers increases due to competition.

Purchase premiums are contrary to Walter Bagehot's seminal principles of central bank crisis intervention (Bagehot 1873), which are often summarized as "[T]o avert panic, central banks should lend early and freely (i.e., without limit), to solvent firms, against good collateral, and at 'high rates'." (Madigan 2009). More recent work by Rochet and Vives (2004) provides a theoretical framework to describe solvent but illiquid banks and derives policy implications consistent with Bagehot. The principle of high, or penalty, rates is meant to discourage excessive use of central bank facilities and encourage the return of private intermediaries to markets. Our model suggests that penalty rates may inhibit the efficacy of central bank intervention if pass-through to market participants without direct access to the central bank is impaired.

In an extension to the model, we consider a central bank that suffers a disutility from large interventions, which we refer to as risk aversion.[3] In our model, if the central bank is sufficiently risk averse, it may not intervene at a high enough price to incentivize dealers to act as if they are unconstrained. Put another way, the central bank may only offer to purchase assets below common value, lowering the impact on its balance sheet. The level of risk aversion necessary to discourage central banks from intervening at a premium is increasing in the liquidity shock and dealer balance sheet costs, but decreasing in dealer inventories, number of dealers and asset risk. This suggests that central banks may be less willing to fully alleviate frictions in risky markets with a large number of small dealers, but may be more willing to do so when markets have a high demand for liquidity and when dealer balance sheet costs are high.

[3]Central banks may incur costs from large interventions, including concerns about moral hazard, operational costs, reputational risk and the possibility of asset default.

We contrast asset purchases with secured central bank lending. Secured central bank lending can create similar market outcomes to asset purchases; absent other constraints, optimal asset purchases induce dealers to buy and sell equal amounts from other market participants as an optimal lending program. Following the 2007–2008 financial crisis, financial regulators implemented the Basel III framework, which includes a leverage ratio. Our model shows that in the presence of a leverage constraint, lending programs lose effectiveness compared to asset purchases.

Finally, we consider clients who hold two correlated assets, but where the central bank only intervenes in one. In practice, central banks may be constrained to intervene only in certain markets due to operational constraints and legal constraints. When assets are positively correlated, central bank purchases improve prices in both assets as dealers substitute one asset for another, selling quantities of one to the central bank and purchasing quantities of the other from clients. However, when assets are negatively correlated, dealers are induced to sell both assets; one to the central bank and one to market participants. While mispricing is reduced in the asset bought by the central bank, mispricing is worsened in the other.

1.1 Related Literature

We build on prior work that describes liquidity crises in OTC markets. Weill (2007) develops a search-theoretic model of dynamic liquidity provision with competitive capital-constrained dealers, which describes how price dislocations occur when there is an imbalance between buyers and sellers. Similarly, Lagos et al. (2011) describe how well-capitalized dealers may withdraw from providing liquidity in a crisis. Both works imply a role for central banks to provide liquidity through asset purchases. Our work complements these existing models by explicitly deriving optimal central bank policies under a number of circumstances, given an imperfectly competitive, balance sheet-constrained dealer sector.

Our paper complements work on optimal interventions by focusing on the channel of dealer balance sheet constraints. Philippon and Skreta (2012) use mechanism design to characterize optimal central bank interventions in the context of asymmetric information between borrowers and lenders. They find optimal interventions are achieved through lending. Similarly, Tirole (2012) studies optimal intervention mechanisms under adverse selection and heterogeneity of firms' asset holdings and finds that the regulator optimally purchases the worst-quality assets from the market to partially restore a market freeze.

Our paper contributes to analyses of how dealer balance sheet constraints affect market liquidity. In a related model, Cimon and Garriott (2019) show that when regulatory balance sheet constraints bind, welfare may decrease as dealers resort to agency trading. An and Zheng (2020) and Li and Li (2020) find similar results. With respect to central bank intervention, Acharya et al. (2017) describe the conditions of dealers that participated in the US Federal Reserve's lender of last resort programs and that dealers with greater leverage and less-liquid collateral tended to participate more. This suggests dealers with weaker balance sheets self-select into facilities and that such facilities are therefore effective for alleviating balance sheet constraints. Drechsler et al. (2016) show analogous results for European banks and the lender of last resort facilities of the European Central Bank and also find that banks who access the facilities purchase distressed assets.

Our model contributes to understanding the liquidity channel of quantitative easing (QE), where central banks purchase sovereign bonds to lower long-term interest rates. Our results are consistent with Bailey et al. (2020), who show that QE purchases may be especially effective during a period of market dysfunction, enhancing monetary policy transmission by improving market liquidity. Similarly, Acharya et al. (2020) shows that the announcement of sovereign bond purchases by the European Central Bank (ECB) in 2012 reduced the risk of sovereign bond fire sales by investors, whereas a prior ECB lending program targeted at banks increased the risk of sovereign bond fire sales. Pelizzon et al. (2020) find that QE in government bond markets may cause asset mispricing in the presence of trading frictions.

Our model is consistent with findings on central bank purchase programs of non-sovereign assets. Falato et al. (2020) find that the US Federal Reserve's purchases of corporate bonds helped mitigate stress faced by corporate bond funds and, in turn, improved conditions for primary bond issuance. Mäkinen et al. (2020) empirically examine the effect of corporate bond purchases by the ECB and find no evidence of yields decreasing relative to ineligible bonds, consistent with the substitution effects in our model.

While our paper studies central bank asset purchases from dealers, central banks have engaged in a wide range of programs, including direct purchases from issues and purchases from the secondary market. Our paper provides results that are also inline with work evaluating these other types of policy intervention during the COVID-19 crisis, which show improved prices and improvement in liquidity (Boyarchenko et al. 2020; D'Amico et al. 2020; O'Hara and Zhou 2021; Gilchrist et al. 2020).

The growth of NBFI means, increasingly, a large class of investors rely on market liquidity rather than traditional bank lending. Buiter and Sibert (2008) argue that as credit is increasingly provided by NBFIs, central banks should have the ability to conduct open market operations on a wide range of securities, acting as "market maker of last resort." In practice, central banks have correspondingly taken a greater role in supporting market liquidity in crises (Grad et al. 2011). d'Avernas et al. (2020) show that traditional central bank facilities are not sufficient to reduce liquidity risk in the presence of a large NBFI sector, but that central bank asset purchase programs reduce risk without directly providing liquidity to NBFIs. Jeanne and Korinek (2020) find that when macroprudential policy is ineffective due to a large NBFI sector, it is optimal for the government to intervene less generously as an imperfect substitute for regulation.

2 Model

We construct a three-period model with a single risky asset, in the style of Cimon and Garriott (2019). At the beginning of $t = 0$ the asset has a common value $v = 1$, which is publicly known to all agents. The asset's payoff, which is realized at the the of $t = 2$, is normally distributed with expected value $v = 1$ and standard deviation σ. One client wishes to buy the asset, while another wishes to sell the asset. The two clients are unable to interact and, instead, must buy and sell from a group of N Cournot-competitive dealers. There is a central bank who may intervene with the dealers.

2.1 Client Demand

The two clients, one who wishes to buy the asset and one who wishes to sell the asset, each have a utility function that reflects their role. The client who wishes to buy the asset has a utility function:

$$U_S = (\Gamma - \epsilon + v - P_S) \sum_{n=1}^{N} S_n - \frac{\lambda}{2} (\sum_{n=1}^{N} S_n)^2, \tag{1}$$

while the client who wishes to sell the asset has a utility function:

$$U_B = (P_B + \Gamma + \epsilon - v) \sum_{n=1}^{N} B_n - \frac{\lambda}{2} (\sum_{n=1}^{N} B_n)^2. \tag{2}$$

$\sum_{n=1}^{N} B_n \geq 0$ and $\sum_{n=1}^{N} S_n \geq 0$ represent the quantities bought and sold by all dealers.[4] Γ represents the size of the clients' gains from trade, while ϵ, referred to as the liquidity shock, represents the relative liquidity preference for sellers versus buyers. Each client maximizes their utility function by selecting a total quantity of the asset to either sell or buy from all dealers combined. Once maximized, these utility functions correspond to inverse demand functions at which dealers can sell or buy the asset:

$$P_S = v + \Gamma - \epsilon - \lambda \sum_{n=1}^{N} S_n, \tag{3}$$

$$P_B = v - \Gamma - \epsilon + \lambda \sum_{n=1}^{N} B_n. \tag{4}$$

[4] The parameter restrictions $\sum_{n=1}^{N} B_n \geq 0$ and $\sum_{n=1}^{N} S_n \geq 0$ are able to generate corner solutions where buyers or sellers trade a quantity 0. However, they cannot reverse their role from buyer to seller, or vice versa.

The inverse demand curves further clarify the interpretation to the values of Γ and ϵ. Were clients able to trade directly with each other at a price of $P_S = P_B$, the prevailing price would be given $v - \epsilon$ with a quantity traded of $\frac{\Gamma}{\lambda}$. The value of ϵ can then be seen directly as a liquidity preference by clients, who may favour holding cash over the asset. Similarly, the value of Γ can be seen as the relative gain from trading between the clients versus their disutility from conducting large transactions.

Assumption 1a: There is a relatively higher liquidity demand by sellers, $\epsilon > 0$.

Assumption 1b: The gains from trade are large compared to the liquidity shock, $\Gamma > 2\epsilon$.

We focus on the case where $\epsilon > 0$, in which clients have a preference to sell assets to dealers rather than to buy them. This also represents the case where clients have a preference for cash over holding the asset at common value. In addition, this best represents the environment in which a central bank would engage in an asset purchase program. The alternative, where clients have a relative preference to buy assets from dealers rather than sell them, is unlikely to be one in which central bank asset purchases are either necessary or beneficial.[5]

We also focus on the case where $\Gamma > 2\epsilon$, in which gains from trade are large compared to the value of the liquidity shock. This assumption allows us to focus on the balance sheet constraints of the dealer banks. In the opposite case, dealers may optimally choose not to intermediate trades, even absent balance sheet constraints.

2.2 Dealers

Each dealer begins with a quantity I of the asset in their inventory. Dealers are able to trade the asset in any quantity with the clients. That is to say, initially, the dealers have no short-selling constraint or inventory limitation.[6]

[5] The reverse case, where $\epsilon < 0$, could be used to represent a case of a central bank reducing the size of its holdings in the presence of excess market demand for assets.

[6] In Section 5 we address lending programs and dealers with borrowing constraints.

Each dealer views the demand functions of the clients, P_B and P_S. Each dealer also views the central bank purchase premium ζ, which is to say that the central bank purchases assets from the dealer at a price of $\zeta + v$.

Each dealer simultaneously selects a quantity to buy B_n and sell S_n to the clients, and a quantity F_n to sell the central bank to maximize their profit function:

$$\pi_n = (P_S - v)S_n + (v - P_B)B_n - \frac{\kappa\sigma^2}{2}(I - F_n + B_n - S_n)^2 + \zeta F_n \tag{5}$$

where κ is the cost of the dealer's balance sheet space.

2.3 Central Bank

A central bank is able to undertake interventions by offering to purchase inventory from each dealer at a price $(\zeta + v)$. This action is referred to as an asset purchase, where ζ is the asset purchase premium.

If a central bank purchases assets, their profit function is given by:

$$\pi_C = -\zeta \sum_{n=1}^{N} F_n - \frac{\kappa_C \sigma^2}{2}(\sum_{n=1}^{N} F_n)^2, \tag{6}$$

where F_n is the amount purchased from each dealer n and κ_C is a term representing the central bank's disutility for purchasing from dealers. For simplicity, we refer to this as the central bank's "risk aversion" term.

