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IMPACT: The Bank of Canada's International Model for Projecting Activity

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Abstract

We present the structure and features of the International Model for Projecting Activity (IMPACT), a global semi-structural model used to conduct projections and policy analysis at the Bank of Canada. Major blocks of the model are developed based on the rational error correction framework of Kozicki and Tinsley (1999), which allows the model to strike a balance between theoretical structure and empirical performance. IMPACT divides the world economy into six regions: United States, the euro area, Japan, China, oil-importing emerging-market economies and oil-exporting rest of the world. The model features a rich set of cross-border trade and financial linkages that have been shown in the literature to be crucial to explaining global co-movements in business cycles. It is also globally consistent in the sense that both net foreign assets and net exports must be equal to zero at the global level. These cross-region linkages and the global stock-flow consistency allow IMPACT to generate a rigorous and more complete picture of the evolution of the global economy to better inform policy.

Bank topics: Business fluctuations and cycles; International topics; Econometric and statistical methods; Economic models
JEL codes: C, C6, C68, E, E2, E27, E3, E37, F, F0, F01, F32, F4, F47

Résumé

Nous présentons la structure et les caractéristiques d'IMPACT (*International Model for Projecting Activity*), un modèle semi-structurel de l'économie mondiale utilisé à la Banque du Canada pour l'élaboration de projections et l'analyse de politiques. La constitution des principaux blocs d'IMPACT repose sur le modèle à correction rationnelle des erreurs proposé par Kozicki et Tinsley (1999), qui réussit à concilier contenu théorique et comportement empirique. Le modèle IMPACT découpe le globe en six blocs régionaux (États-Unis, zone euro, Japon, Chine, pays émergents importateurs de pétrole et autres pays exportateurs de pétrole). Il se distingue par ses nombreux liens commerciaux et financiers interrégionaux, que la littérature considère comme un facteur explicatif déterminant de la synchronicité des cycles économiques. Il présente en outre une cohérence sur le plan mondial, en ce sens où les actifs étrangers nets et les exportations nettes doivent équivaloir à zéro à l'échelle mondiale. Grâce à cette prise en compte de l'interdépendance régionale et à la cohérence stock-flux à l'échelle mondiale, le modèle dresse un portrait rigoureux et plus complet de l'évolution de l'économie mondiale, ce qui permet de mieux éclairer la prise de décisions.

Sujets : Cycles et fluctuations économiques, questions internationales, Méthodes économétriques et statistiques, Modèles économiques
Codes JEL : C, C6, C68, E, E2, E27, E3, E37, F, F0, F01, F32, F4, F47

1. Introduction

In recent decades, the world has undoubtedly become more integrated. Cross-border flows of trade, investment and capital have surged, while business cycles have become more synchronized across countries. It is thus important to account for cross-regional spillovers when constructing the outlook for an economy or conducting scenario analysis. At the Bank of Canada, efforts to develop global macroeconomic models that capture these spillovers have proceeded along two lines: dynamic stochastic general equilibrium (DSGE) models with explicit microfoundations that are used for simulation-based structural policy analysis (e.g., BoC-GEM-FIN, the Bank of Canada's Global Economic Model with Financial Frictions) and semi-structural models that are better suited to generate conditional forecasts.[1]

We introduce the International Model for Projecting Activity (IMPACT), a new semi-structural global projection model that aims to produce mutually consistent outlooks for different key regions around the world. As advocated by Blanchard (2018) and Wren-Lewis (2018), IMPACT's semi-structural form yields a good compromise between theory and empirics because it "… combines enough theory to 'tell a story,' but gives the policy-maker the confidence that the models' predictions are consistent with past evidence." IMPACT builds on previous work by Bank staff (e.g., Gosselin and Lalonde 2005; Bailliu, Blagrave and Rossiter 2010) and aims to extend these frameworks to better account for trade and financial linkages across countries.[2]

IMPACT features empirically grounded theoretical properties and can generate globally consistent forecasts for standard demand, trade and price variables. Moreover, given its semi-structural form, it is relatively easy to adapt, and improvements to parts of the model can be introduced without needing to re-estimate the whole model. IMPACT also satisfies three conditions not often met by global projection models. First, it is globally consistent in the sense that exports and imports must be equated at the global level.[3] Its detailed trade equations include several novel features, such as an endogenous measure of third-party price competitiveness. Second, the stock-flow equilibrium for the external sector of each economy is fully articulated: each region has a steady-state target for its ratio of net foreign assets (NFA) to gross domestic product (GDP), and current account adjustments (facilitated by movements in the real effective exchange rate) take place to achieve this target in the long run. Third, it features a rich set of cross-border spillover mechanisms to capture the co-movement of business cycles and the central role played by the United States in the global business cycle, including trade and financial channels.[4]

[1] BoC-GEM-FIN is a multi-region DSGE model augmented with a banking system that includes an interbank market and cross-border lending. For more details on this model, see De Resende and Lalonde (2011).

[2] The Bank has a long history of building semi-structural models for conducting projections and policy analysis. In 2004, the Bank introduced a large-scale semi-structural projection model of the US economy called MUSE (Gosselin and Lalonde 2005). The Bank enhanced MUSE in 2009 by incorporating blocks for the economies in the euro area, Japan, China and the rest of the world. The resulting model, BoC-GPM, helped staff construct projections for different regions of the world by formalizing the interaction of key macro variables, such as output, inflation, interest rates and the exchange rate in non-US economies, and by capturing some global spillover effects through the trade and exchange-rate channels (Bailliu, Blagrave and Rossiter 2010). BoC-GPM was used until the first version of IMPACT, known as GMUSE, was implemented in 2011.

[3] This rules out black hole effects, wherein growth in a given economy can be driven by exports, without these necessarily having to be imported by anyone.

[4] See, for example, Cesa-Bianchi, Imbs and Saleheen (2016), Devereux and Yetman (2010), Di Mauro et al. (2007) and Kollmann (2012) for a comprehensive discussion of business-cycle co-movement and its drivers.

IMPACT comprises blocks for the United States (US), the euro area (EA), Japan (JA), China (CH), oil-importing emerging-market economies (EMEs) and oil-exporting rest of the world (RW) to assure global completeness.[5] The level of detail varies by block, with the United States being the most detailed. Most behavioural equations in IMPACT are governed by the rational error correction approach presented in Kozicki and Tinsley (1999). We adopt this approach because it strikes a balance between theoretical structure and empirical performance. It has proven successful in modelling both the Canadian economy (Gervais and Gosselin 2014) and the US economy at the Bank of Canada (Gosselin and Lalonde 2005) and the Federal Reserve (Brayton and Tinsley 1996).

IMPACT joins a growing community of semi-structural global policy models, which includes the Global Projection Model (GPM) (Carabenciov et al. 2013), the Flexible System of Global Models (FSGM) (Andrle et al. 2015) and ECB-Global (Dieppe et al. 2017). As with other institutional policy models, IMPACT is a living model that undergoes continuous development to improve its ability to explain the global economy and enhance its narrative capabilities. As such, the remainder of the report provides a snapshot of the model, which is subject to change. Section 2 presents the modelling approach. Section 3 details the model structure and estimation results. Section 4 presents analysis for the key shocks in the model. Section 5 concludes.

2. Modelling approach

2.1 Rational error correction

IMPACT is composed of a series of behavioural equations and identities. The model's core behavioural equations rely extensively on the rational error correction (REC) framework of Tinsley and Kozicki (1999). This framework allows the dynamic behaviour of economic variables to be driven by changes that are induced by expectations and those that are delayed responses to previous decisions. The relative importance of these changes in shaping economic dynamics is estimated.

Under REC, agents face costs of a decision variable diverging from its target level and costs of modifying this variable to return to its target. Expected future costs are discounted such that costs in the distant future have less bearing on current decisions than those in the near future. Then the optimizing behaviour approach results in the minimization of a cost function specified in terms of the discounted current and future costs of a decision variable. In the case of aggregate domestic demand, for example, agents choose the level of domestic demand to minimize the following cost function:

$$E_{t-1}\left\{\sum_{i=0}^{\infty}\beta^{i}[\varphi_0(DD_{t+i}-DD^{*}_{t+i})^2+\varphi_1\,(\Delta DD_{t+i})^2\;+\varphi_2(\Delta^2 DD_{t+i})^2+\;\dots]\right\}. \tag{1}$$

where *DD* is the logarithm of domestic demand and *DD** is the logarithm of the target level of domestic demand. The target level can be considered the optimal level in the absence of adjustment costs. The first squared expression is the expected cost of deviating from the target level of domestic demand over time. The other expressions are the adjustment costs agents face to make changes to DD_t to eventually reach DD^{*}_{t}. REC models permit a general specification of adjustment costs beyond the standard quadratic cost $\varphi_1(\Delta DD_{t+i})^2$ (e.g., Rotemberg 1982), with φ_2

[5] See appendix A for a complete definition of IMPACT's regional aggregates.

capturing the unit cost of changing the rate of change in DD_t, φ_3 the cost of changing the rate of acceleration in DD_t, and so on. By minimizing equation (1), we derive the following decision rule for domestic demand:

$$\Delta DD_t = -a_0(DD_{t-1} - DD^*_{t-1}) + \sum_{j=1}^{m-1} a_j \Delta DD_{t-j} + E_{t-1}\{\sum_{i=0}^{\infty} f_i \Delta DD^*_{t+i}\} \tag{2}$$

This decision rule stipulates that the cost-minimizing adjustment at time t, ΔDD_t, depends on the percent difference between the previous period's domestic demand and its target level, $(DD_{t-1} - DD^*_{t-1})$; previous percentage changes in the level of DD_t, ΔDD_{t-j}; and a weighted sum of expected percentage changes in the target level of domestic demand, ΔDD^*_{t+i}. The f_i coefficients are a function of the discount rate, β_i, and the cost parameters, φ_i, while m represents the order of the adjustment costs, which determines the number of lags in the decision rule.[6]

To allow some flexibility in estimating the importance of the last term in equation (2), we multiply this term by coefficient γ, which is estimated. When γ =0, equation (2) becomes a basic error correction model. When $\gamma = 1$, our estimates of the degree of forward-lookingness of equation (2) are fully consistent with the prediction of the REC approach.

2.2 Estimation and model calibration

REC equations used in IMPACT are estimated in three steps.