The central bank's risk aversion relates to the value of risk in its portfolio and can be interpreted in the context of the central bank wishing to avoid losses from its interventions. A central bank may view these interventions as encouraging moral hazard on behalf of the financial system, may not wish to be seen as favouring commercial banks or may simply have concerns over expanding the size of its own balance sheet. The risk aversion term, κ_C, can be seen as a combinations of these motives.

Figure 1: Model Timing

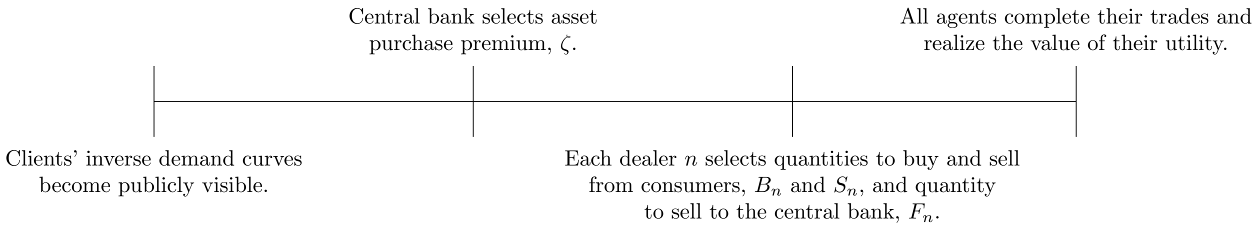

Figure 1 illustrates the timing of the model. In $t = 0$, clients formulate their inverse demand functions, which become visible to all agents. The central bank then selects ζ, the asset purchase premium. In $t = 1$, each dealer selects quantities B_i and S_i to buy and sell from clients, along with F_i, the quantity to sell to the central bank. Finally in $t = 2$, all trades are completed and agents realize the value of their utility.

2.4 Timing

In $t = 0$, client gains from trade Γ and the liquidity shock ϵ become public knowledge, and dealers are each endowed with an identical publicly visible inventory $I \geq 0$. Given these variables, the central bank sets the price ζ of its intervention. In $t = 1$, the dealers trade with clients and choose whether to sell to the central bank. In $t = 2$, all agents realize the value of their trades. The timing of the model is illustrated in Figure 1.

3 Baseline Equilibrium with No Interventions

First, we solve a model in which the central bank does not intervene.

Assumption 2: Each dealer sells a quantity $F_n = 0$ to the central bank.

This section serves as a baseline equilibrium in which dealers may become constrained without intervention. We relax this assumption in Section 4. In this section, each dealer has a profit function given by Equation 5 with $F_n = 0$.

We solve for a symmetric Cournot-competitive equilibrium. Each dealer selects quantities B_n and S_n, taking the actions of each other dealer as given. We then impose symmetry, such that each dealer buys an identical quantity, and each dealer sells an identical quantity.

The equilibrium quantity bought by each dealer is given by:

$$B^0 = \begin{cases} \dfrac{\Lambda(\Gamma + \epsilon - KI) + 2\Gamma K}{\Lambda(\Lambda + 2K)} & \text{if } I < \hat{I}, \\ 0 & \text{if } I \geq \hat{I}. \end{cases} \tag{7}$$

The equilibrium quantity sold by each dealer is given by:

$$S^0 = \begin{cases} \dfrac{\Lambda(\Gamma - \epsilon + KI) + 2\Gamma K}{\Lambda(\Lambda + 2K)} & \text{if } I < \hat{I}, \\ \dfrac{KI + \Gamma - \epsilon}{K + \Lambda} & \text{if } I \geq \hat{I}. \end{cases} \tag{8}$$

For notational simplicity, we define:

$$K = \kappa\sigma^2, \tag{9}$$

$$\Lambda = \lambda(N+1) \tag{10}$$

where K is the balance sheet cost of the risky asset and Λ is the pricing concession from dealer Cournot competition.

In both the dealers' supply and demand functions, there is a discontinuity around $\hat{I}$, which is given by:

$$\hat{I} = \frac{\Lambda(\Gamma + \epsilon) + 2\Gamma K}{\Lambda K}. \tag{11}$$

For initial inventories below $\hat{I}$, dealers will both buy and sell some quantity of the asset. For higher inventories, dealers will be unwilling to buy any positive quantity.[7] We refer to this latter case as a "market breakdown," as the dealers are no longer performing an intermediary role. In this section, we focus on the case where the dealers both buy and sell the asset, but discuss the market breakdown further in Section 3.1, below.

[7] Were $\Gamma < 2\epsilon$, a similar discontinuity, where dealers with a sufficiently low inventory would be unwilling to sell any positive quantity, could exist.

When dealers both buy and sell assets, the equilibrium prices are given by:

$$P_B^0 = v - \Gamma - \epsilon + \lambda N \frac{\Lambda(\Gamma + \epsilon - KI) + 2\Gamma K}{\Lambda(\Lambda + 2K)} \tag{12}$$

$$P_S^0 = v + \Gamma - \epsilon - \lambda N \frac{\Lambda(\Gamma - \epsilon + KI) + 2\Gamma K}{\Lambda(\Lambda + 2K)}. \tag{13}$$

The resulting bid-ask spread is:

$$P_S^0 - P_B^0 = \frac{2\Gamma}{N+1}. \tag{14}$$

This bid-ask spread has a feature present in Cimon and Garriott (2019); namely, that it is independent of the dealers' balance sheet constraints. The dealer's balance sheet constraint κ acts by moving prices higher or lower. However, the difference between the price bought and sold depends only on the gains from trade between clients and on the number of dealers in the model. This result will flow into the results of any intervention, where the bid-ask spread will remain constant, but where prices to both buy and sell will adjust.

3.1 Policy Evaluation

We study possible interventions based on four factors: (i) market breakdown, (ii) pricing deviation from common value, (iii) pricing deviation from an unconstrained market, and (iv) dealer risk positions.

We define market breakdown as an equilibrium in which dealers are unwilling to purchase any quantity of the asset. Were clients able to trade directly with each other, a similar state would not occur unless clients had no gains from trade ($\Gamma = 0$). However, because of dealer inventory constraints, clients who wish to sell may only be able to trade a quantity of 0, representing a welfare cost imposed by the intermediaries. Market breakdown occurs at $I \geq \hat{I}$, defined by Equation 11. The factors influencing $\hat{I}$ are illustrated in Figure 2.

We define pricing deviation from common value as the difference between the average of the buying and selling price $P_B + P_S$ and twice the asset's common value v. The equilibrium pricing deviation from common value is given by:

$$PDC^0 = P_B^0 + P_S^0 - 2v,$$
$$= -2\left(\frac{NI\lambda K + \lambda\epsilon + 2\epsilon K}{\Lambda + 2K}\right). \tag{15}$$

Pricing deviation from an unconstrained market represents the difference between the average of the buying and selling price $P_B + P_S$ and the hypothetical buying and selling price were dealers to have no balance sheet costs ($\kappa = 0$). This unconstrained price is given by $P_B(\kappa = 0) + P_S(\kappa = 0) = 2\frac{(N+1)v-\epsilon}{(N+1)}$. The equilibrium pricing deviation from an unconstrained dealer market is given by:

$$PDU^0 = P_B^0 + P_S^0 - (P_B^0(\kappa = 0) + P_S^0(\kappa = 0)),$$
$$= -2NK\left(\frac{I\Lambda + 2\epsilon}{(N+1)(\Lambda + 2K)}\right). \tag{16}$$

The dealers' net risk position is the total volume of the asset held on the dealers' balance sheets at the end of trading, $I - F + B - S$. The equilibrium risk position is given by:

$$RP^0 = N(I + B^0 - S^0 - F^0),$$
$$= N\left(\frac{I\Lambda + 2\epsilon}{\Lambda + 2K}\right). \tag{17}$$

For all four measures, we focus on cases where the liquidity demand from sellers is greater than that of buyers ($\epsilon > 0$). This represents periods where the market is attempting to sell assets back to dealers, on net, which is usually the case during a financial crisis.

Without intervention, assets trade at prices below both their common and their unconstrained values, and dealers take on positive risk positions. If dealers are sufficiently endowed with an initial inventory, they may not choose to engage with asset sellers at any price. In Section 4, we show that by purchasing a positive quantity of assets a central bank can effectively lower the initial inventory and create desirable impacts on all four measures above.

The desirability of purchases comes with an important caveat. Increasing prices and reducing the dealer's net risk position is valuable if prices are below their common value and if dealers hold excess inventory. If prices are above their common values or dealers are short, these outcomes would likely be considered undesirable. Thus, it is not simply true that larger asset purchases uniformly result in more favourable outcomes, and the size and price of a central bank's intervention is of primary importance.

4 Risk-Neutral Central Bank

In this section, we relax Assumption 2 and allow the central bank to intervene. We solve a model in which the central bank is risk neutral by imposing Assumption 3.

Assumption 3: The central bank is risk neutral, such that $\kappa_C = 0$

We solve the model through backward induction. First, each dealer n selects B_n, S_n and F_n given the asset purchase premium (ζ) and each other dealer's best response. Second, the central bank selects ζ, given the dealer's best response functions. We search for a symmetric Cournot-competitive equilibrium.

When ζ is such that $-\Gamma - \epsilon < \zeta < \Gamma - \epsilon$,[8] each dealer's best responses are given by:

$$B^*(\zeta) = \frac{\Gamma + \epsilon + \zeta}{\Lambda}, \tag{18}$$

$$S^*(\zeta) = \frac{\Gamma - \epsilon - \zeta}{\Lambda}, \tag{19}$$

$$F^*(\zeta) = \frac{\Lambda(IK + \zeta) + 2K(\zeta + \epsilon)}{\Lambda K}. \tag{20}$$

[8] For $-\Gamma - \epsilon > \zeta$, dealers would choose to purchase a quantity of zero, while for $\zeta > \Gamma - \epsilon$, dealers would sell a quantity of zero. For expositional simplicity, we do not focus on these cases, as they do not occur in optimal risk-neutral central bank interventions.

The best response functions correspond to market prices of:

$$P_B^*(\zeta) = v - \Gamma - \epsilon + N\frac{\Gamma + \epsilon + \zeta}{\Lambda}, \tag{21}$$
$$P_S^*(\zeta) = v + \Gamma - \epsilon - N\frac{\Gamma - \epsilon - \zeta}{\Lambda}. \tag{22}$$

A feature of an intervention is the simplification of the dealers' supply and demand of the asset to clients. Equations 18 and 19 contain only variables representing the clients' demand functions, the price of the intervention and the number of dealers. All other factors, such as initial dealer inventories, asset risk and balance sheet costs, are only present in the amount sold to the central bank, seen in Equation 20. Put another way, our model predicts that a central bank that intervenes at a fixed price should expect market outcomes that are independent of initial dealer inventories and asset risk. Instead, the change in behaviour from these factors will be seen by the central bank in the amount sold to them.

4.1 Interventions at Common Value

Before considering the central bank's optimal intervention, we first consider an arbitrary intervention of particular interest: one in which the central bank is willing to purchase the asset at its common value $\zeta = 0$.

In practice, common value interventions may appeal to both a central bank and to market participants. Such an intervention may be viewed by these parties as not distorting asset prices, while also not providing either a premium or a penalty to those involved. While this is not an optimal intervention, we show that it does have a number of appealing properties. When the central bank intervenes at $\zeta = 0$, the dealers' optimal purchases and sales are then given by:

$$B^*(\zeta = 0) = \frac{\Gamma + \epsilon}{\Lambda}, \tag{23}$$
$$S^*(\zeta = 0) = \frac{\Gamma - \epsilon}{\Lambda}, \tag{24}$$

which are equal to the dealers' optimal purchases and sales if they had no balance sheet costs ($\kappa = 0$).