1. The target level of a variable is estimated using Stock and Watson's (1993) dynamic ordinary least squares (OLS) methodology. This level is best viewed as a cointegrating relationship between the variable of interest and its long-run determinants (X_t). In the case of domestic demand (DD^*_t), for instance, X_t could include permanent income or some measure of wealth.
2. The expected future level of the variable is computed using the coefficients estimated in step 1 and forecasts of X. These forecasts are produced by way of a vector autoregression (VAR) containing all information relevant to X_t.
3. The associated dynamic equation is estimated using iterative ordinary least squares. This step first combines the VAR coefficients from step 2, a calibrated discount rate (β from equation (1)) and starting point values for a_j to construct initial values for $\hat{f}_i$. The estimation of equation (2) using linear regression with these parameters yields an estimated value of $\hat{\gamma}$ and updated $\hat{a}_j$ parameters. The procedure is repeated until convergence is achieved.

In most cases, parameters presented in this paper were estimated and found to be statistically significant at the 5 percent confidence level or better. Some parameters are calibrated whenever data is lacking or uninformative (we document these instances in Section 3).

3. Model structure

IMPACT includes six regional blocks describing the United States (US), the euro area (EA), Japan (JA), China (CH), oil-importing emerging-market economies (EME) and oil-exporting rest of the world (RW). The theory and structure of the model's US block is well documented in Gosselin and Lalonde (2005). We describe the US block's trade, inflation,

[6] The order of adjustment costs is determined empirically using the Ljung-Box test and is typically chosen to ensure that residuals are not serially correlated.

monetary policy reaction function and financial accelerator, with recent updates to US domestic demand equations described in Appendix C.[7]

All regions feature similar equations governing inflation dynamics and the monetary policy reaction function. Moreover, all regions except RW feature similar equations governing trade variables. However, regions other than the United States feature a simple specification for domestic demand, with the aggregate of consumption, investment and government expenditures modelled as a single variable, using REC equations to govern dynamics around its desired path. Lastly, net exports and net foreign assets in the RW block are determined to ensure that both net exports and net foreign assets at the global level are equal to zero. Although the structure of behavioural equations is common across regions, we exploit the flexibility inherent in the model's semi-structural nature to include additional explanatory variables and mechanisms to capture unique features of a region's business cycle.[8] For instance, we find that including real money growth in the Chinese domestic demand equation improves its explanatory power.

In the sections that follow, we proceed equation by equation, presenting and contrasting the regional specifications for domestic demand, inflation, interest rates, and the price and volume of exports and imports. Because exchange rates, net foreign assets and current-account positions are modelled symmetrically across all blocks of the model, we present and discuss these equations at the end of the section. In the model, all trending variables are detrended by the level of regional output to achieve stationarity. Stationarity is required to solve the model and has several desirable properties. Most importantly, stationarity allows us to adopt non-linear specifications for certain equations, such as the dynamics of capital accumulation in the United States.[9] For simplicity and without loss of generality, we report the model's behavioural equations in levels before the detrending process.

3.1 Domestic demand

In all the non-US blocks of IMPACT, real GDP is disaggregated into domestic demand and net exports. Following the methodology outlined in Section 2, we specify a desired path of domestic demand and a dynamic REC equation that governs the convergence of the domestic demand to the desired path.

Target level of domestic demand

Domestic demand comprises household consumption, investment and government expenditures. The desired path of domestic demand depends on income and wealth variables that we expect to influence consumption according to the permanent income hypothesis (or a milder version thereof) as well as real interest rates and the price of oil, which we expect to affect both consumption and investment. Therefore, we specify the desired equation of domestic demand as follows with the respective regional coefficients presented in **Table 1**:

$$log\, DD^*_{i,t} = \theta_1\, log\, W^{Human}_{i,t} + \theta_2\, log\, W^{other}_{i,t} + \theta_3 rr_{i,t} + \theta_4\, log\, OIL_t + C_{i,1} + \theta_5 Specific_{i,t}, \quad (4)$$

where $DD^*_{i,t}$ is the desired level of domestic demand; $W^{Human}_{i,t}$ is human wealth proxied by the level of potential output; $W^{other}_{i,t}$ is a measure of real financial and/or housing wealth; $rr_{i,t}$ is an average of risky and safe interest rates capturing the influence of short- and long-term interest rates on the consumer's desired level of durable goods and

[7] Since its introduction in March 2004, the size and the richness of MUSE has substantially increased relative to the version described in Gosselin and Lalonde (2005).

[8] As discussed in Brayton, Laubach and Reifschneider (2014), the eclectic approach of the paradigm based on REC allows for a more flexible model structure than DSGE models.

[9] This approach departs from the non-stationary balanced growth path approach taken in MUSE and the Large Empirical and Semi-Structural Model (LENS).

residential capital as well as the firm's desired level of productive capital; and OIL_t is the real price of oil defined as the price of Brent deflated by the US GDP deflator.[10] We also include some region-specific variables to improve the overall fit of the target level of the domestic demand equation.[11]

Table 1: Estimated coefficients of the target level of domestic demand

Region	Human wealth ϑ_1	Other wealth[1] ϑ_2	Real interest rates[2] ϑ_3	Oil prices ϑ_4	Constant $C_{i,1}$
Euro area	1.00	0.27	-0.56	-0.02	0.00
Japan	0.90	0.10	-0.57	-0.01	-0.47
China	1.00	N/A	-0.36	-0.00	-0.08
Emerging-market economies	1.00	N/A	-0.80	-0.01	0.01
Rest of the world	1.00	N/A	-0.40	$OIL_{DEMAND,t}$: 0.09 $OIL_{SUPPLY,t}$: -0.02	-0.01

Sample period: 2002Q1–17Q2

Notes: 1. The specification of real household net worth varies across regions. In general, the sum of coefficients on the non-stationary *Human* and *Other wealth* variables is restricted to 1 to ensure a stable ratio of wealth to output in steady state. For the euro area, we stationarize the *Other wealth* variable using a deterministic trend, requiring only that the coefficient on *Human wealth* be restricted to 1. In other blocks, real financial net worth is proxied by an unobserved residual term due to data availability.

2. Measures of real interest rates differ across advanced and emerging economies. For the EA and JA, we proxy the real interest rate using a weighted average of the short-term policy rate, long-term government bond rate and some regional risk premiums (see section 3.4). For CH, EME and RW, we use the sum of the real short-term policy rate and regional risk-premium as real interest rate measure.

The estimation results presented in **Table 1** suggest that there is a meaningful dispersion in the sensitivity of target domestic demand to real interest rates. This sensitivity is highest in oil-importing EMEs and lowest in China, with that of advanced economy regions falling somewhere in between. The sensitivity of demand to shifts in the level of oil prices is calibrated in oil-importing regions to match empirical evidence presented in Gervais (2019) and Ellwanger (2019).

[10] We introduce two additional variables in RW to account for the region's unique status as a net oil exporter. The wealth effect in RW generated by changes in oil prices will differ depending on the underlying source of the oil-price shock. Specifically, movements in oil prices could stem from supply-driven shifts in RW oil production or demand-driven shifts in global economic activity. To obtain a time series of global oil-price supply drivers ($OIL_{SUPPLY,t}$), we average the cumulative log contribution of supply shocks to oil prices coming from four different structural oil-price decomposition models. The residual demand drivers ($OIL_{DEMAND,t}$) capture developments in oil prices unrelated to supply, including stronger global economic activity and speculative demand for crude oil. Empirically, we find that both oil-price coefficients are statistically significant, with the large positive coefficient on the demand driver consistent with the large domestic wealth effects generated by terms of trade shocks in commodity-exporting regions. The negative coefficient on the supply driver, meaning that a positive supply shock, although negative for prices, is positive for the region's domestic demand, is somewhat surprising, given that price effects dominate volume effects in most macro models (e.g., ECB-Global, FSGM and Cashin et al. 2014). However, it aligns well with recent empirical efforts to model the global consequences of region-specific oil supply shocks (Mohaddes and Pesaran 2015). Other macro models might also find different results because they model global supply shocks, while, in IMPACT, oil supply shocks are assumed to be originating from the RW region.

[11] We also include a dummy for the euro crisis in the euro area, a demographic trend for Japan, an urbanization trend for China and a dummy to capture a structural break in the RW region.

Dynamic specification of domestic demand

As for all the dynamic REC equations of IMPACT, we choose an order of adjustment costs (m) that is sufficient to remove any autocorrelation of the residuals. Domestic demand is thus primarily determined by the REC term, lags of domestic demand growth and expected changes in the target level of domestic demand. To ensure convergence to a stable ratio of domestic demand to GDP in the steady state, the sum of the coefficients on lagged domestic demand growth and expected changes in the target level of domestic demand are constrained to unity in estimation. Thus, the dynamics of domestic demand in the non-US regions are given by:

$$\Delta log\, DD_{i,t} = \theta_1\left(log\, DD_{i,t-1} - logDD^*_{i,t-1}\right) + \sum_{k=1}^{n} \theta_{2k}\Delta logDD_{i,t-k} + \theta_3 E_t\{\sum_{k=0}^{m} f_k \Delta DD^*_{t+k}\} + \theta_4 Specific_{i,t} + \varepsilon_{DD,i,t}\,, \quad (8)$$

where $E_t\{\sum_{k=0}^{m} f_k \Delta DD^*_{t+k}\}$ is the expected discounted future growth rate of the target level of domestic demand.

Table 2: Estimated coefficients of the dynamic domestic demand equations

Region	Gap ϑ_1	Lags ϑ_{2k}	Expectations ϑ_3	Region-specific factors ϑ_4
Euro area	-0.24	$\Delta logDD_{t-1}$:0.17 $\Delta logDD_{t-2}$: 0.20	0.63	--
Japan	-0.21	$\Delta logDD_{t-1}$: 0.47	0.53	--
China	-0.29	$\Delta logDD_{t-1}$: 0.38	0.62	$\Delta MONEY_{real,CH,t}$: 0.13
Emerging-market economies	-0.17	$\Delta logDD_{t-1}$:0.37 $\Delta logDD_{t-2}$: -0.06 $\Delta logDD_{t-3}$: 0.15	0.54	--
Rest of the world[1]	-0.19	$\Delta logDD_{t-1}$: 0.49	0.51	$\Delta logOIL_t$: 0.02

Sample period: 2002Q1–17Q2

[1] The oil-price variable in the rest of the world block is the gap between Brent prices and the forward, eight-quarter moving average.

In two cases, we augment the dynamic domestic demand specification with region-specific factors.