Proposition 1 (Central Bank Asset Purchases at Common Value) *If the central bank purchases assets from dealers at their common value, dealers will take a risk position of zero ($RP = 0$) and prices will be equal to their unconstrained value ($PDU = 0$).*

A key feature of purchasing assets at common value is that both the equilibrium risk position and the pricing deviation from an unconstrained dealer market will be zero. That is to say that if the central bank purchases assets from dealers at their common value, the dealers will trade with the market at prices that would prevail were they to have no balance sheet costs. The dealers will sell a sufficient quantity of the asset to the central bank such that they have no asset risk on their balance sheet. Instead of risk being borne by dealers, the risk will be borne by the central bank.

4.2 Optimal Central Bank Interventions

Next, we consider the central bank's welfare maximization decision. The central bank selects ζ to maximize the surplus of the entire economy, given by:

$$\begin{aligned} W = {} & N\left(B^*(\zeta) + S^*(\zeta)\right)\Gamma + N\left(B^*(\zeta) - S^*(\zeta)\right)\epsilon - \frac{\lambda}{2}\left(\left(NB^*(\zeta)\right)^2 + \left(NS^*(\zeta)\right)^2\right) \\ & - N\frac{K}{2}(I - F^*(\zeta) + B^*(\zeta) - S^*(\zeta))^2. \end{aligned} \tag{25}$$

The central bank considers the dealers' best response functions given by Equations 18 and 19, resulting in an optimal asset purchase price:

$$\zeta^* = \frac{2\epsilon K}{\Lambda(N+1) + 2KN}. \tag{26}$$

Figures 3 and 4 illustrate the impact of the central bank's optimal asset purchases on quantities and prices, respectively. When the central bank purchases assets from the dealers,

dealers optimally buy more from and sell less to clients. In turn, as dealers purchase more, prices adjust upwards. Seized markets and depressed prices are two common features of markets during a financial crisis. In our model, asset purchases by the central bank can alleviate both these issues.

Another way to view the change in traded quantities is through the lens of principal and agency trading. Kargar et al. (2020) show that during the COVID-19 crisis, dealers shifted to agency trading, despite principal trading being preferred by their clients. We define the quantity of agency trading as the minimum of the amounts that the dealer both buys and sells—that is to say, the quantity that the dealer buys and then immediately sells to other clients. Alternatively, principal trading is the absolute value of the difference between the dealer's purchases and sales. Figure 5 illustrates how intervention impacts the balance of these two trading types.

Asset purchases lead dealers to take on larger principal positions and to intermediate through agency trading less. However, dealers do not retain these larger positions on their balance sheets. Instead, dealers sell a large volume to the central bank. The central bank acts, in effect, as another client willing to buy from dealers.

Figure 6 illustrates the impact of the the intervention on total welfare. The welfare improvement depends on the underlying factors of the model, most interestingly, the balance sheet cost. Without intervention, balance sheet costs reduce welfare by penalizing dealers more heavily if they take large positions. With intervention, balance sheet costs play a different role, allowing the central bank more refined control over dealer actions through asset purchases. This same process will play an important role when assessing market outcomes.

Proposition 2 (Intervention Outcomes) *(i) When $\epsilon > 0$, optimal post-intervention risk positions are negative.*

(ii) Higher balance sheet costs move dealer post-intervention risk positions and pricing deviation from common value, closer to zero.

Figure 7 illustrates the impact of intervention on policy outcomes. The central bank's intervention not only increases welfare, but also reduces the total risk position taken by dealers and raises prices towards their common value. In fact, when there is excess selling, it is optimal for the central bank to move dealers to end trading with a short position, encouraging them to buy larger quantities from the market.

The pricing deviation under an optimal central bank regime is also of note. After the central bank intervenes, the average price is below the asset's common value but above the value that would prevail were dealers unconstrained. That is to say that a central bank's asset purchases will push prices above what they would be if investor demand were identical but dealers had no balance sheet constraint. This presents a challenge for the central bank, who may view such an action as distortionary, and offers a case for why a central bank may choose not to engage in welfare-optimizing asset purchase.

Balance sheet costs play a useful role in improving market outcomes. Higher balance sheet costs enable a central bank to more directly control dealer actions through changes to their inventory. In the extreme case, if balance sheet costs are zero, asset purchases by the central bank have no impact on dealer decisions.

Proposition 3 (Central Bank Balance Sheet) *The welfare-maximizing amount purchased by the central bank ($NF^*(\zeta^*)$) is increasing in balance sheet costs (κ), asset risk (σ^2), the liquidity shock (ϵ) and the number of dealers (N). The amount purchased per dealer ($F^*(\zeta^*)$) is decreasing in the number of dealers (N).*

The factors influencing the size of asset purchases are illustrated in Figure 8. Generally, the quantity purchased and price purchased move in tandem. Increases in price make the purchases more attractive to dealers, increasing use of the purchase facility. Because of this, the central bank is able to control the size of the asset purchase through the pricing mechanism.

5 Lending Programs

Lending programs by central banks often have similar objectives to asset purchase programs. These programs are both intended to provide banks and dealers with cash to support their core functions. Indeed, lending programs were likely beneficial during the COVID-19 crisis (O'Hara and Zhou 2021). In this section, we consider a lending program and its impact on dealer activities.

In practice, dealers often finance bond purchases through repo transactions (Huh and Infante 2021). We treat dealers' inventories as in Cimon and Garriott (2019) and assume that net dealer holdings $(I + B_n - S_n)$ are financed through leveraged repo transactions. In the model, the central bank is able to lend the dealer cash in exchange for their bonds, at a rate ζ_L.[9] We treat these loans as a separate risk on the dealers' balance sheets, with a correlation of one to the dealers' inventory values. A correlation of one represents the fact that sources of risk on the value of repos from a central bank are likely highly similar to repos from other sources.

We assume that these loans carry less risk to dealers' balance sheets than the normal repo transactions used to finance their purchases, carrying a factor $(1 - \mu)$ of the risk. If the dealer obtains a loan of size L_n from the central bank, their balance sheet cost is:

$$\frac{K}{2}\left((I - L_n + B_n - S_n)^2 + (1-\mu)^2 L_n^2 + 2(1-\mu)(I - L_n + B_n - S_n)L_n\right) \tag{27}$$

where $(I - L_n + B_n - S_n)^2$ is the risk from assets financed by private repos, $(1-\mu)^2 L_n^2$ is the risk from assets financed by central bank lending and $2(1-\mu)(I - L_n + B_n - S_n)L_n$ is the covariance term between the two.

[9] A positive value of ζ_L suggests that the dealer is paying a negative interest rate on the loan from the central bank. An alternative interpretation is that $\zeta_L > 0$ represents a loan that is below the dealer's usual cost of funding.

For a given intervention ζ_L, each dealer's profit function, after algebraic manipulation, is then given by:

$$\pi_n = (P_S - v)S_n + (v - P_B)B_n - \frac{K}{2}(I - \mu L_n + B_n - S_n)^2 + \zeta_L v L_n. \tag{28}$$

Dealers are subject to a leverage constraint that includes debt to the central bank, thus lending programs suffer from limits. We assume that net dealer holdings $(I + B_n - S_n)$ are financed through repo transactions, which provides the most beneficial treatment of the impact of central bank lending on dealers' total leverage. Further, we assume that the dealer is able to lend out more bonds than are available on its balance sheet. Put another way, the dealer is not constrained, such that $L_n \leq (I + B_n - S_n)$. We assume that selecting $L_n > (I + B_n - S_n)$ also does not increase leverage; instead, the excess loans replace other sources of leverage on the dealer's balance sheet.

Under these treatments, pledging the assets to the central bank in exchange for cash simply changes the source of leverage. Thus, lending bonds to the central bank does not add any new leverage, but also does not reduce it. The same friction may not be present if the central bank purchases assets outright in exchange for new central bank cash.[10] Alternative treatments, where either the central bank's loans represent new leverage for the dealer or where they are unable go short through loans, produce even more constrained lending outcomes. Therefore, this section presents a conservative case for the benefits of purchases over loans.

The dealer's leverage constraint is then a function of its total transactions with the private market, given by:

$$\Psi \geq I + B_i - S_i \tag{29}$$

where $\Psi > I$.

[10] During the COVID-19 crisis, some banking regulators exempted central bank reserves and similar holdings from leverage ratio requirements. One example is OSFI in Canada; see https://www.osfi-bsif.gc.ca/Eng/osfi-bsif/med/Pages/20200409-nr.aspx.

When ζ_L is such that $-\Gamma - \epsilon < \frac{\zeta_L}{\mu} < \Gamma - \epsilon$,[11] the dealers' best response functions are:

$$B_L^*(\zeta_L) = \begin{cases} \dfrac{\left(\Gamma + \epsilon + \dfrac{\zeta_L}{\mu}\right)}{\Lambda} & \text{if } \zeta_L < \hat{\zeta_L} \\ \dfrac{(\Psi - I)\Lambda + 2\Gamma}{2\Lambda} & \text{if } \zeta_L \geq \hat{\zeta_L} \end{cases} \tag{30}$$

$$S_L^*(\zeta_L) = \begin{cases} \dfrac{\left(\Gamma - \epsilon - \dfrac{\zeta_L}{\mu}\right)}{\Lambda} & \text{if } \zeta_L < \hat{\zeta_L} \\ \dfrac{(I - \Psi)\Lambda + 2\Gamma}{2\Lambda} & \text{if } \zeta_L \geq \hat{\zeta_L} \end{cases} \tag{31}$$

and

$$L^*(\zeta_L) = \begin{cases} \dfrac{\Lambda\left(IK + \dfrac{\zeta_L}{\mu}\right) + 2K\left(\dfrac{\zeta_L}{\mu} + \epsilon\right)}{\mu\Lambda K} & \text{if } \zeta_L < \hat{\zeta_L} \\ \dfrac{\mu K\Psi + \zeta_L}{\mu^2 K} & \text{if } \zeta_L \geq \hat{\zeta_L} \end{cases} \tag{32}$$

where $\hat{\zeta_L}$ is given by:

$$\hat{\zeta_L} = \frac{\mu((\Psi - I)\Lambda - 2\epsilon)}{2}. \tag{33}$$

The central bank internalizes the dealers' leverage constraint when setting a lending price. That is to say that the central bank knows that if it sets a price $\zeta_L \geq \hat{\zeta_L}$, the dealer will operate in a constrained manner. The risk-neutral central bank's welfare function is:

$$\begin{aligned} W_L = {} & N\left(B_L^* + S_L^*(\zeta)\right)\Gamma + N\left(B_L^* - S_L^*\right)\epsilon - \frac{\lambda}{2}\left((NB_L^*(\zeta_L))^2 + (NS_L^*(\zeta_L))^2\right) \\ & - N\frac{K}{2}(I - \mu L^*(\zeta_L) + B_L^*(\zeta_L) - S_L^*(\zeta_L))^2. \end{aligned} \tag{34}$$

[11] This condition, which simplifies exposition, is similar to the one for central bank asset purchases and holds under optimal risk-neutral central bank lending interventions.

The central bank's best response is then given by:

$$\zeta_L^* = \begin{cases} \dfrac{2\mu\epsilon K}{\Lambda(N+1)+2KN} & \text{if } \epsilon < \hat{\epsilon_1} \\ \dfrac{\mu((\Psi - I)\Lambda - 2\epsilon)}{2} & \text{if } \hat{\epsilon_1} \leq \epsilon < \hat{\epsilon_2} \\ 0 & \text{if } \epsilon \geq \hat{\epsilon_2} \end{cases} \tag{35}$$

where

$$\hat{\epsilon_1} = \frac{\lambda(\Psi - I)\left(\Lambda(N+1)+2KN\right)}{2\Lambda + 4K} \tag{36}$$

$$\hat{\epsilon_2} = \frac{(\Psi - I)\Lambda}{2}. \tag{37}$$

We analyze the impact of lending in two parts. First, we examine dealers who are not impacted by their leverage constraint ($\epsilon < \hat{\epsilon_1}$).

Proposition 4 (Unconstrained Lending Interventions) *Consider dealers who are not impacted by their leverage constraint ($\epsilon < \hat{\epsilon_1}$). When the central bank optimally intervenes with loans, the amounts dealers buy and sell from clients are identical to when the central bank optimally intervenes with purchases ($B_L^*(\zeta_L^*) = B^*(\zeta^*)$, $S_L^*(\zeta_L^*) = S^*(\zeta^*)$). The intervention takes places at a lower price and larger size, scaled by the amount of risk reduction μ ($\zeta_L^* = \mu\zeta^*$, $L^*(\zeta_L^*) = \dfrac{F^*(\zeta^*)}{\mu}$).*

Proposition 4 outlines the results of a lending program, compared to that of an asset purchase program. When dealers have not yet reached their leverage constraint, a risk-neutral central bank is able to achieve identical outcomes for the dealers' clients using either lending programs or asset purchase programs. The difference, however, is that dealers retain risk when they engage in lending.

Dealers take larger loans from the central bank to compensate for the risk they retain. Indeed, dealers wish to lend out an excess of the asset, entering what is effectively a short position. As a result, the central bank is able to offer a lower benefit to dealers for these loans than if they were to purchase assets outright. The price offered by the central bank and the size of the loan scales by the amount of risk that loans are able to remove from the dealers' balance sheets (μ). When loans from the central bank offer a relatively small reduction in risk, μ is closer to zero. In these cases, loans are offered with fewer benefits to dealers and in very large size. When loans from the central bank offer a larger reduction in risk, closer to that of an outright purchase, μ is closer to one. In this case, the central bank offers loans of similar size to those of asset purchases.

Next, we examine the case where the leverage constraint does impact dealer and central bank decisions ($\epsilon \geq \hat{\epsilon_1}$). The leverage constraint poses a problem for the central bank. When there is excess market selling, the central bank wishes for the dealer to purchase more. However, when the dealer is balance sheet constrained, if the central bank intervenes with loans, the dealer may be unable to take on further inventory.

Proposition 5 (Lending Interventions under Leverage Constraints) *When dealers are impacted by the leverage constraint ($\epsilon \geq \hat{\epsilon_1}$), optimal interventions become fixed when the liquidity shock (ϵ) is sufficiently high. Further increases in the liquidity shock do not affect the size or price of the intervention.*

For a sufficiently high liquidity shock, the central bank's approach becomes inelastic: they offer the dealer loans at $\zeta_L^* = 0$, independent of any further increasing in market selling. This is because the dealers are unable to purchase any more assets, regardless of the intervention price offered by the central bank, as they have become fully leverage constrained. At this point, further price concessions by the central bank only encourage dealers to lend more bonds to them, but do not spur any market purchases.

Proposition 5 offers two possible avenues for a concerned regulator, depending on their area of influence. A central bank that controls the nature of the intervention may opt for asset purchases, rather than loans, in situations where dealers are constrained by leverage. These asset purchases may not be similarly affected by leverage constraints, since they add central bank cash to the dealers' balance sheets, which may be excluded from leverage ratio calculations. Alternatively, a banking regulator may consider changes to leverage constraints, specifically regarding loans from the central bank, during a liquidity crisis. If loans from the central bank do not cause the dealer to become leverage constrained, then they perform similar functions to an asset purchase, the one caveat being that they may be of larger size than a purchase program.

6 Risk-Averse Central Bank

In this section, we again focus on asset purchases and relax Assumption 3 and consider a central bank who has a disutility from purchasing assets ($\kappa_C > 0$), also referred to in this paper as risk aversion. This risk aversion can be generated by a number of concerns, such as the moral hazard of interventions, operational costs or concerns about independence. These concerns are often cited as reasons why central banks may be reluctant to engage in interventions, or cause them to engage in interventions at unfavourable prices. Our analysis provides a means of framing the impact of these concerns.

The dealers' best response functions to any intervention ζ are identical to Section 4 and are given by Equations 18 and 19. Given the dealers' best response functions, the central bank selects ζ to maximize total welfare, given by:

$$\begin{aligned} W = {} & N\left(B^*(\zeta_{RA}) + S^*(\zeta_{RA})\right)\Gamma + N\left(B^*(\zeta_{RA}) - S^*(\zeta_{RA})\right)\epsilon \\ & - \frac{\lambda}{2}\left((NB^*(\zeta_{RA}))^2 + (NS^*(\zeta_{RA}))^2\right) \\ & - N\frac{K}{2}(I - F^*(\zeta_{RA}) + B^*(\zeta_{RA}) - S^*(\zeta_{RA}))^2 - \frac{\kappa_C\sigma^2}{2}(NF^*(\zeta_{RA}))^2. \end{aligned} \tag{38}$$

When the central bank has a disutility from purchasing excess quantities of the asset, it offers a lower purchase price and purchases a lower quantity. Ultimately, this dampens the impact of the asset purchase program. Dealer positions decrease and prices increase compared to the case where a central bank does not intervene; however, they do not do so to the same magnitude as when a risk-neutral central bank intervenes. The differences between a risk-neutral and risk-averse central bank are illustrated in Figure 9.

Proposition 6 (Asset Purchases by a Risk-Averse Central Bank) *Consider interventions ζ_{RA} where dealers choose to buy assets after the intervention has occurred ($-\Gamma - \epsilon < \zeta_{RA}$). A central bank who has a disutility from purchasing assets may choose an optimal purchase price below the asset's common value ($\zeta_{RA} < 0$) if its risk aversion coefficient is sufficiently large ($\kappa_C > \hat{\kappa_C}$). The value $\hat{\kappa_C}$ is increasing in ϵ and κ and decreasing in I, N and σ.*

Of special interest is the central bank's willingness to purchase assets at common value ($\zeta_{RA} = 0$), the value at which it is able to encourage dealers to act as if they are unconstrained. If the central bank's risk aversion coefficient is above $\hat{\kappa_C}$ it does not optimally set ζ_{RA} sufficiently high to achieve this outcome. Thus, when $\hat{\kappa_C}$ is higher, the central bank is able to be more risk averse and still achieve an outcome where $\zeta_{RA} \geq 0$. The value of $\hat{\kappa_C}$ is increasing in the size of the liquidity shock (ϵ) and the dealers' balance sheet costs (κ), but decreasing in initial dealer inventories (I), the number of dealers (N) and the asset's risk (σ). This suggests that central banks with concerns about moral hazard, their own balance sheet costs or their independence from the market may be more willing to fully remove the impact of dealer balance sheet costs on markets when (i) markets are experiencing a high liquidity demand from sellers, (ii) dealer balance sheet costs are high, (iii) pre-crisis dealer inventories are low, (iv) there are few dealers and (v) assets are of low risk.

7 Cross-Asset Impacts

In this section, we consider two identical assets. The assets, denoted $i = 1$ and $i = 2$, each have identical common values $v_1 = v_2 = v = 1$. The assets have identical variances, $\sigma_1^2 = \sigma_2^2 = \sigma^2$, and have a covariance given by $\sigma_{1,2} = \sigma^2\rho$. Each dealer is endowed with an identical quantity of both assets, I. For simplicity, we re-impose Assumption 3 and assume that the central bank is risk neutral ($\kappa_C = 0$).

To model the cross-asset impacts of purchases, we model a central bank who is only able to intervene in Asset 1. In practice there are a number of reasons why a central bank may not simultaneously intervene in all assets. A central bank may wish to coordinate interventions around highly traded assets, such as benchmark bonds, or may only wish to intervene in certain asset classes. Alternatively, a central bank may face operational constraints that prevent it from intervening over a large number of assets.

In this section, each asset is demanded by a separate type of client, with dealers able to sell and buy at prices:

$$P_{S,i} = v + \Gamma - \epsilon - \lambda \sum_{n=1}^{N} S_{i,n}, \tag{39}$$

$$P_{B,i} = v - \Gamma - \epsilon + \lambda \sum_{n=1}^{N} B_{i,n}. \tag{40}$$

The dealers' profit functions are then given by:

$$\pi_n = \sum_{i=1}^{2} \left((P_{S,i} - v) S_{i,n} + (v - P_{B,i}) B_{i,n} \right) - K(I - F_{1,n} + B_{1,n} - S_{1,n})^2 + \zeta v F_{1,n}$$
$$- K(I + B_{2,n} - S_{2,n})^2 - K\rho(I - F_{1,n} + B_{1,n} - S_{1,n})(I + B_{2,n} - S_{2,n}). \tag{41}$$

As before, we search for a symmetric equilibrium through backwards induction. First, each dealer n selects quantities $B_{i,n}$ and $S_{i,n}$ to buy and sell in each asset i, along with a quantity $F_{1,n}$ of Asset 1 to sell to the central bank for a given asset purchase premium ζ. We then impose symmetry such that each dealer buys and sells identical quantities $B_{i,M}$ and $S_{i,M}$ for each asset i and an amount $F_{1,M}$ to the central bank. In each case, the subscript M is used to denote the symmetric equilibrium value with multiple assets. Finally, the central bank selects the asset purchase premium ζ for asset 1, given the dealers' best response functions.

The central bank selects ζ to maximize total welfare in both markets, given by:

$$W = \sum_{i=1}^{2} \left((\Gamma + \epsilon) N B_{i,M}(\zeta) + (\Gamma - \epsilon) N S_{i,M}(\zeta) - \frac{\lambda}{2} \left((N B_{i,M}(\zeta))^2 + (N S_{i,M}(\zeta))^2 \right) \right)$$
$$- N \frac{K}{2} \left((I - F_{1,M}(\zeta) + B_{1,M}(\zeta) - S_{1,M}(\zeta))^2 + (I + B_{2,M}(\zeta) - S_{2,M}(\zeta))^2 \right)$$
$$- N K \rho (I - F_{1,M}(\zeta) + B_{1,M}(\zeta) - S_{1,M}(\zeta))(I + B_{2,M}(\zeta) - S_{2,M}(\zeta)). \tag{42}$$

Proposition 7 (Cross-Asset Impacts) *Consider interventions where dealers choose to both buy and sell positive quantities of both assets after the intervention has occurred ($B_{i,M}(\zeta_M^*) > 0$, $S_{i,M}(\zeta_M^*) > 0$).*

(i) When $\rho > 0$, a central bank that conducts optimal purchases of Asset 1 causes dealer purchases of Asset 2 to increase and dealer sales of Asset 2 to decrease. When $\rho < 0$, the reverse is true.

(ii) When $\rho > 0$, a central bank that conducts optimal purchases of Asset 1 causes dealer risk positions in Asset 2 to increase and pricing deviation of Asset 2 to move upwards. When $\rho < 0$, the reverse is true.

The effects of asset purchases on other assets traded by dealers are illustrated in Figure 10 and are described by Proposition 7. When the other asset is positively correlated with the one purchased by the central bank, prices in both assets move upwards towards their common value. Thus, purchasing one asset can improve price efficiency in other similar assets traded by the same dealers. This finding is consistent with Boyarchenko et al. (2020), who show a decrease in spreads of bonds that were eligible, as well as those that were ineligible, for corporate credit facilities.

Alternatively, if the assets are negatively correlated, central bank purchases of one asset cause the dealers to increase net sales of the other. Dealers may use fixed-income assets to hedge other positions on their balance sheet. If the central bank purchases these bonds, the dealers will wish to exit their hedged positions as well. Thus, central bank purchases may incentivize dealers to exacerbate net selling in other markets during times of stress.