First, in the China specification, we include the difference between the real growth of M2 money and a notion of equilibrium real money growth. This difference is defined as:

$$\Delta MONEY_{real,CH,t} = \left[\left(\Delta MONEY_{CH,t} - \pi^{4qma}_{CH,t}\right) - \left(\Delta MONEY^*_{CH,t} - \pi^*_{CH}\right)\right], \quad (9)$$

where $\Delta MONEY_{CH}$ is the nominal growth rate of M2 in the Chinese economy, π^{4qma}_{CH} is the four-quarter moving average of quarterly Chinese inflation, $\Delta MONEY^*_{CH}$ is a measure of the equilibrium growth rate for money (equivalently, the velocity of money in the economy) and π^*_{CH} is the Chinese inflation target. The nominal growth rate of M2 is given by:

$$\Delta MONEY_{CH,t} - \Delta MONEY^*_{CH,t} = 0.03(\Delta MONEY_{CH,t-1} - \Delta MONEY^*_{CH,t}) + 0.48\,(\pi^{4qma}{}_{CH,t+3} - \pi^*) + \frac{0.48}{4}\left(\sum_{k=0}^{3} logY^{gap}_{CH,t+k}\right)] + \varepsilon_{MONEY,CH,t}, \quad (10)$$

Given the low coefficient on the lag of money growth in this equation (0.03), shocks to money growth are extremely short-lived, which is what we observe in the data. Moreover, all else equal, both higher average inflation and higher average output gap raise money growth.

Second, we add the change in the logarithm of real oil prices ($\Delta logOIL_t$) to the RW specification primarily to improve the properties of the model. More specifically, adding this variable implies that RW domestic demand responds positively on impact to increases in oil prices regardless of the driver of the oil-price shock.[12] Moreover, adding this variable increases the equation's statistical fit.

3.2 Inflation and short-term nominal interest rates

IMPACT adopts a semi-structural version of the New Keynesian Phillips curve (NKPC), where current inflation is determined by expected future inflation, past inflation and the output gap.[13] Monetary policy is modelled as an inertial Taylor-type rule that sets the short-term nominal interest rates. The Phillips Curve is estimated simultaneously with the monetary policy rule and an IS curve that approximates the behaviour of output. This approach also allows us to impose Bayesian priors to help generate parameter values supported by macroeconomic theory.

3.2.1 Phillips curves

The NKPC for region *i* that determines inflation in period *t* is given by:

$$\pi_{i,t}=\theta_1\pi_{i,t}^{exp} + \theta_2\pi_{i,t}^{*} + \theta_3\sum_{k=1}^{n}\pi_{i,t-k} + \theta_4\Delta logOIL_t + \theta_5 logY_{i,t}^{gap} + \sum_{i=1}^{n}\theta_{6i}\,Specific_{i,t} + \varepsilon_{\pi,i,t}, \tag{11}$$

where π is the quarter-over-quarter rate of inflation, π^{exp} is expected inflation, π^{*} is the central bank's inflation target and Y^{gap} is the output gap.[14] Expected inflation is modelled differently for each region.[15]

As with the domestic demand specifications, we exploit the model's flexible structure to best fit the inflation data across regions. In particular, we consider the following variables when estimating the Phillips curve in some regions: relative price of imports, oil prices, agricultural prices and the real effective exchange rate.

[12] Gervais (2019) suggests that all types of oil price shocks that raise the price of oil cause a near-term rise RW domestic demand.

[13] Microfounded derivations of the NKPC from optimizing price-setting firms facing nominal price rigidities can be found in Gali and Gertler (1999).

[14] See section 3.7 for more details on potential output in IMPACT.

[15] For the United States, expected inflation is given by $\pi_{i,t}^{exp} = 0.2\,\pi_{i,t-1}^{exp} + (1-0.2) * \pi_{i,t}^{*} + \varepsilon_{\pi,i,t}$. NKPC for the US block incorporates survey-based inflation expectations from the Michigan household survey. For other regions, $\pi_{i,t}^{exp} = (\frac{1}{n})\sum_{k=1}^{n}\pi_{i,t+k}$, where n = 2 for the euro area, n = 4 for Japan, n=1 for China, EME and RW blocks.

Table 3: Estimated coefficients of the Phillips curve

Region	Measure of inflation	Expected inflation ϑ_1	Inflation target ϑ_2	Past inflation ϑ_3	Oil prices ϑ_4	Output gap ϑ_5	Region-specific factors ϑ_{6i}
United States	Core PCE	0.24	0.22	$\pi_{(t-1)}$: 0.38 $\pi_{(t-2)}$: 0.16	--	0.04 if $Y_{i,t}^{gap} < 0$ 0.05 otherwise	$\Delta log\ PM_{xoil,US,t}$: 0.025
Euro area	Core HICP	0.07	0.26	$\pi_{(t-1),}\pi_{(t-2)}: 0.34$	--	0.09	--
Japan	Core CPI	0.13	0.26	$\pi_{(t-1),}\pi_{(t-2),}\pi_{(t-3),}$ $\pi_{(t-4)}$: 0.15	--	0.06	--
China	Total CPI	0.14	0.21	0.65	0.02	0.07	$\Delta logER\_GAP_{CH}$: -0.04 $\Delta AGRI_t$: 0.01
Emerging-market economies	Total CPI	0.14	0.21	0.65	0.03	0.05	$\Delta logER\_GAP_{EM}$: -0.08 $\Delta AGRI_t$: 0.02
Rest of the world	Total CPI	0.12	0.28	0.60	0.02	0.05	$\Delta logER\_GAP_{RW}$: -0.04 $\Delta AGRI_t$: 0.01

Sample period: 2002Q1–17Q2

The estimation results presented in **Table 3** suggest the backward-looking component of inflation is more important than the forward-looking one in all model blocks. Moreover, we find that there is a positive relationship between the output gap and inflation, as theory suggests. However, there is heterogeneity in the quantitative relationship between these two variables across regions.

Given that we are modelling total consumer price index (CPI) in China, EMEs and RW, we include an explicit role for variations in oil prices. Similarly, the change in agricultural prices ($\Delta AGRI_t$), defined as the average quarterly change over the past year in the log of the Bank of Canada agricultural commodity price sub-index deflated by the US GDP deflator, is found to improve the fit in the China, EME and RW Phillips curve specifications. Lastly, to capture the influence of import price inflation on consumer prices, we include the change in the logarithm of the relative price of imports excluding oil in the US Phillips curve ($\Delta log\ PM_{xoil,US,t}$) and the gap between past and future expected real exchange rate movements in the Phillips curve specifications of China, EME and RW.[16]

[16] The gap between past and future expected movements in the exchange rate is used to reduce the persistence of the impact of exchange rate movements on domestic price inflation. This gap is given by $\Delta logER\_GAP_{i,t} = \Delta logER_{i,t-1} - (\sum_{i=0}^{7} \Delta logER_{i,t+i})/8$.

3.2.2 Monetary policy reaction functions

The reaction functions in IMPACT are given by the following Taylor-type rules:

$$i_{i,t} = \theta_1 i_{i,t-1} + (1-\theta_1)[r^*_{i,t} + \pi^*_{i,t} + \theta_2(\pi^{exp}_{i,t} - \pi^*_{i,t}) + \theta_3 logY_{gap,i,t}] + \varepsilon_{i,i,t}, \quad (12)$$

where i_t is the short-term nominal interest rate, $r^*_{i,t}$ is the neutral real rate and the other variables are as defined above. Each regional equation is estimated jointly with the NKPC and an IS curve using Bayesian techniques. Instead of estimating the neutral real rate along with the remainder of the model, we use an estimated series of the global neutral real rate.[17]

Table 4: Estimated monetary policy reaction functions

Region	Lag ϑ_1	Inflation ϑ_2	Output gap ϑ_3
United States	0.84	2.39	0.90
Euro area	0.87	3.05	0.72
Japan	0.90	2.52	0.62
China	0.80	1.90	0.18
Emerging-market economies	0.69	2.60	0.38
Rest of the world	0.79	2.98	0.66

Sample period: 2002Q1–17Q2

Strong priors are placed on the θ_1 coefficient to keep it from approaching a random walk. Policy rates exhibit the greatest amount of persistence in advanced economies. Compared with interest rates in other EMEs, the Chinese interest rate displays more inertia. This likely reflects institutional constraints because Chinese authorities need to obtain permission from the State Council before adjusting the policy rate, which tends to limit changes in this rate. Moreover, monetary policy in EMEs tends to be less reactive to the output gap than that of advanced economies.

3.3 Trade

One of our key goals in constructing IMPACT was to ensure that the model *explicitly* captured spillovers through the trade channel. We accomplish this by modelling exports and imports separately from domestic demand. The regional export and import equations for the United States, the euro area, Japan, China and oil-importing EMEs are akin to other recently developed empirical trade models, where the main drivers are measures of income, relative prices and propensity to trade (Alexander, Cayen and Proulx 2017; Bussière et al. 2013; Morel 2012). However, the model also includes several additional features to enhance its empirical fit and narrative capabilities. These include a measure of price competitiveness that considers third-party competition and a measure of China's intermediate imports to proxy its transition from processing toward higher value-added exports. The RW block is treated as a residual, with net exports and the current account in this block adjusting to ensure that both net exports and the current account are zero at the global level. Like the demand equations, all trade volume equations, as well as their matching relative trade price equations, are modelled using the REC framework. Most of the equations in the trade block include real oil prices, which are modelled as a function of drivers of both demand and supply of oil.

[17] The global neutral real interest rate is estimated as an unobserved component driving the five-year forward five-year real rates of 11 advanced economies using a state-space model. In this model, idiosyncratic country-specific risks account for deviations of country-specific real rates from the global neutral real interest rate in the medium to long run.

3.3.1 Relative import prices

Relative import prices play an important role in IMPACT because they help determine the target volume of imports.

Target level of import prices

The target level of relative import prices is defined as:

$$log\, PM^*_{i,t} = \theta_1 log\, ER_{i,t} + \theta_2\, log\, \boldsymbol{OIL_t} + \, C_{i,1} + \theta_3\, TREND_{i,t}, \tag{13}$$

where $PM^*_{i,t}$ is the desired level of the relative price of imports (deflated by the GDP deflator), $ER_{i,t}$ is the real effective exchange rate, OIL_t is the real price of Brent oil as above, and $TREND_{i,t}$ is a time trend to proxy for the downward trend in tradeable goods prices relative to those of non-tradeable goods. As expected, an appreciation of the real effective exchange rate leads to a fall of the relative price of imports, while an increase in real prices of oil raises the relative price of imports (**Table 5**). The estimated exchange rate elasticities are consistent with other empirical studies (Bussière, Gaulier and Steingress 2017; Boz, Gopinath and Plagborg-Møller 2017; and others). In particular, the much lower exchange rate elasticity in the United States is consistent with the findings of Gopinath, Itskhoki and Rigobon (2010). The $TREND_{i,t}$ variable is included only when required by the estimation.