Another contrast comes in the size of the dealer's position. When the assets are positively correlated, dealers increase purchases and reduce sales of both assets. However, since the central bank is only purchasing a single asset, the size of dealer positions differs. Net positions decrease in the asset being purchased, but increase in the other asset. Alternatively, when the assets are negatively correlated, dealers reduce net positions in both assets. Ultimately, dealers will hold less risk in the assets being purchased by the central bank, but more in similar unpurchased assets and less in dissimilar unpurchased assets.

8 Conclusion

During the COVID-19 crisis, balance sheet constrained dealers became unable to make markets in normally liquid assets. Asset purchases from dealers by central banks became a key tool in keeping these markets functional. We construct a model of a central bank that purchases assets from balance sheet constrained dealers.

We show that when markets experience a high demand for liquidity by sellers, central bank asset purchases encourage dealers to buy more and sell less from the market. That is to say that dealers take on larger principal positions and rely less on agency trading. This increase in dealer buying creates upwards price pressure. The end result is smaller dealer risk positions, market prices closer to their common values and improved welfare, at the cost of a larger central bank balance sheet.

We analyze an alternative model in which the central bank intervenes with secured loans, rather than purchases. A central bank who intervenes with loans is able to achieve a larger intervention at a lower price while producing identical outcomes for the dealers' clients. However, if dealers are leverage constrained, intervention with loans becomes ineffective compared to asset purchases.

In reality, a central bank may be concerned by its own balance sheet size and risk. We extend the model to include a risk-averse central bank, which purchases assets at lower prices and lower quantities than a risk-neutral central bank. This risk-averse central bank is unable to achieve the same welfare outcomes as their risk-neutral counterpart, but does achieve improvements over non-intervention.

Impacts on other assets traded by dealers depend on their correlation with the assets purchased by the central bank. Assets with positive correlation to the one purchased by the central bank are bought from the market in higher quantities and sold to the market in lower quantities by dealers. The result is that dealers take on larger risk positions in these assets; however, prices move towards their common value.

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A Proofs

Proof of Proposition 1 The unconstrained amounts bought and sold by a dealer can be found by evaluating Equations 7 and 8 at $\kappa = 0$. Evaluating Equations 18 and 19 at $\zeta = 0$ results in Equations 23 and 24, which are equal to the unconstrained amounts bought and sold.

The policy outcomes of interventions at price ζ are given by:

$$\begin{aligned} PDC(\zeta) &= P_B^*(\zeta) + P_S^*(\zeta) - 2v, \\ &= 2\left(\frac{N\zeta - \epsilon}{N+1}\right), \end{aligned} \tag{43}$$

$$\begin{aligned} PDU(\zeta) &= P_B^*(\zeta) + P_S^*(\zeta) - (P_B^0(\kappa = 0) + P_S^0(\kappa = 0)), \\ &= \frac{2N\zeta}{N+1}, \end{aligned} \tag{44}$$

$$\begin{aligned} RP(\zeta) &= N(I + B^*(\zeta) - S^*(\zeta) - F^*(\zeta)), \\ &= -\frac{N\zeta}{K}. \end{aligned} \tag{45}$$

At $\zeta = 0$, $PDU = 0$ and $RP = 0$, representing a pricing deviation of 0 from the unconstrained price and a risk position of 0.

Proof of Proposition 2 Given $B^*(\zeta^*)$, $S^*(\zeta^*)$ and $F^*(\zeta^*)$, the total risk position taken on by dealers with optimal asset purchases is given by:

$$RP(\zeta^*) = -\frac{2N\epsilon}{\Lambda(N+1) + 2KN}. \tag{46}$$

Evaluating Equation 46 at $\epsilon > 0$ results in $RP(\zeta^*) < 0$.

Given $B^*(\zeta^*)$, $S^*(\zeta^*)$, the total pricing deviation from common value is given by:

$$PDC(\zeta^*) = -\frac{2\epsilon\Lambda}{\Lambda(N+1) + 2KN}. \tag{47}$$

First consider the case where $\epsilon > 0$. Equations 46 and 47 are such that $RP(\zeta^*) < 0$ and $PDC(\zeta^*) < 0$. Under these circumstances, the first order conditions are such that:

$$\frac{\partial RP(\zeta^*)}{\partial \kappa}(\epsilon > 0) > 0, \tag{48}$$
$$\frac{\partial PDC(\zeta^*)}{\partial \kappa}(\epsilon > 0) > 0. \tag{49}$$

Thus an increase in κ increases these values towards zero. The case when $\epsilon < 0$ is the inverse.

Proof of Proposition 3 Any inventory purchased by the central bank ($NF^*(\zeta^*)$) is held on their balance sheet. In equilibrium, this is:

$$NF^*(\zeta^*) = N\left(\frac{I\Lambda^2 + 2\lambda(N(IK + \epsilon) + 2\epsilon) + 4\epsilon K}{\lambda(\Lambda(N+1) + 2KN)}\right). \tag{50}$$

By taking the first order conditions of Equation 50 and applying algebraic manipulation, it can be shown that when $\epsilon > 0$, $\frac{\partial NF^*(\zeta^*)}{\partial \kappa}(\epsilon > 0) > 0$, $\frac{\partial NF^*(\zeta^*)}{\partial \sigma}(\epsilon > 0) > 0$, $\frac{\partial NF^*(\zeta^*)}{\partial \epsilon}(\epsilon > 0) > 0$, $\frac{\partial NF^*(\zeta^*)}{\partial N}(\epsilon > 0) > 0$ and $\frac{\partial F^*(\zeta^*)}{\partial \kappa}(\epsilon > 0) < 0$.

Proof of Proposition 4 First consider the value of ζ_L given by Equation 35 when $\epsilon < \hat{\epsilon_1}$. It is equal to the value of $\zeta^* * \mu$, where ζ^* is given by Equation 26. Next consider Equations 30, 31 and 32 when $\epsilon < \hat{\epsilon_1}$. In each case, replacing $\frac{\zeta_L}{\mu}$ with ζ results in supply equal to those in Section 4 given by Equations 18, 19 and 20.

Proof of Proposition 5 Consider Equation 30, representing the function $B^*_L(\zeta_L)$. For $\zeta_L < \hat{\zeta_L}$, $B^*_L(\zeta_L)$ is increasing in ζ_L. For $\zeta_L > \hat{\zeta_L}$, $B^*_L(\zeta_L)$ does not increase further in ζ_L.

From Equation 33, $\hat{\zeta_L}$ is decreasing in ϵ. Consider the case where $\zeta_L < \hat{\zeta_L}$. Increases in ϵ will increase $B^*_L(\zeta_L)$ and decrease $\hat{\zeta_L}$ until $\zeta_L = \hat{\zeta_L}$, at which point further increases in ϵ do not increase $B^*_L(\zeta_L)$. Second, consider the case where $\zeta_L > \hat{\zeta_L}$. Increases in ϵ do not increase $B^*_L(\zeta_L)$ and further decrease $\hat{\zeta_L}$. Thus, for ϵ sufficiently high, further increases do not impact $B^*_L(\zeta_L)$.

The proof for Equation 31, representing the function $S^*_L(\zeta_L)$, is identical.

Proof of Proposition 6 Optimizing Equation 38 with respect to ζ results in an optimal purchase price given by:

$$\zeta_{RA}^{*} = -\frac{K(N\Lambda^2 I\kappa_C + 2\lambda(N(N+1)\kappa_C(IK+\epsilon) - \kappa\epsilon) + 4NK\kappa_C\epsilon)}{\Lambda^2(\kappa_C N + \kappa) + 2\lambda NK(\kappa + 2(N+1)\kappa_C) + 4N\kappa_C K^2}. \tag{51}$$

The asset is purchased at a discount if Equation 51 is less than 0. This occurs when:

$$K(N\Lambda^2 I\kappa_C + 2\lambda(N(N+1)\kappa_C(IK+\epsilon) - \kappa\epsilon) + 4NK\kappa_C\epsilon)) > 0 \tag{52}$$

which can be rewritten,

$$\kappa_C > \frac{2\lambda K\epsilon}{N(\Lambda^2 I + 2\Lambda(KI+\epsilon) + 4K\epsilon)} = \hat{\kappa_C}. \tag{53}$$

Algebraic manipulation can show that $\frac{\partial\hat{\kappa_C}}{\partial N} < 0$, $\frac{\partial\hat{\kappa_C}}{\partial\epsilon} > 0$, $\frac{\partial\hat{\kappa_C}}{\partial I} < 0$, $\frac{\partial\hat{\kappa_C}}{\partial\kappa} > 0$ and $\frac{\partial\hat{\kappa_C}}{\partial\sigma} < 0$.

Proof of Proposition 7 Part (i) First, consider dealers who are unable to sell any assets to the central bank ($F_{1,n} = 0$). Dealers maximize their profit with purchases and sales of Asset 2:

$$B_{2,M}^0 = \frac{\Lambda(\Gamma + \epsilon - IK(\rho+1)) + 2\Gamma K(\rho+1)}{\Lambda(\Lambda + 2K(\rho+1))}, \tag{54}$$

$$S_{2,M}^0 = \frac{\Lambda(\Gamma - \epsilon + IK(\rho+1)) + 2\Gamma K(\rho+1)}{\Lambda(\Lambda + 2K(\rho+1))}. \tag{55}$$

Next consider dealers who are able to sell assets to the central bank. These dealers maximize their profit with purchases and sales of Asset 2:

$$B_{2,M}(\zeta) = \frac{\Lambda(IK(\rho^2-1) + \rho\zeta + \Gamma + \epsilon) - 2\Gamma K(\rho^2-1)}{\Lambda(\Lambda + 2K(1-\rho^2))}, \tag{56}$$

$$S_{2,M}(\zeta) = \frac{\Lambda(IK(1-\rho^2) - \rho\zeta + \Gamma - \epsilon) - 2\Gamma K(\rho^2-1)}{\Lambda(\Lambda + 2K(1-\rho^2))}. \tag{57}$$

The value ζ_M^* is then determined by maximizing Equation 42 with respect to ζ.

Next, through algebraic manipulation we are able to solve for:

$$B^0_{2,M} - B_{2,M}(\zeta^*_M) = -\rho\frac{\Phi_1\Phi_2}{\Phi_3\Phi_4} \tag{58}$$

$$S^0_{2,M} - S_{2,M}(\zeta^*_M) = \rho\frac{\Phi_1\Phi_2}{\Phi_3\Phi_4} \tag{59}$$

where

$$\Phi_1 = N\Lambda K(\Lambda + 2K(1-\rho))(\rho+1) \tag{60}$$

$$\Phi_2 = I\Lambda^2 + 2\lambda(NI + K(\rho+1) + \epsilon(N+2)) + 4K\epsilon(\rho+1) \tag{61}$$

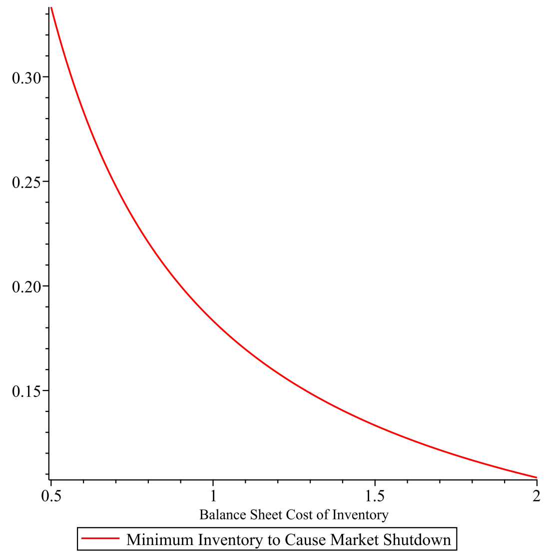
$$\Phi_3 = \Lambda + 2K(\rho+1) \tag{62}$$

$$\begin{aligned}\Phi_4 =& \Lambda^3(N+1) + 2K\Lambda^2(3N + 2 - N\rho^2 - 2\rho^2)\\ &+ 12K^2\Lambda(N+\frac{1}{3})(\rho+1)(1-\rho) + 8NK^3(\rho-1)^2(\rho+1)^2\end{aligned} \tag{63}$$

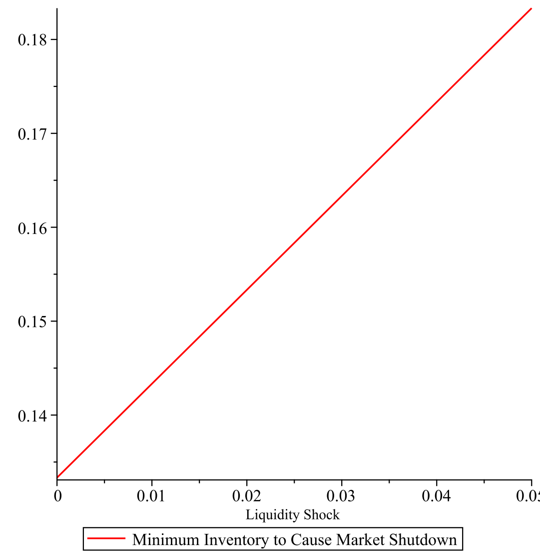
Since $\lambda > 0$, $\kappa > 0$, $\sigma > 0$, $N \geq 2$, $\epsilon > 0$, $I \geq 0$ and $-1 \leq \rho \leq 1$, then $\Phi_1 \geq 0$, $\Phi_2 \geq 0$, $\Phi_3 \geq 0$ and $\Phi_4 \geq 0$. Therefore, if $\rho > 0$, $B^0_{2,M} < B_{2,M}(\zeta^*_M)$ and $S^0_{2,M} > S_{2,M}(\zeta^*_M)$. The reverse is true if $\rho < 0$.

Proof of Proposition 7 Part (ii) From Proposition 7 Part (i), $B^*_{2,M}(\zeta^*_M) \geq B^0_{2,M}$ and $S^*_{2,M}(\zeta^*_M) \leq S^0_{2,M}$ when $\rho \geq 0$. Therefore, $(I + B^*_{2,M}(\zeta^*_M) - S^*_{2,M}(\zeta^*_M)) \geq (I + B^0_{2,M} - S^0_{2,M})$, $P^*_{B,2}(\zeta^*_M) \geq P^0_{B,2}$ and $P^*_{S,2}(\zeta^*_M) \geq P^0_{S,2}$. The reverse is true for $\rho \leq 0$.

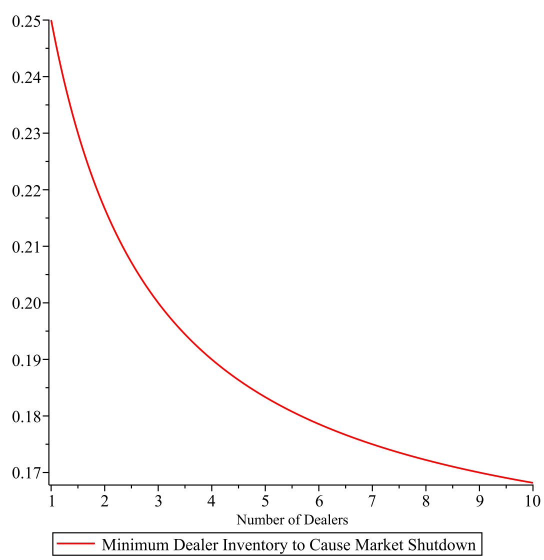
Figure 2: Initial Inventory and Market Breakdown



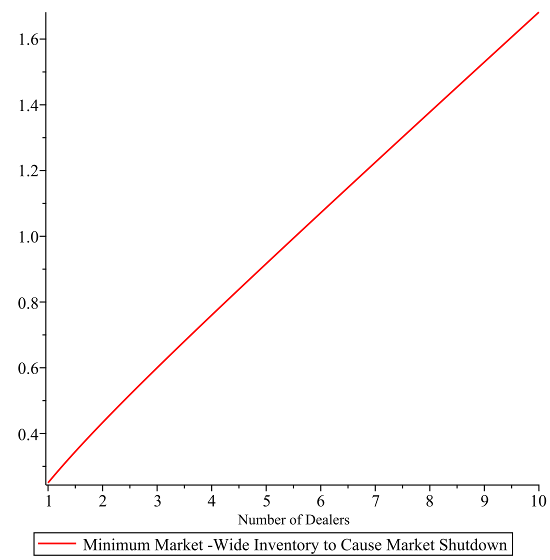
(a) Balance Sheet Costs



(b) Liquidity Shock



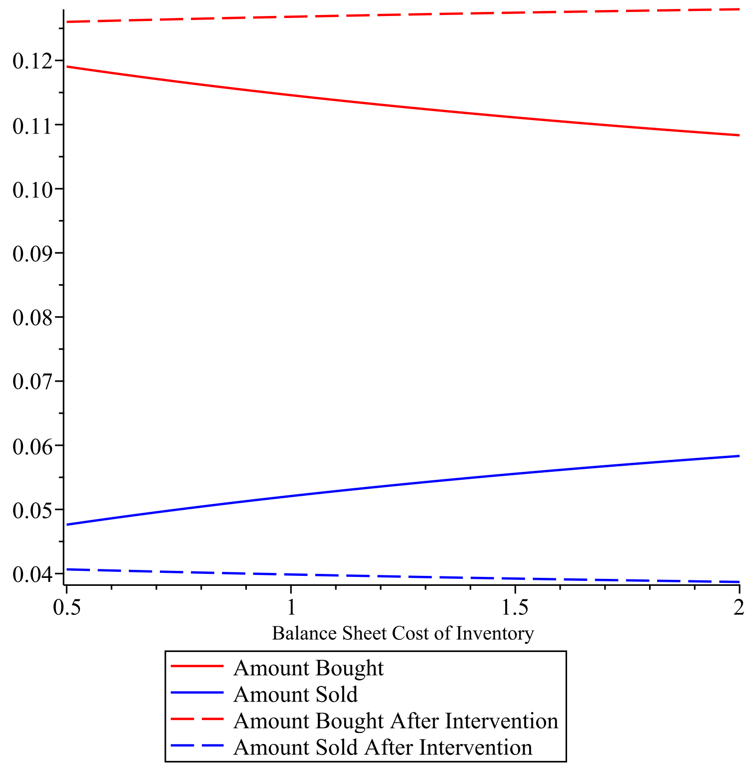
(c) Inventory per Dealer



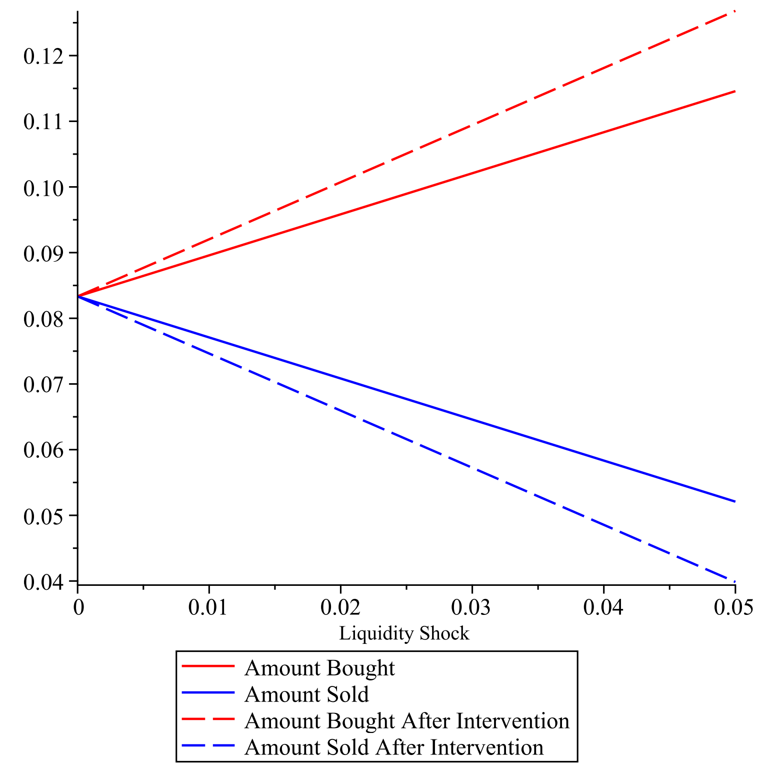
(d) Total Market Inventory

Figure 2 illustrates the minimum initial inventory that will cause a market breakdown without intervention. Panel A illustrates the effect of balance sheet costs (κ), Panel B illustrates the effect of the seller's liquidity shock (ϵ) and Panel C illustrates the effect of the number of dealers. In all four panels, $\sigma^2 = 1$, $\Gamma = 0.1$, $\lambda = 1$, $v = 1$ and $I = 0$. In Panel A, $\epsilon = 0.05$ and $N = 5$; in Panel B, $N = 5$ and $\kappa = 1$; while in Panels C and D, $\epsilon = 0.05$ and $\kappa = 1$.