Table 5: Estimated coefficients of the desired import price equations

Region	Exchange rate ϑ_1	Oil prices ϑ_2	Constant $C_{i,1}$	Trend θ_3
United States—Non-oil imports	-0.38	--	1.98	0.004
United States—Oil imports	--	1.00	-4.14	--
Euro area	-0.62	0.05	1.11	--
Japan	-0.59	0.06	-1.75	--
China	-0.64	0.08	-0.20	--
Emerging-market economies	-0.60	0.08	-1.17	0.01

Sample period: 2002Q1–17Q2

Dynamic specification

The dynamic equation for relative import prices is:

$$\Delta log\, PM_{i,t} = \theta_1\big(log\, PM_{i,t-1} - logPM^*_{i,t-1}\big) + \theta_2 \Delta logPM_{i,t-1} + \theta_3\{\textstyle\sum_{k=0}^{n} f_k \Delta PM^*_{i,t+k}\} + \theta_4 \Delta logOil_t \; + \; \varepsilon_{PM,i,t}\,. \tag{14}$$

The estimated coefficients for each region can be found in **Table 6** along with those for the US block for comparison. To purge the residual term of autocorrelation according to the REC procedure, we use an order of adjustment costs equal to 2 in all regions except for EMEs, where an order of 3 is required. The coefficients on oil-price changes are calibrated to ensure adequate properties in response to a global oil-supply shock.

Table 6: Estimated coefficients of the dynamic import price equations

Region	Gap ϑ_1	Lag ϑ_2	Expectations ϑ_3	Oil-price changes ϑ_4
United States—Non-oil imports	-0.24	0.43	0.57	--
United States—Oil imports	-0.66	--	--	$\Delta logOil_{(t)}$: 0.70 $\Delta logOil_{(t-1)}$:0.13
Euro area	-0.20	0.37	0.53	0.04
Japan	-0.59	0.175	0.83	0.05
China	-0.13	0.46	0.43	0.04
Emerging-market economies*	-0.24	0.13	0.60	0.04

Sample period: 2002Q1–17Q2

* The dynamic import price equation contains two lags.

3.3.2 Import volumes

Target level of import volumes

The specification of the trade volume equations in IMPACT is almost identical to the approach taken in MUSE. The desired equations capture an income and a price effect. Specifically, the target level of imports is defined as:

$$log\, M^{*}_{i,t} = \theta_1 log\, PM_{i,t} + \theta_2 log\, DD_{i,t} + \theta_3 TRADE_{i,t} + C_{i,1} + \theta_4\, (logOIL_t\text{-}logOIL_{ss}), \quad (15)$$

where M^* is the desired level of real imports, PM is the relative price of imports described above, DD is the real level of domestic demand, OIL is the real price of oil, OIL_{ss} is the steady-state value of the real price of oil, and $TRADE$ is a proxy for openness to global trade.[18]

In a stock-flow model, the long-run income elasticity must be equal to 1 to ensure convergence to a stable and well-specified ratio of imports to GDP (**Table 7**). For all IMPACT trade-volume equations, we find that we cannot reject the unitary income elasticity if we include a trade-openness variable in the specification of the desired path. Also, we include several dummies and region-specific trends to best fit the data over the estimation sample and to address the many structural breaks in the income elasticity of global trade over the past several decades.[19] The inclusion of these variables allows us to recover a stable relationship between trade volume, price and demand that, in the absence of further unexpected structural breaks, will provide unbiased model-based forecasts.

[18] Like the one included MUSE, this proxy is the log of the volume of exports over GDP in member economies of the Organisation for Economic Co-operation and Development.

[19] Two notable structural breaks are China's entry to the World Trade Organization in 2001 and the global financial crisis (e.g., Poloz 2016; Constantinescu, Mattoo and Ruta 2015). We included other dummies for the creation of the European monetary union in 1998 and a statistically identified structural break in 2013. We also include two region-specific trends. First, we include a trend in the US oil-import equation to capture the large structural decline in the relative share of oil imports in US GDP ($TREND_{US,t}$). Second, we add to the Chinese equation a measure of intermediate import intensity to explicitly capture the move away from processing toward higher value-added exports ($M_{Intensity,CH,t}$).

Table 7: Estimated coefficients of the desired import volume equations

Region	Import prices ϑ_1	Domestic demand ϑ_2	Trade ϑ_3	Constant $C_{i,1}$	Oil prices ϑ_4
United States—Non-oil imports	-0.70	Demand, excluding investment: 0.80 Investment—software and equipment: 0.20	0.69	-0.28	--
United States—Oil imports	--	1.00	--	-3.30	-0.01
Euro area	-0.28	1.00	1.08	0.15	0.03
Japan	-0.25	1.00	1.17	0.09	0.02
China	-0.53	1.00	1.00	1.59	0.01
Emerging-market economies*	-0.18	1.00	1.90	1.83	0.02

Sample period: 2002Q1–17Q2

* EM equation includes PM + 0.16*exchange rate separately to better fit the data. See dynamic PM table.

Dynamic specification

We estimate the following REC model for the dynamic path of imports:

$$\Delta log\, M_{i,t} = \theta_1\left(log\, M_{i,t-1} - log M^*_{i,t-1}\right) + \theta_2 \sum_{k=0}^{n} \Delta log M_{i,t-k} + \theta_3\{\sum_{k=0}^{m} f_j \Delta M^*_{t+k}\} + \theta_4\left(\Delta log DD_{i,t} - \Delta log DD^{3qma}_{i,t-1}\right) + \sum_{i=1}^{n} \theta_5 Specific_i + \varepsilon_{M,i,t}. \quad (16)$$

To remove autocorrelation in the residuals, we use an order of adjustment costs equal to 2 in all regions. The error-correction terms range from -0.10 to -0.17, which suggests that adjustment costs are low for import volumes (**Table 8**). As in MUSE, we include the growth rate of real domestic demand $\left(\Delta log DD_{i,t} - \Delta log DD^{3qma}_{i,t-1}\right)$ to capture the fact that the short-term income elasticity tends to be greater than its long-run value (Hooper, Johnson and Marquez 2000). Lastly, we add region-specific drivers to improve the empirical fit of the import volume equations. These include the growth rate of import prices ($\Delta log\, PM_{US,t}$) in the US non-oil imports equation and the growth of exports ($reexports_i$), relative to its steady state, for China and oil-importing EMEs to capture the imported content of their exports. [20, 21]

[20] The difference with the steady state is used to ensure convergence to a stable ratio.

[21] There is a large literature on trade processing in China, but there are fewer published papers about this phenomenon in other EMEs. Two notable ones are Thornton and Goglio (2002) and Bun, Klaassen and Tan (2009).

Table 8: Estimated coefficients of the dynamic import price equations

Region	Gap ϑ_1	Lag ϑ_{2k}	Expectations ϑ_3	Demand growth gap ϑ_4	Region-specific drivers ϑ_5
United States—Non-oil Imports	-0.17	0.30	0.70	1.00	$\Delta log\ PM_{US,t}$: 0.50
United States—Oil Imports	-0.38	$\Delta \log M_{t-1}$: 0.12 $\Delta \log M_{t-2}$:-0.08	0.95	--	--
Euro area	-0.12	0.30	0.70	2.00	--
Japan	-0.11	0.30	0.70	1.50	--
China	-0.15	0.23	0.77	2.00	$reexports_{CH}$: 0.52
Emerging-market economies	-0.10	$\Delta \log M_{t-1}$:0.30	0.70	2.00	$reexports_{EME}$: 0.20

Sample period: 2002Q1–17Q2

3.3.3 Export prices

Target level of export prices

The specification of the desired path of the relative price of exports is identical to that for the relative price of imports. In particular, the target level of relative export prices is defined as:

$$log\ PX^*_{i,t} = \theta_1\ log\ ER_{i,t} + \theta_2\ log\ OIL_t +\ C_{i,1} + \theta_3 TREND_{i,t}, \tag{17}$$

where $PX^*_{i,t}$ is the desired level of relative export prices, and $ER_{i,t}, OIL_t$ and $TREND_{i,t}$ are as defined above. An appreciation of the exchange rate makes the import content of exports less expensive and encourages firms to lower their prices to compete globally, thereby reducing the price of exports (**Table 9**). In contrast, a rise in the price of oil increases the price of exports because it increases the cost of their energy content. As with the relative price of imports, the estimated exchange rate elasticities are consistent with other estimates found in the literature.

Table 9: Estimated coefficients of the desired export price equations

Region	Exchange rate ϑ_1	Oil prices ϑ_2	Constant C_1	Trend ϑ_3
United States	-0.13	0.03	0.26	0.001
Euro area	-0.48	0.04	0.89	--
Japan	-0.51	0.02	-1.22	0.004
China	-0.27	0.07	-0.25	--
Emerging-market economies	-0.44	0.08	-0.96	0.007

Sample period: 2002Q1–17Q2

Not surprisingly, the price of exports reacts less than the price of imports to fluctuations in the price of oil in oil-importing regions. Therefore, an increase in the price of oil leads to a deterioration of the terms of trade for oil-importing regions. Similarly, an appreciation of the currency has a smaller effect on export prices compared to import prices, leading to an improvement of the terms of trade.

Dynamic specification

The dynamic specification for the relative price of exports is given by:

$$\Delta log\ PX_{i,t} = \theta_1\left(log\ PX_{i,t-1} - logPX^*_{i,t-1}\right) + \theta_2 \Delta logPX_{i,t-1} + \theta_3\{\textstyle\sum_{k=0}^{n} f_k \Delta PX^*_{i,t+k}\} + \theta_4 \Delta logOIL_t + \varepsilon_{PX,i,t}. \tag{18}$$

The change in the price of oil is added to the US, euro area and Japan specifications to improve the model's short-term properties in response to a global oil-supply shock.

Table 10: Estimated coefficients of the dynamic export price equations

Region	Gap ϑ_1	Lag ϑ_2	Expectations ϑ_3	Oil-price changes ϑ_4
United States	-0.25	0.30	0.66	0.05
Euro area	-0.30	0.10	0.81	0.02
Japan	-0.53	0.17	0.82	0.02
China	-0.69	0.12	0.88	--
Emerging-market economies	-0.65	0.15	0.85	--

Sample period: 2002Q1–17Q2

3.3.4 Export volumes

Target level of export volumes

The target level of exports is primarily a function of foreign domestic demand and export price competitiveness:

$$log\ X^*_{i,t} = C_1 + \theta_1\left(log\ Y^{Foreign}_{i,t} + log\ M\_prop^*_{i,t}\right) + \theta_2(log\ PX_{i,t} + log\ ER_{i,t} - log\ PM^*_{i,t}). \tag{19}$$

The first term represents the foreign demand for exports and is a function of the trade-weighted average of foreign domestic demand ($Y^{Foreign}_t$) and the weighted average of foreign propensities to import out of domestic demand (M_prop_t), which is given by: [22]

$$log\ Y^{Foreign}_{i,t} = \sum_j exp_{i,j} * log(\ dem_j) + C_{i,1} \tag{20}$$

$$log\ M\_prop^*_{i,t} = \textstyle\sum_j exp_{i,j} * \left(log(m_j) - log(\ dem_j)\right) + C_{i,1}, \tag{21}$$

where $exp_{i,j}$ is the share of region i's exports going to region j, dem_j is the level of domestic demand in region *j and* m_j is the level of import of region *j*.