Figure 3: Quantities Traded



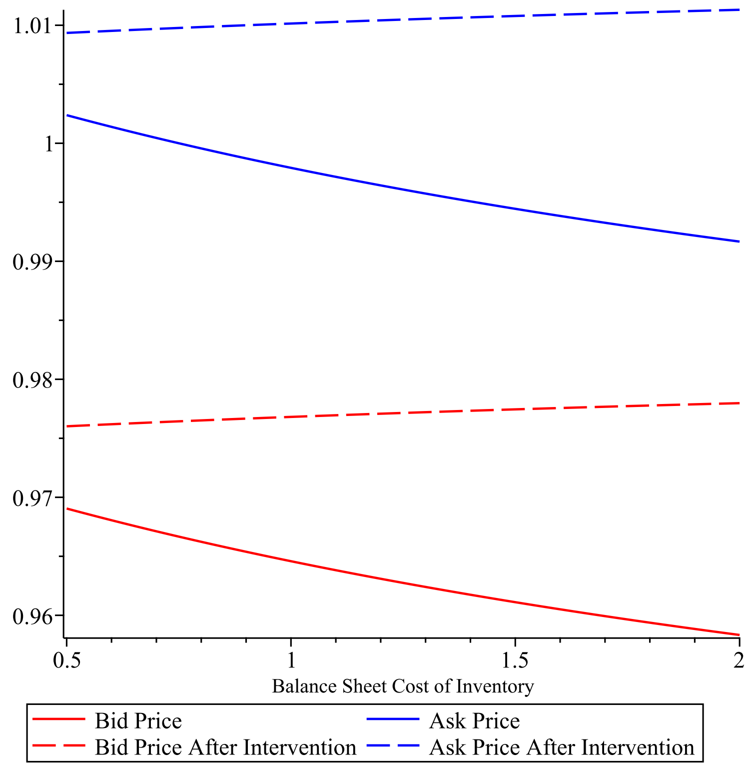
(a) Balance Sheet Costs



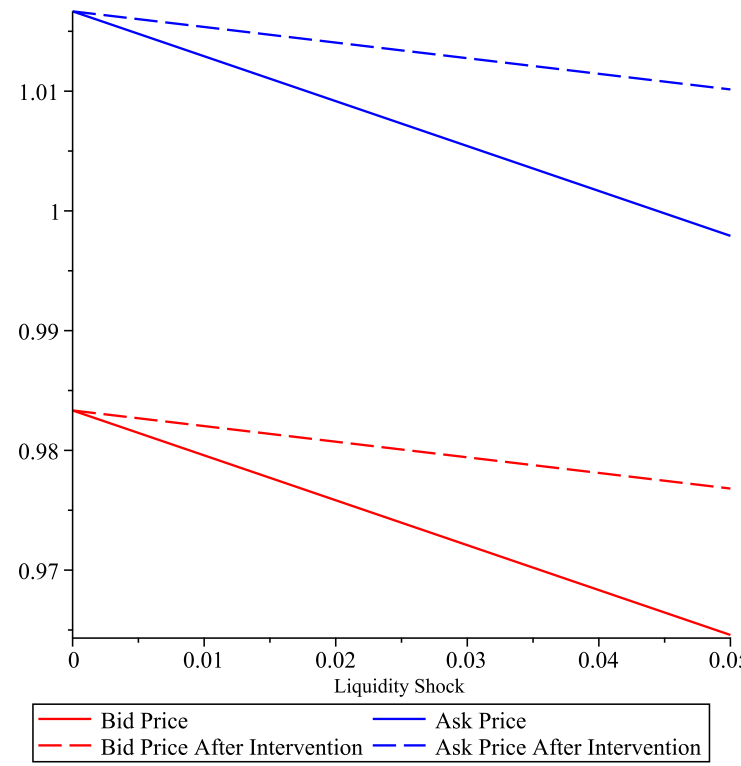
(b) Liquidity Shock

Figure 3 illustrates the equilibrium quantities traded, with and without optimal asset purchases. Panel A illustrates the effect of balance sheet costs (κ) and Panel B illustrates the effect of the seller's liquidity shock (ϵ). In both panels $N = 5$, $\sigma^2 = 1$, $\Gamma = 0.1$, $\lambda = 1$, $v = 1$ and $I = 0$. In Panel A, $\epsilon = 0.05$ and in Panel B, $\kappa = 1$.

Figure 4: Prices



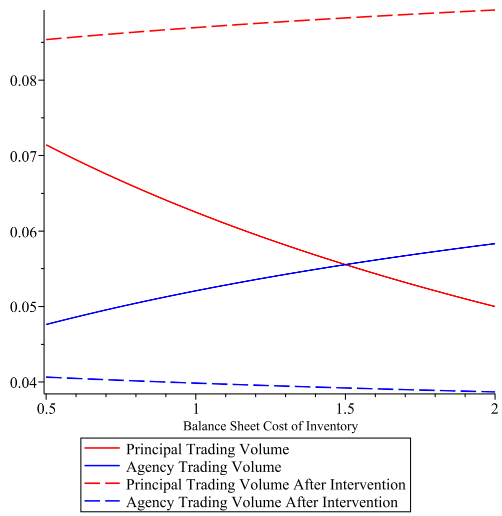
(a) Balance Sheet Costs



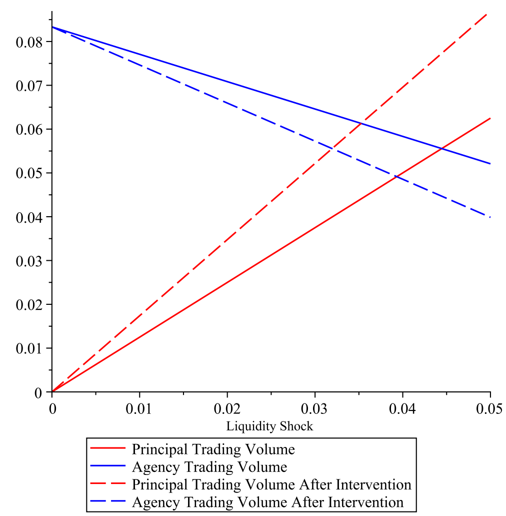
(b) Liquidity Shock

Figure 4 illustrates the equilibrium bid and ask prices, with and without optimal asset purchases. Panel A illustrates the effect of balance sheet costs (κ) and Panel B illustrates the effect of the seller's liquidity shock (ϵ). In both panels $N = 5$, $\sigma^2 = 1$, $\Gamma = 0.1$, $\lambda = 1$, $v = 1$ and $I = 0$. In Panel A, $\epsilon = 0.05$ and in Panel B, $\kappa = 1$.

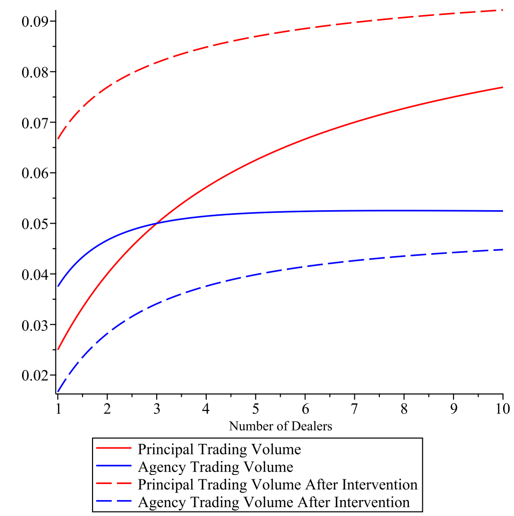
Figure 5: Principal and Agency Trading



(a) Balance Sheet Costs



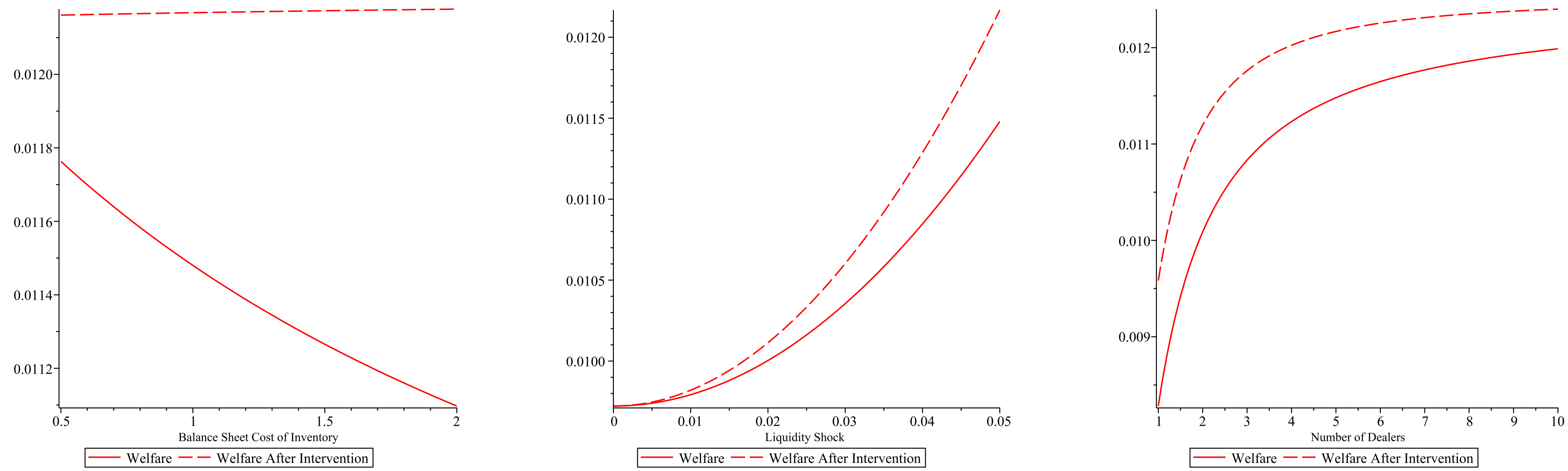
(b) Liquidity Shock



(c) Number of Dealers

Figure 5 illustrates the mix of principal trading and agency trading. Panel A illustrates the effect of balance sheet costs (κ), Panel B illustrates the effect of the seller's liquidity shock (ϵ) and Panel C illustrates the effect of the number of dealers. In all three panels, $\sigma^2 = 1$, $\Gamma = 0.1$, $\lambda = 1$, $v = 1$ and $I = 0$. In Panel A, $\epsilon = 0.05$ and $N = 5$; in Panel B $N = 5$ and $\kappa = 1$; while in Panel C, $\epsilon = 0.05$ and $\kappa = 1$.

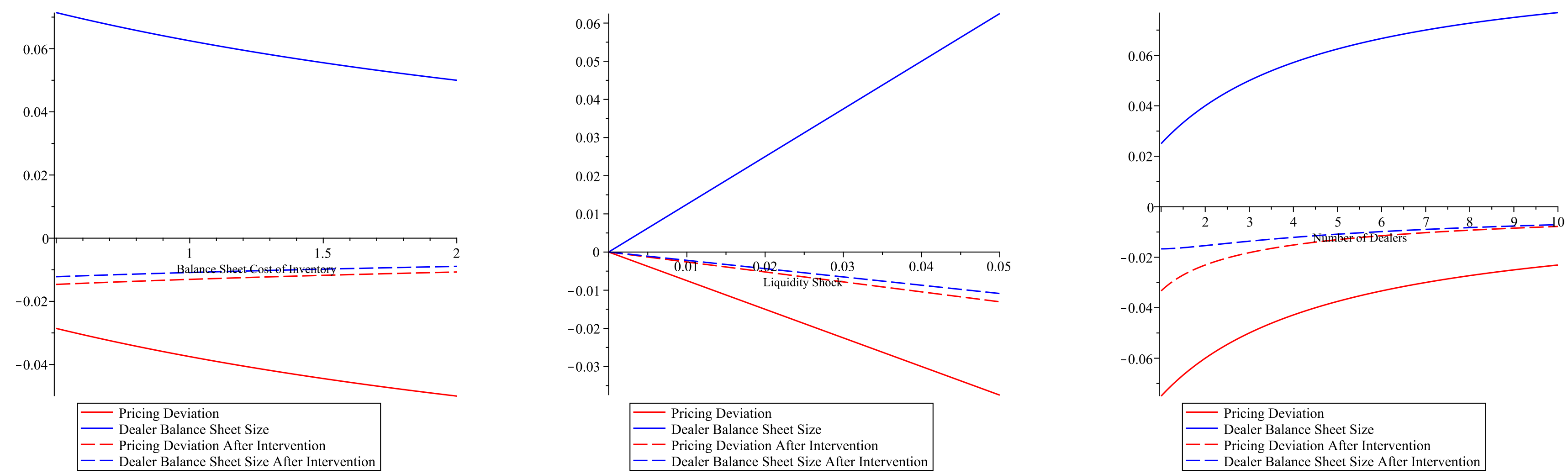
Figure 6: Welfare



(a) Balance Sheet Costs **(b)** Liquidity Shock **(c)** Number of Dealers

Figure 6 illustrates the realized welfare, with and without optimal asset purchases. Panel A illustrates the effect of balance sheet costs (κ), Panel B illustrates the effect of the seller's liquidity shock (ϵ) and Panel C illustrates the effect of the number of dealers. In all three panels, $\sigma^2 = 1$, $\Gamma = 0.1$, $\lambda = 1$, $v = 1$ and $I = 0$. In Panel A, $\epsilon = 0.05$ and $N = 5$; in Panel B $N = 5$ and $\kappa = 1$; while in Panels C, $\epsilon = 0.05$ and $\kappa = 1$.