The coefficient on foreign demand ($Y^{Foreign}_t$) in equation (19) is calibrated to 1 to ensure a stable export to output ratio in the steady state (**Table 11**). The second term in this equation represents a measure of export price competitiveness. This measure first expresses export prices in foreign currency and then captures the effect of third-

[22] In the case of China and oil-importing EMEs, this variable is adjusted to reflect the historical increase in market share experienced by these regions.

party competition (using the series PM_t^*). $PM_{i,t}^*$ is the trade-weighted average import prices in a region *i* target markets:[23]

$$\log PM_{i,t}^* = \frac{(\sum_j exp_{i,j} * log(pm_j))}{(1-exp_{i,rw})} + C_{i,1}, \tag{22}$$

where pm_j is the import prices in region *j*. As with the level of imports, we allow for regional flexibility in the use of dummies to best fit the data over the estimation sample. As expected, an appreciation of the currency leads to a decrease in the target level of exports, all else being equal. The introduction of third-party price competition requires that a region's export prices be falling relative to that of its competitors to experience a boost in exports.

Table 11: Estimated coefficients of the desired export volume equations

Region	Constant C_1	Foreign demand ϑ_1	Price competitiveness ϑ_2
United States	6.11	1.00	-0.17
Euro area	12.65	1.00	-0.19
Japan	2.33	1.00	-0.31
China	12.07	1.00	-0.69
Emerging-market economies	7.19	1.00	-0.40

Sample period: 2002Q1–17Q2

Dynamic specification

$$\Delta log\, X_{i,t} = \theta_1 (log\, X_{i,t-1} - log X_{i,t-1}^*) + \theta_2 \Delta log X_{i,t-1} + \theta_3 \{\sum_{k=0}^{4} f_k \Delta X_{i,t+k}^*\} + \theta_4 \Delta log Y_{i,t-1}^{Foreign,Gap} + \varepsilon_{X,i,t} \tag{23}$$

In addition to the error-correction term, lag and leads, we add a demeaned proxy for the growth rate of foreign demand to increase the short-term income elasticity of exports:

$$\Delta log Y_{i,t-1}^{Foreign,Gap} = \Delta log Y_{i,t}^{Foreign} + \alpha_5 \Delta log M_{i,t}^* - (\Delta log Y_{i,t-1}^{Foreign} + \Delta log M_{i,t-1}^*)^{3q\, ma}. \tag{24}$$

We calibrate the error-correction term (θ_1) for the euro area and Japan to generate reasonable dynamic properties for export volumes.

[23] This approach is comparable to the real effective exchange rate indicator developed for Canada by Barnett, Charbonneau and Poulin-Bellisle 2016.

Table 12: Estimated coefficients of the dynamic export volume equations

Region	Gap ϑ_1	Lag ϑ_2	Expectations ϑ_3	ΔForeign growth gap ϑ_4
United States	-0.20	0.47	0.53	1.00
Euro area	-0.20	0.42	0.58	1.00
Japan	-0.20	0.35	0.65	1.00
China	-0.19	0.49	0.51	1.00
Emerging-market economies	-0.13	0.46	0.54	1.00

Sample period: 2002Q1–17Q2

3.3.5 Oil prices

In IMPACT, real oil prices are modelled as a function of drivers of both demand and supply:

$$logOIL_t = OIL_{SUPPLY,t} + OIL_{DEMAND,t} \tag{29}$$

where $OIL_{SUPPLY,t}$ is the exogenous sequence of global supply driver of the price of oil defined in section 3.1, which, in the context of IMPACT, is assumed to represent unanticipated expansions or contractions of oil production in the oil-exporting RW block.

The residual $OIL_{DEMAND,t}$ driver is modelled as:

$$OIL_{DEMAND,t} = -1.22 + 10 * Y_{wld,t}^{gap} + OIL_{DEMAND,t}^{ss} + 0.004 * TREND_{OILD,t} + oil\_specific\_demand_{shk} \tag{30}$$

$$OIL_{DEMAND,t}^{ss} = 12 * (\Delta logY_{wld,t}^{pot} - \Delta logY_{wld}^{pot,ss}) + 0.8 * OIL_{DEMAND,t-1}^{ss}, \tag{31}$$

where $Y_{wld,t}^{gap}$ is the global output gap, $Y_{wld,t}^{pot}$is the global potential output, $OIL_{DEMAND,t}^{ss}$ is a proxy for productivity-led increases in global oil demand, $TREND_{OILD,t}$ is a time trend and $oil\_specific\_demand_{shk}$ is a residual term that captures exogenous shifts in oil-specific demand as well as unanticipated supply shocks occurring in regions outside of the RW.[24] Oil prices react endogenously to changes in global demand, which are captured through movements in the global output gap. The coefficient on the output gap is calibrated based on the results from the local-projection analysis of global demand shocks on oil prices.[25]

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[24] We add $demand_{ss}$ to allow positive productivity shocks that raise the level of activity and oil consumption to generate an increase in the price of oil. Otherwise, a positive productivity shock would generate a decline in oil prices as the output gap turns briefly negative.

[25] Global oil flow demand shocks are obtained from a quarterly version of the Kilian and Murphy (2013) structural vector autoregression (SVAR) of the oil market using IMPACT global GDP growth as a proxy for global activity. We then perform a local-projection analysis of the incidence of those shocks on the real price of oil.

3.4 Financial spillovers

Several studies have shown that capturing asset-price co-movement is essential to explaining global business cycle co-movements.[26] To capture these financial spillovers, IMPACT incorporates a common global component in each regions' risk premium. Regional risk premiums are modelled as a combination of (i) a domestic financial accelerator according to Bernanke, Gertler and Gilchrist (1998, henceforth BGG), and (ii) a global risk premium, proxied by the US corporate bond spread.[27] By doing this, we aim to mimic international portfolio rebalancing toward riskier assets when there is a fall in the global risk premium.

3.4.1 US corporate bond spread

The corporate bond spread (bp_t) is primarily a function of current and expected output gaps, which proxy for the unobserved US financial accelerator effect ($finacc_{US,t}$).[28] More specifically, the US corporate bond spread is given by:

$$bp_t = 0.001 + 0.78bp_{t-1} + finacc_{US,t} + 0.0004 * d(vix_t) + shk_t, \tag{25}$$

where

$$finacc_{US,t} = -0.24\left(\frac{0.75\sum_{k=0}^{3} log\ y_{gap,US,t+k}}{4} + \frac{0.25\sum_{k=0}^{3} log\ y_{gap,EA,t+k}}{4}\right). \tag{26}$$

We include both US and EA output gaps in $finacc_{US,t}$ to capture the role of the United States and Germany in driving the global risk premium, weighted by the size of these economies. The smaller weight on the EA output gap is consistent with the empirical literature, which finds larger spillovers to foreign economies from the United States than from the euro area.[29] The coefficient on $finacc_{US,t}$ is calibrated to balance a suitable response of bp_t to US demand and policy rate shocks and provides a good historical fit required to ensure reasonable forecasting properties.[30] In addition to the financial accelerator, we include a measure of stock market volatility (the VIX) to capture movements in the corporate spread related to exogenous financial shocks.[31]

3.4.2 Regional risk premiums

In regions outside of the United States, risk premiums ($prem_{i,t}$) are influenced by the region's financial accelerator ($finacc_{i,t}$) and the US corporate spread (bp_t) in the following way:

$$prem_{i,t} = (1-\beta_1) * finacc_{i,t} + \beta_1(bp_t - \overline{bp}) + \beta_2 * prem_{i,t-1} + premshk_{i,t}, \tag{27}$$

[26] See, for example, Bayoumi (2016), Jordà, Schularick and Taylor (2017) and Devereux and Yetman (2010). Asset price co-movements can be modelled in several ways, including by adding a banking channel (loan flows) or by modelling search for yield effects (portfolio flows).

[27] The US corporate spread embodies most—if not all—of the financial information relevant to the macroeconomy. First, it incorporates the cyclical probability of default; namely, the BGG effect. Second, the excess bond premium (over and above the default risk) has been shown to cause substantial and protracted contractions in the US economic activity (Gilchrist and Zakrajšek 2012). Moreover, VAR regressions suggest that an increase in the US excess bond premium decreases output gaps and increases financial spreads significantly in all regions. Excess bond premium can be linked to credit shocks (Gertler and Kiyotaki 2010) and news about productivity/future profitability (Gortz, Tsoukalas and Zanetti 2016).

[28] The International Monetary Fund and the European Central Bank take a similar approach in their respective semi-structural policy models FSGM (Andrle et al. 2015) and ECB-Global (Dieppe et al. 2017).

[29] Estimating the source and the size of spillovers across industrialized countries, Bayoumi and Swiston (2009) show that the US shocks generate significant spillovers, while those from the euro area are small.

[30] The selected coefficients generate responses of bp_t that are consistent with the demand (liquidity preference) shock in FRBNY (Del Negro et al. 2013) and in the range of admissible responses to the monetary-policy shocks in Gertler and Karadi (2015).

[31] VIX dynamics are driven by the following estimated autoregressive process: $vix_t = 4.6 + 0.76 * vix_{t-1} + vix_{shk}$.

where

$$finacc_{i,t} = -\frac{0.24\left(\sum_{k=0}^{3} ly_{gap,i,t+k}\right)}{4}. \tag{28}$$

Equation (27) represents a blend of traditional open-economy macro models that assume an external finance premium that depends only on domestic fundamentals and no-arbitrage finance models in which the regional risk premium is equal to the global risk premium (Bauer and Diez de los Rios 2012). The coefficient for the sensitivity of regional premium to the global premium (β_1) is calibrated to match average regional spread responses to US monetary policy and risk premium shocks found in the VAR evidence and other semi-structural models such as ECB-Global (**Table 13**).

Table 13: Estimated coefficients of risk premia equations

Region	Financial accelerator β_1	Lag β_2
United States	1.00	0.78
Euro area	0.10	0.80
Japan	0.12	0.70
China	0.05	0.78
Emerging-market economies	0.28	0.74
Rest of the world	0.08	0.77

Sample period: 2002Q1–17Q2

We estimate that the EME block is the most sensitive region to global financial developments. This estimate is consistent with empirical global vector autoregressive (GVAR) analysis (e.g., Mohaddes and Pesaran 2015) and responses from other large semi-structural models, such as ECB-Global (Dieppe et al. 2017). We estimate that China is the least sensitive to the global premium. This result likely reflects China's partially managed capital account.

3.5 Current accounts, net foreign assets and exchange rates

We turn now to a discussion of the current account positions, NFA positions and exchange-rate equations.

Current account and NFA positions

Current account position for each region (CA_i) is defined as follows:

$$CA_{i,t} = INT\_NFA_{i,t} + X_{i,t} * PX_{i,t} - M_{i,t} * PM_{i,t} \tag{32}$$

$$INT\_NFA_{i,t} = 4(NFA_{\,i,t-1} * R_{NFA_{i,t}}) \tag{33}$$

$$R_{NFA_{i,t}} = R_{GOV,t} + \beta\left(R_{NFA_{i,t-1}} - R_{GOV\,i,t-1}\right), \tag{34}$$

where INT_NFA is the value of interest payments on NFA positions, NFA is the NFA position in local currency units, R_{NFA} is the interest rate paid on the NFA and R_{GOV} is the US government bonds rate.

Exchange rate

Target level of exchange rate

In the steady state, the ratio of NFAs to GDP in all (non-RW) blocks of the model converges to a target ratio. To ensure the stationarity of the NFA positions, the convergence is facilitated by the real effective exchange rate, which is in part driven by a premium on foreign debt (Schmitt-Grohe and Uribe 2003). Thus, if a region finds itself in a position where its NFA-to-GDP ratio is below the steady-state target, then the real exchange rate will depreciate, thereby generating an improvement in the nominal trade balance that is sufficient to attain the target NFA ratio. The desired value of the exchange rate in each block of the model is governed by:

$$log\ ER^{*}_{i,t} \ = log\ ER_i^{SS} + \theta_1 \left[\left(\frac{NFA_{i,t}}{Y_{i,t}} \right) - \left(\frac{NFA_i}{Y_i} \right)^{*} \right], \tag{35}$$

where ER_i^{SS} is the steady-state value for the real effective exchange rate, and $\left(\frac{NFA_i}{Y_i}\right)^*$ is the long-run target for the ratio of NFAs to GDP. The former variable (ER_i^{SS}) is endogenous and corresponds to the value of the real effective exchange rate that is compatible with the target ratio of NFAs to GDP. Given the steady-state version of the model, there is a unique value of the exchange rate (ER_i^{SS}) such that the ratio of NFAs to GDP converges to its target level. The steady state value of the real effective exchange rate (ER_i^{SS}) reacts to permanent shocks, such as productivity shocks.

The REC terms assigned to the gap between the actual and targeted ratio of NFAs to GDP are calibrated to generate a reasonable rate of adjustment for this ratio when the economy is faced with shocks. The speed of adjustment is estimated to be slower for China than it is for the other countries. This result probably reflects the fact that Chinese authorities partially manage the exchange rate.

Dynamic specification

The dynamic exchange rate equation is a modified hybrid uncovered interest parity (UIP) specification, similar to the equation in BoC-GPM. In the short run, the exchange rate follows a combination of policy rate differentials ($rdiff_{i,t}$) and regional risk premium differentials ($premdiff_{i,t}$). There is also an error-correction term slowly pulling the exchange rate toward its targeted value, $log\ ER_t^*$.

For a regional block, the hybrid-UIP equation governing real exchange rate dynamics is given by:[32]

$$\begin{aligned} rdiff_{i,t} = 4\left(log\ ER_{i,t}^{Expected} - log\ ER_{\ i,t}\right) - \theta_2\left(log\ ER_{\ i,t-1} - log\ ER_{i,t}^{*}\right) + \theta_3 premdiff_{i,t} \\ + \ dlershk_{i,t} \end{aligned} \tag{36}$$

where:

$$rdiff_{i,t} = \ (rr_{i,t} - \overline{rr_{i,t}}) - \left[\sum_{j=1}^{4} exp_{i,j} * (rr_{j,t} - \overline{rr_{j,t}}) + (1 - \sum_{j=1}^{4} exp_{i,j}) * (rr_{rw,t} - \overline{rr_{rw,t}}) \right] \tag{37}$$

[32] For the United States, equation (38) becomes $premdiff_{us,\,t} = \sum_j \ exp_{us,j}(prem_{j,t})$.

$$premdiff_{i,t} = \sum_j exp_{i,j}(prem_{i,t} - prem_{j,t}) + exp_{i,us}(prem_{i,t}) \tag{38}$$

$$log\, ER^{Expected}_{i,t} = \theta_4\, log\, ER_{i,t+1} + (1 - \theta_4) logER_{i,t-1} \tag{39}$$

Table 14: Estimated coefficients of policy rate differentials

Region	Error correction on NFA θ_1	Exchange-rate gap θ_2	Regional risk-premium differential θ_3	Exchange rate θ_4
United States	1.50	-0.15	0.50	0.30
Euro area*	2.50	-0.15	1.00	0.40
Japan*	2.75	-0.15	0.50	0.60
China	1.00	-0.12	-0.20	0.10
Emerging-market economies	1.50	-0.14	-0.60	0.35

*In the euro area and Japan, an increase in the exchange rate marks a depreciation of the currency. In other regions, an increase is an appreciation.

As per the UIP, the coefficient on the policy rate differential is set to 1. For the coefficient on the $premdiff_{i,t}$, we estimate the UIP equations for each region with the premium differentials. We then calibrate the coefficients on $premdiff_{i,t}$ to match some target responses (**Table 14**). The negative sign on the $premdiff_{i,t}$ coefficient (θ_3) in China and EMEs is counterbalanced by a positive sign on the $premdiff_{i,t}$ coefficient in the US, Japan and EA UIP equations. The difference in signs is consistent with the estimates achieved from the UIP regressions and allows the model to capture flight to safety effects. In addition, when calculating the premium differentials, we do not subtract US corporate spread from regional spreads to ensure exchange rate depreciations during financial stress for currencies that are sensitive to the global premiums, such as EME currencies.

The inclusion of $premdiff_{i,t}$ in the UIP equation implies a ranking of the exchange rate responses to fundamental shocks. As will be clearer in the review of model properties in Section 4, safer currencies, such as the US dollar, the euro and the Japanese yen, move in tandem in real effective terms while they move opposite to riskier currencies such as EME currencies. The parameters of these equations were calibrated to generate a reasonable adjustment path for the exchange rate. For this reason, the coefficient for China was set lower than for the other economies to represent the slow adjustment of Chinese exchange rate because the Chinese authorities partly manage their exchange rate.

3.6 Global consistency

Global balance of trade

Given that net exports must be zero at the global level (in exchange-rate-adjusted terms), the net-export position in the RW block of the model is precisely determined by the net-export positions of the other blocks of the model. More specifically, net exports in the RW block are given by:

$$- \, Net \; Exports^{Real}_{RW,t} = \left(X_{US,t} - M_{US,t} + \frac{X_{EA,t} - M_{EA,t}}{Z_{EA_{2012}}} + \frac{X_{JA,t} - M_{JA,t}}{Z_{JA_{2012}}} + \frac{X_{CH,t} - M_{CH,t}}{Z_{CH_{2012}}} + \frac{X_{EME,t} - M_{EME,t}}{Z_{EME_{2012}}} \right) \Big/ Z_{RW_{2012}}, \tag{40}$$

where $Z_{i,2012}$ is the average of bilateral real exchange rate between block i and the United States in 2012, and X, M are as defined elsewhere in the model. This equation expresses the net-export position of the rest-of-the-world block, in real 2012 RW currency ($Net \; Exports^{Real}_{RW,t}$) as the sum of all foreign net-export positions, in real terms, transformed in 2012 RW currency. This is done by multiplying real exports and real imports of each region (which are in real domestic currency) by the real RW bilateral exchange rate in 2012 and dividing it by the real bilateral exchange rate of the region in 2012.

Global balance of net foreign asset positions

To ensure global consistency, we require the global NFA position to be zero in all periods. This implies that the current account at the global level is also zero. To see this, note that NFA is given by:[33]

$$NFA_t \equiv NFA_{t-1} + CA_t. \tag{41}$$

To satisfy that the global current account is zero, the current account in the RW block is determined by:

$$\begin{aligned} -CA_{RW,t} = {} & \left(CA_{US,t} * Z_{RW,2012}\right) + \left(CA_{EA,t} * \frac{Z_{RW,2012}}{Z_{EA,2012}}\right) + \left(CA_{JA,t} * \frac{Z_{RW,2012}}{Z_{JA,2012}}\right) \\ & + \left(CA_{CH,t} * \frac{Z_{RW,2012}}{Z_{CH,2012}}\right) + \left(CA_{EME,t} * Z_{RW,2012}/Z_{EME,2012}\right). \end{aligned} \tag{42}$$

In this equation, $CA_{i,t}$ corresponds to the real current-account position (in domestic currency) of block *I,* and $Z_{j,2012}$ is the real bilateral exchange rate of region j in 2012.

3.7 Potential gross domestic product

Potential output in IMPACT is exogenous in all regions, except in the United States, where we allow endogenous variations in the capital stock to affect potential output. When IMPACT is used to prepare the global projection, staff estimates for potential output in each region are taken as given. These estimates are obtained using a growth accounting framework that decomposes potential output into trend total factor productivity (TFP), capital deepening and trend labour input (TLI), as described in Beard et al. (2018).

4. Model properties

In this section, we examine the properties of IMPACT by considering the responses of key economic variables to some noteworthy shocks in the model. We first review the response of each regional block to domestic demand, inflation and monetary policy shocks (figures B.1 to B.3 in Appendix B). We then review the model's spillover properties by examining regional responses to some shocks emanating from the United States (figures B.4 to B.6 in Appendix B).

[33] Revaluation effects are considered in the current account that enters the equation for the NFA position.

Lastly, we investigate the effects of an exogenous supply-driven increase in oil prices stemming from RW (Figure B.7 in Appendix B).

4.1 Domestic demand shock

We first examine the response of each region to its own domestic demand shock. The size of the shock in each region is chosen such that the level of domestic demand in the respective region increases by 1 percent on impact.[34] Figure B.1 overlays the responses of the individual regions to facilitate a cross-region comparison of domestic properties. The temporary demand shock gives rise to a modestly smaller increase in GDP. The difference between the *initial* GDP and domestic demand responses is almost exclusively determined by the income elasticity of imports as well as the region's propensity to import. Net exports experience a sudden deterioration as real imports increased by more than 1 percent, consistent with a short-run elasticity of imports to demand above 1. Because of the sudden increase in the output gap attributable to the domestic-demand shock, headline inflation rises immediately after the shock occurs. In response to increases in the output gap and inflation, monetary policy tightens over the next several quarters, with short-term interest rates increasing by up to 60 basis points (bps) at their peak, which occurs about six quarters after the initial shock to domestic demand. Because of the increase in interest rates, the currency appreciates, with a peak response of about 1.1 percent in the euro area, about seven quarters after the shock. This appreciation causes real exports to fall, which puts further downward pressure on net exports and the current-account-to-GDP ratio. In the medium term, for the equilibrium NFA-to-GDP ratio to be restored, the currency must depreciate, which forces a current-account surplus to emerge, thereby offsetting the current-account deficit that arose in the first few years following the domestic-demand shock.