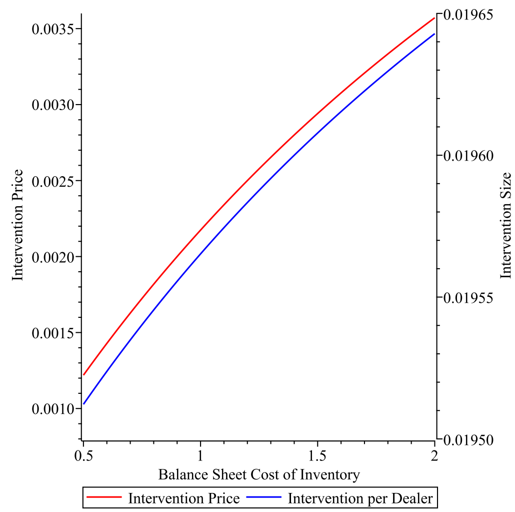
Figure 7: Policy Outcomes



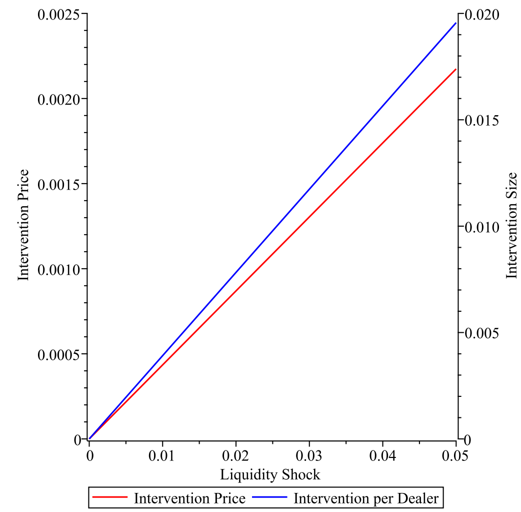
(a) Balance Sheet Costs **(b)** Liquidity Shock **(c)** Number of Dealers

Figure 7 illustrates the realized policy outcomes, with and without optimal asset purchases. In each panel the total dealer risk position $(N * (I + B - S))$ and pricing deviation $(P_B + P_S - 2v)$ are illustrated. Panel A illustrates the effect of balance sheet costs (κ), Panel B illustrates the effect of the seller's liquidity shock (ϵ) and Panel C illustrates the effect of the number of dealers. In all three panels, $\sigma^2 = 1$, $\Gamma = 0.1$, $\lambda = 1$, $v = 1$ and $I = 0$. In Panel A, $\epsilon = 0.05$ and $N = 5$; in Panel B $N = 5$ and $\kappa = 1$; while in Panel C, $\epsilon = 0.05$ and $\kappa = 1$.

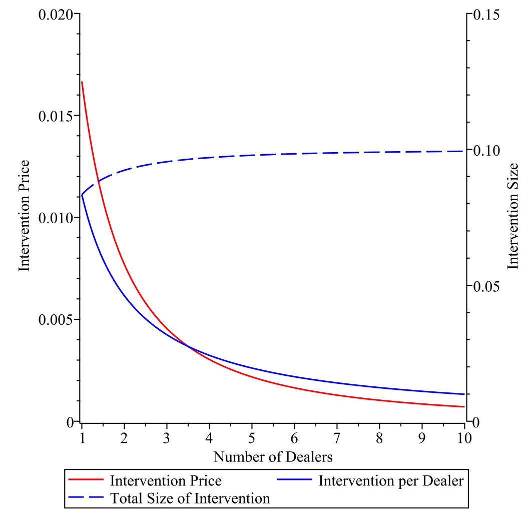
Figure 8: Optimal Asset Purchases



(a) Balance Sheet Costs



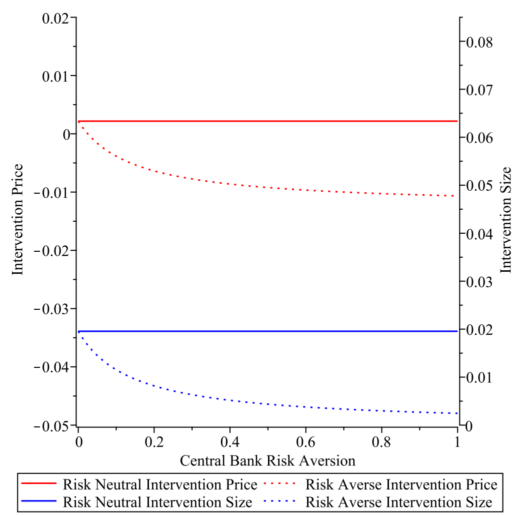
(b) Liquidity Shock



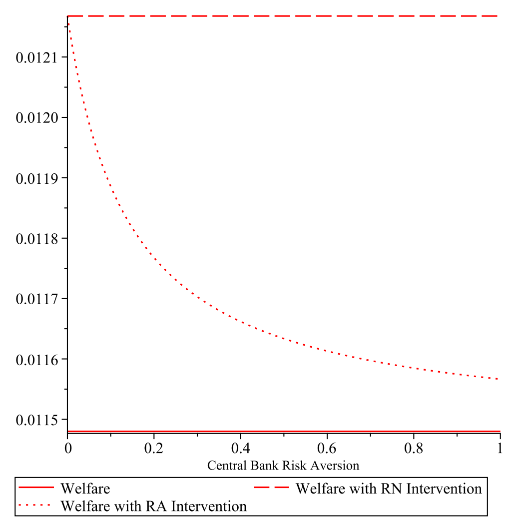
(c) Number of Dealers

Figure 8 illustrates the optimal asset purchase size and premium paid by the central bank for asset purchases (ζ). Panel A illustrates the effect of balance sheet costs (κ), Panel B illustrates the effect of the seller's liquidity shock (ϵ) and Panel C illustrates the effect of the number of dealers. In all three panels, $\sigma^2 = 1$, $\Gamma = 0.1$, $\lambda = 1$, $v = 1$ and $I = 0$. In Panel A, $\epsilon = 0.05$ and $N = 5$; in Panel B $N = 5$ and $\kappa = 1$; while in Panels C, $\epsilon = 0.05$ and $\kappa = 1$.

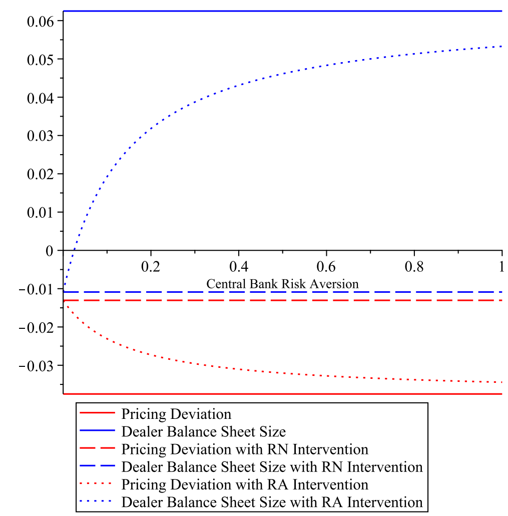
Figure 9: Central Bank Risk Aversion



(a) Optimal Asset Purchase



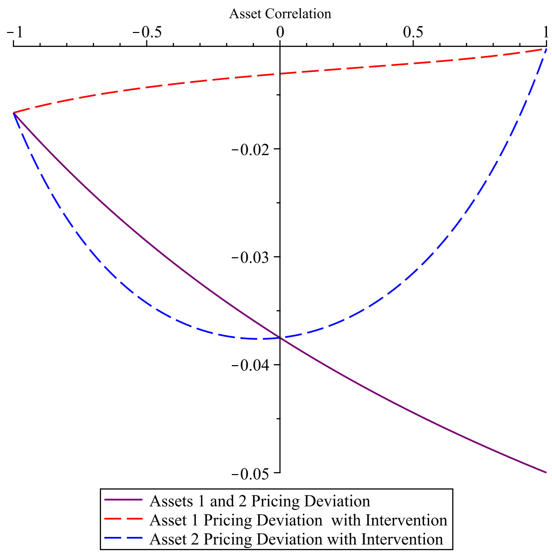
(b) Welfare



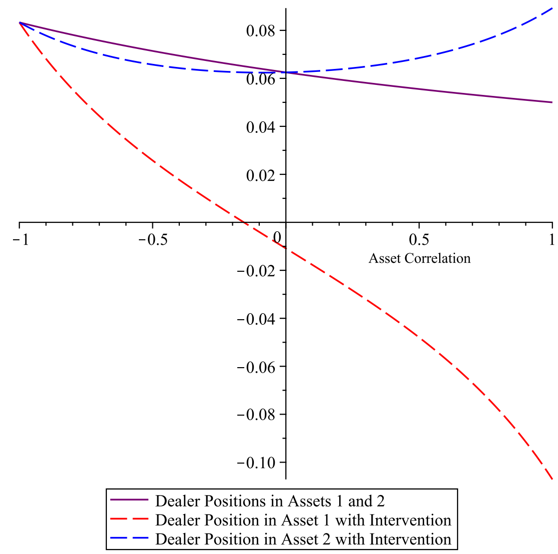
(c) Market Outcomes

Figure 9 illustrates the impact of central bank risk aversion (ζ_C). Panel A illustrates the impact of central bank risk aversion on the optimal asset purchase size and premium, Panel B illustrates the impact of central bank risk aversion on welfare and Panel C illustrates the impact of central bank risk aversion on market outcomes. In all three panels, $N = 5$, $\sigma^2 = 1$, $\kappa = 1$, $\Gamma = 0.1$, $\epsilon = 0.05$, $\lambda = 1$, $v = 1$ and $I = 0$.

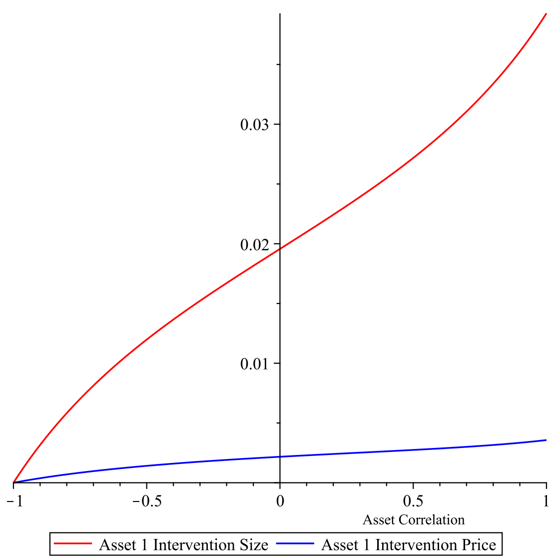
Figure 10: Cross-Asset Effects



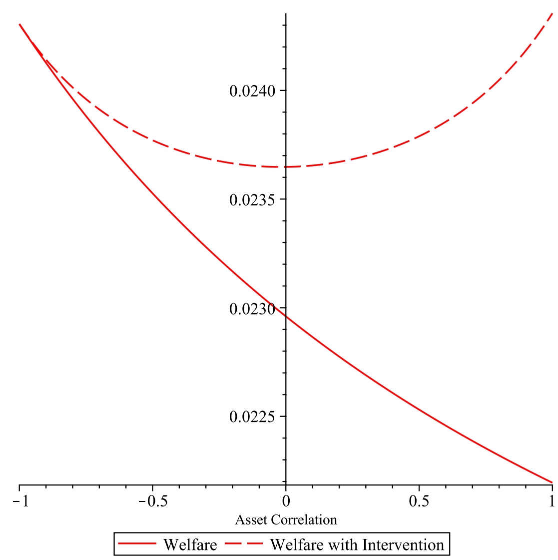
(a) Pricing Deviation



(b) Net Dealer Position



(c) Optimal Intervention



(d) Welfare

Figure 10 illustrates cross-asset impacts of intervention in Asset 1 based on its correlation coefficient with Asset 2. Panel A illustrates pricing deviation, Panel B illustrates dealer positions, Panel C illustrates the central bank's optimal intervention in Asset 1 and Panel D illustrates welfare. In all four panels, $N = 5$, $\sigma^2 = 1$, $\kappa = 1$, $\Gamma = 0.1$, $\epsilon = 0.05$, $\lambda = 1$, $v = 1$ and $I = 0$.