Domestic demand shocks in IMPACT reveal several differences in regional properties. First, US domestic demand shocks have a more persistent impact on US variables than comparable shocks in other regions. This reflects the unique role played by endogenous capital formation in the US supply-side equations. Stronger business investment feeds into potential output, which has long-lasting positive effects on national income. As a result, the US monetary policy response to the US demand shock is much larger and persistent than in other regions. In contrast to the US responses, demand shocks in China are the least persistent, likely reflecting the government's ability to stabilize output growth around its mandated targets. Moreover, the Chinese currency experiences a muted appreciation relative to other regions, in keeping with the region's managed exchange rate regime.

4.2 Inflation shock

Examining the response of various blocks of the model to a domestic inflation or cost-push shock provides insights about the behaviour of the inflation process itself (specifically, the persistence of inflation shocks), and the aggressiveness of monetary policy in response to these shocks.

Inflation responses to cost-push shocks tend to be short-lived in most regions, except for the euro area and Japan, as can be seen in Figure B.2 of Appendix B. This reflects in part the aggressive response of monetary policy, with real interest rates immediately rising after the shock. The increase in interest rates operates through the same channels as the demand shock discussed above. The contractionary impact of interest rates leads to a decline in domestic demand, while the appreciation of the currency negatively affects net exports, which amplifies the output contraction.

A comparison of responses across regions reveals that EMEs are much more sensitive to cost-push shocks. In China, in particular, the outsized response of Chinese domestic demand to this shock leads to a large temporary fall in

[34] For the US block, a 1 percent shock to domestic demand is defined as a shock to private consumption that yields a 1 percent increase in domestic demand on impact.

imports, which generates a noticeable improvement in net exports. More broadly, this greater sensitivity combined with the important role played by supply shocks in driving the EM business cycle[35] (Aguiar and Gopinath, 2007) can explain a large part of their relatively more volatile business cycle.

4.3 Nominal interest rate shock

Following a temporary 100-bps positive shock to the nominal interest rate, domestic demand falls, reaching its trough approximately six quarters after the interest-rate shock (Figure B.3 in Appendix B). This delayed response illustrates the transmission lag for changes in monetary policy to the real economy. The shock to interest rates also causes the currency to appreciate in all regions except the rest of the world, where the demand-induced decline in global oil prices offsets the positive effect of the interest-rate differential. Currency appreciation lowers the level of exports. Meanwhile, because of the lower output gap and the appreciation of the currency, inflation falls by up to 0.1 percentage point about six to eight quarters after the shock. Overall, GDP is lower by about 0.2 to 0.4 percent at its trough, which is reached about six quarters after the shock. The response of the interest rate to monetary policy shocks tends to be more persistent in advanced economies, which is likely a reflection of the relatively low variance of interest rates over history. As a result, these shocks tend to have more persistent effects on domestic demand in these regions.

4.4 Spillovers from a US domestic demand shock

In Figure B.4 in Appendix B, we analyze the spillovers from a US domestic demand shock. Stronger US domestic demand first spills over to other economies through the trade channel, with stronger foreign demand resulting in higher exports for all regions outside of the United States. Stronger domestic demand also improves firm profitability, which lowers the US financial premium. The fall in this premium then spreads to other regions' risk premiums, which stimulates their domestic demand; the effects of this are largest in EMEs and RW. Due to the inclusion of financial premium differentials in the UIP equation, the US demand shock leads to an appreciation of the EME and RW currencies.[36] All else equal, the appreciation of these currencies then dampens the inflationary pressures from the spillovers, which mutes the response of monetary authorities, thereby contributing to a stronger response of domestic demand in EME and RW. Net exports still improve in these regions because stronger foreign demand outweighs the dampening effects of currency appreciation.

Lastly, the rise in global demand driven by the United States puts upward pressure on oil prices in the presence of inelastic supply. The rise in oil prices leads to a differentiation in spillovers with higher oil prices dampening the positive financial spillovers in oil-importing regions and magnifying the spillovers for the oil-exporting RW region.

4.5 Spillovers from a US monetary policy shock

In Figure B.5 in Appendix B, we analyze the spillovers from a contractionary US monetary policy shock. In response to this shock, financial spreads increase in all regions, reducing output and domestic demand. The resulting lower global economic demand leads to weaker oil prices.

[35] As discussed in Aguiar and Gopinath (2007), the volatile nature of business cycles in EMEs reflects in part the higher volatility of shocks to potential output in these regions.

[36] The EME and RW currencies appreciate mainly because of better global financial conditions despite their negative interest rate differential.

Domestic demand falls more in the EME and RW blocks than in other non-US regions because of higher risk premium spillovers to these regions.[37] While the US dollar appreciates as expected, we also see real effective appreciation of the euro and the Japanese yen. In contrast, the EME and RW currencies depreciate against the US dollar because these EMEs face higher relative risk premiums. Net exports increase in the EME block because of the depreciation of the real exchange rate in EMEs and lower oil prices. On the other hand, given that RW is a net oil exporter, net exports fall in this region due to lower oil prices. This negative effect is partially mitigated by the weaker currency in RW. Lastly, the stronger currencies in euro area and Japan lead to a deterioration of net exports in these regions.

4.6 Spillovers from US financial shock

Figure B.6 in Appendix B reports the impulse responses from a 100-bps increase in the US corporate spread to analyze cross-regional responses to a US-driven financial shock. Following this shock, risk premiums increase in all regions, leading to broad-based reductions in domestic demand. Oil prices are also lower, reflecting the weaker global economic demand. As in the previous two shocks, the EME and RW blocks are the most sensitive to this financial shock. Both regions face a decline in their output similar in magnitude to that of the United States.

Even though the United States is the source of this shock, the US dollar appreciates following the financial shock. This feature represents the flight to safety to US assets occurring during this episode of financial turmoil. Not only does the US dollar appreciate, but the euro and yen also appreciate in trade-weighted terms, representing the safety of their assets relative to EME and RW.[38]

4.7 Oil supply shock

In Figure B.7 in Appendix B, we analyze the effects of an exogenous supply-driven increase in oil prices. Oil-importing regions are unambiguously worse off because the reduction of supply in oil-producing RW increases oil prices globally. This leads to a deterioration in the terms of trade for oil-importing regions. The negative wealth effect that ensues dampens their domestic demand and imports. Oil-importing regions also face weaker foreign demand, which dampens exports in these regions.

Although RW output experiences an initial positive response to an unexpected contraction in its oil production, the impact quickly turns negative because the adverse effects on domestic demand from lower domestic oil production outweigh the initial benefits from improvements in its terms of trade.[39] The adverse medium-term response of demand to a supply-driven increase in oil prices is in line with the response found in BoC-GEM (Lalonde and Muir 2007) and with a more recent empirical analysis of the macroeconomic consequences of oil regional supply shocks using a GVAR (Mohaddes and Pesaran 2015).[40]

The magnitude of the response of output and demand differs across net oil importing regions, with sizeable domestic oil sectors cushioning the adverse demand impacts in the United States and China. US output is even slightly positive

[37] The higher fall in EME output is consistent with Dedola, Rivolta and Stracca (2017), who find higher spillovers from a US monetary policy shock to EMEs relative to advanced economies.

[38] The Chinese real effective exchange rate also appreciates.

[39] This is consistent with recent work using local projections according to Jordà (2005) and sequences of oil-supply shocks obtained from the Kilian and Murphy (2013) and Perez-Segura and Vigfusson (2016). Results point to a statistically significant positive short-run relationship between RW output and supply-driven movements in oil prices, followed by an equally significant negative medium-term relationship. For more details, see Gervais (2019).

[40] In the Mohaddes and Pesaran (2015) model, an unexpected contraction in the Saudi Arabian oil supply that is not offset by other oil producers generates a negative output response for Saudi Arabia as well as for several other economies included in the definition of RW.

initially because investment in structures in the shale oil sector reacts positively to higher oil prices. The global oil supply shock also has different effects on the exchange rates across regions. Currencies in the United States, China and RW appreciate in the medium-run while currencies in the other regions depreciate.

5. Conclusion

We introduce IMPACT, a new semi-structural model from the Bank of Canada to conduct projections for the global economy and policy analysis. The main innovations of the model are its global consistency and its ability to account for realistic spillovers across regions emanating from trade and financial linkages. Moreover, IMPACT provides a richer set of inputs for the Canadian projection. These include explicit forecasts for fast-growing EMEs and endogenously determined US corporate spreads, which have important macroeconomic implications for the Canadian economy (Leboeuf and Hyun 2018).

Bank staff will continue to maintain and enhance IMPACT in the coming years. More specifically, future work will concentrate on improving the model's fit and expanding the set of global issues that can be studied with the model. Potential extensions of IMPACT include introducing an endogenously determined supply side for the oil market, expanding the set of commodity prices and explicitly modelling term premiums to better capture the role of unconventional monetary policy in all regions.

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Appendix A: Data definitions[41]

Euro area

GDP	Gross domestic product euro area 19 (SA/WDA, Mil.Chn.05. Euros), Eurostat
Interest rates	Main refinancing operation: Effective Date (% pa), European Central Bank
Inflation	Harmonized index of consumer prices excluding Energy and Unprocessed Food for euro area 11-19: Total (SA, 2005=100), European Central Bank
Exchange rate	European Monetary Union exchange rates (Euro/USD), European Central Bank
Real effective exchange rate	Bank of Canada calculations using trade-weighted average of the other blocks' real bilateral exchange rates
Housing wealth	Household housing wealth (net, fixed composition), reporting institutional sector households, non-profit institutions serving households (Euros), European Central Bank
Financial wealth	HHs/NPISHs: Liabilities: Net Financial Transactions (NSA,Bil. Euros)
Net foreign assets	Net foreign asset position Euro 19 (Bil. USD), IMF
Current account	Net current account (NSA, Bil. Euros), European Central Bank
Real exports	Exports of goods, Euro 19, trade balance with the rest of the world (SA/WDA, 2000=100), European Central Bank
Real imports	Imports of goods, Euro 19, trade balance with the rest of the world (SA/WDA, 2000=100), European Central Bank
Nominal exports	Exports Euro 19, trade balance with the rest of the world (SA/WDA, Mil. Euros), Statistical Office of the European Communities
Nominal imports	Imports Euro 19, trade balance with the rest of the world (SA/WDA, Mil. Euros), Statistical Office of the European Communities

Japan

GDP	Gross domestic product (SAAR, Bil.Chn.2005, Yen), Cabinet Office of Japan
Interest rates	BoJ target rate: Uncollateralized overnight call rate: Upper limit,[42] Bank of Japan
Inflation	General consumer price index: excluding food and energy (2005=100), Ministry of Internal Affairs and Communications
Exchange rate	Japan spot exchange rates (Yen/ USD), Bank of Japan
Real effective exchange rate	Bank of Canada calculations using trade-weighted average of the other blocks' real bilateral exchange rates
Housing wealth	Nominal Japan urban land price index: Nationwide: Residential areas (2000Q1=100), Japan Real Estate Research Institute

[41] The series listed in this section are presented as reported from the source, without transformations. All series are further transformed into local currency, seasonally adjusted, rebased to 2012 and annualized.

[42] This series is constructed by combining current Japan uncollateralized call rate: upper limit (daily data) with historical uncollateralized overnight call rate: total (quarterly data).

Financial wealth	Japan stock prices (NSA, 2010=100), The Conference Board
Net foreign assets	Net foreign asset position (Bil. USD), IMF
Current account	Current account balance (SA, 100 Mil. Yen), Bank of Japan, Ministry of Finance
Real exports	Exports of goods and services (SAAR, Bil.Chn. 2005, Yen), Cabinet Office of Japan
Real imports	Imports of goods and services (SAAR, Bil.Chn. 2005, Yen), Cabinet Office of Japan
Nominal exports	Exports of goods and services (SAAR, Bil. Yen), Cabinet Office of Japan
Nominal imports	Imports of goods and services (SAAR, Bil. Yen), Cabinet Office of Japan

China

Nominal GDP	Gross domestic product (NSA, Bil. USD), China National Bureau of Statistics
Real GDP	Level created using gross domestic product (PY=100), China National Bureau of Statistics, Bank of Canada calculation
Interest rates	Nominal lending rate: One year (% pa), People's Bank of China
Inflation	Total consumer price index (year-over-year per cent change), China National Bureau of Statistics
Exchange rate	Spot exchange rates (RMB/USD), State Administration of Foreign Exchange
Real effective exchange rate	Bank of Canada calculations using trade-weighted average of the other blocks' real bilateral exchange rates
Net foreign assets	Asset (Mil. USD)- Liabilities (Mil. USD), IMF
Current account	Current account balance (NSA, Mil. USD), State Administration of Foreign Exchange
Nominal exports	Exports of Goods and Services, Balance of Payments, State Administration of Foreign Exchange (NSA, Mil. USD)
Nominal imports	Imports of Goods and Services, Balance of Payments, State Administration of Foreign Exchange (NSA, Mil. USD)
Price of exports	Export prices Index (2005=100), China Customs
Price of imports	Import prices Index (2005=100), China Customs
Money growth	Money supply, M2 (NSA, Bil. RMB), People's Bank of China

Oil-importing emerging markets

The EME block is constructed using the IMF definition of EMEs non-oil exporter, except for China, which is explicitly modelled in IMPACT due to its growing importance to the global economy. The EME block consists of more than 100 countries, representing 33 percent of the world economy. The block covers four regions: the Middle East and Africa (MEA), Latin America and the Caribbean (LAC), emerging Asia (EA) and the Commonwealth of Independent States and Central and Eastern Europe (CISEU).

A full dataset is only available for a subset of 26 countries (**Table A.4**). Nevertheless, these 26 countries represent 83 percent of the block (using purchasing-power parity weights, IMF October 2017 WEO). The region with the least coverage is MEA. For that region, only data for Egypt and South Africa are available. Together, these two economies represent 39 percent of the non-oil exporting emerging economies of MEA. For other regions, the coverage is between 85 and 91 percent.

Rest of the world

The Rest of the world (RW) block accounts for 40 countries, representing 18 percent of the world economy. The block covers residual advanced economies and oil exporters from two sub-aggregate regions: Middle East, Africa and Latin America, and (2) Common Wealth of Independent States and Asia. A full dataset is only available for a subset of 16 countries (**Table A.4**) representing 72 percent of the block (using PPP weights, IMF October 2017 WEO). These 16 countries are heterogeneous, including 10 advanced economies and 6 emerging markets. The block as a whole is a large oil producer and net oil exporter, with its member countries representing about 70 percent of global oil production. All RW historical series are constructed using member country data taken from several sources accessed through HAVER, including IMF and OECD Main Economic Indicators.

Oil price

Oil price	Crude Oil (petroleum), Brent, USD per barrel

Table A1: Bilateral export weights

	US	EA	JA	CH	EME	RW
US		0.13	0.04	0.08	0.40	0.36
EA	0.1		0.02	0.06	0.32	0.50
JA	0.19	0.08		0.18	0.38	0.18
CH	0.17	0.11	0.07		0.48	0.18
EME	0.22	0.22	0.07	0. 23		0.26
RW	0.11	0.52	0.05	0.08	0.25	

Table A2: Bilateral trade weights (average of exports and imports weights)

Variable	US	EA	JA	CH	EME	RW
US		0.13	0.05	0.14	0.35	0.34
EA	0.09		0.02	0.09	0.30	0.50
JA	0.14	0.08		0.20	0.31	0.28
CH	0.12	0.10	0.08		0.40	0.30
EME	0.19	0.21	0.07	0.23		0.30
RW	0.10	0.55	0.04	0.08	0.23	

Table A3: **Steady-state parameters**

Variable	US	EA	JA	CH	EME	RW
Inflation (%)	2.0	2.0	0.0	3.0	5.5	4.6
Nominal interest rate (%)	3.0	2.75	0.15	6.3	6.3	4.7
NFA (% of GDP)	-50.0	10.0	100	24.5	0.0	-15.0

Table A4: **Countries explicitly included in aggregate regions**

Euro area	Emerging-market economies		Rest of the world
Austria	Argentina	Mexico	Australia
Belgium	Brazil	Paraguay	Bahrain
Cyprus	Bulgaria	Peru	Canada
Estonia	Chile	Philippines	Czech Republic
Finland	Costa Rica	Poland	Denmark
France	Croatia	Romania	Great Britain
Germany	Egypt	Singapore	Iceland
Greece	Hong Kong	South Africa	Israel
Ireland	Hungary	Taiwan	New Zealand
Italy	India	Thailand	Nigeria
Latvia	Indonesia	Turkey	Norway
Lithuania	Malaysia	Ukraine	Qatar
Luxembourg			Russia
Malta			Saudi Arabia
Netherlands			Sweden
Portugal			Switzerland
Slovakia			
Slovenia			
Spain			

Appendix B: Model properties

Figure B.1: Response of each region to its own domestic demand shock

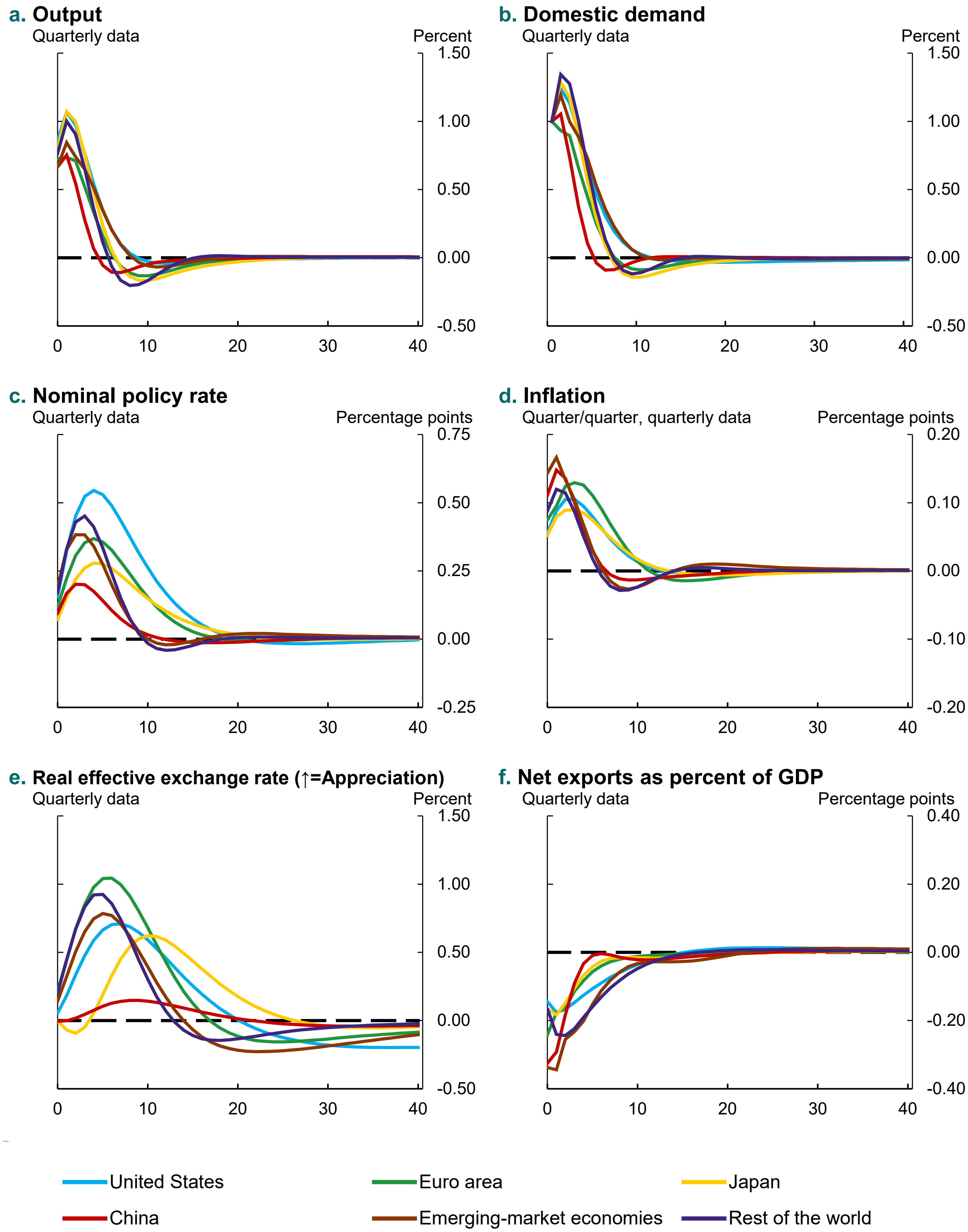


Figure B.2: Response of each region to its own inflation shock

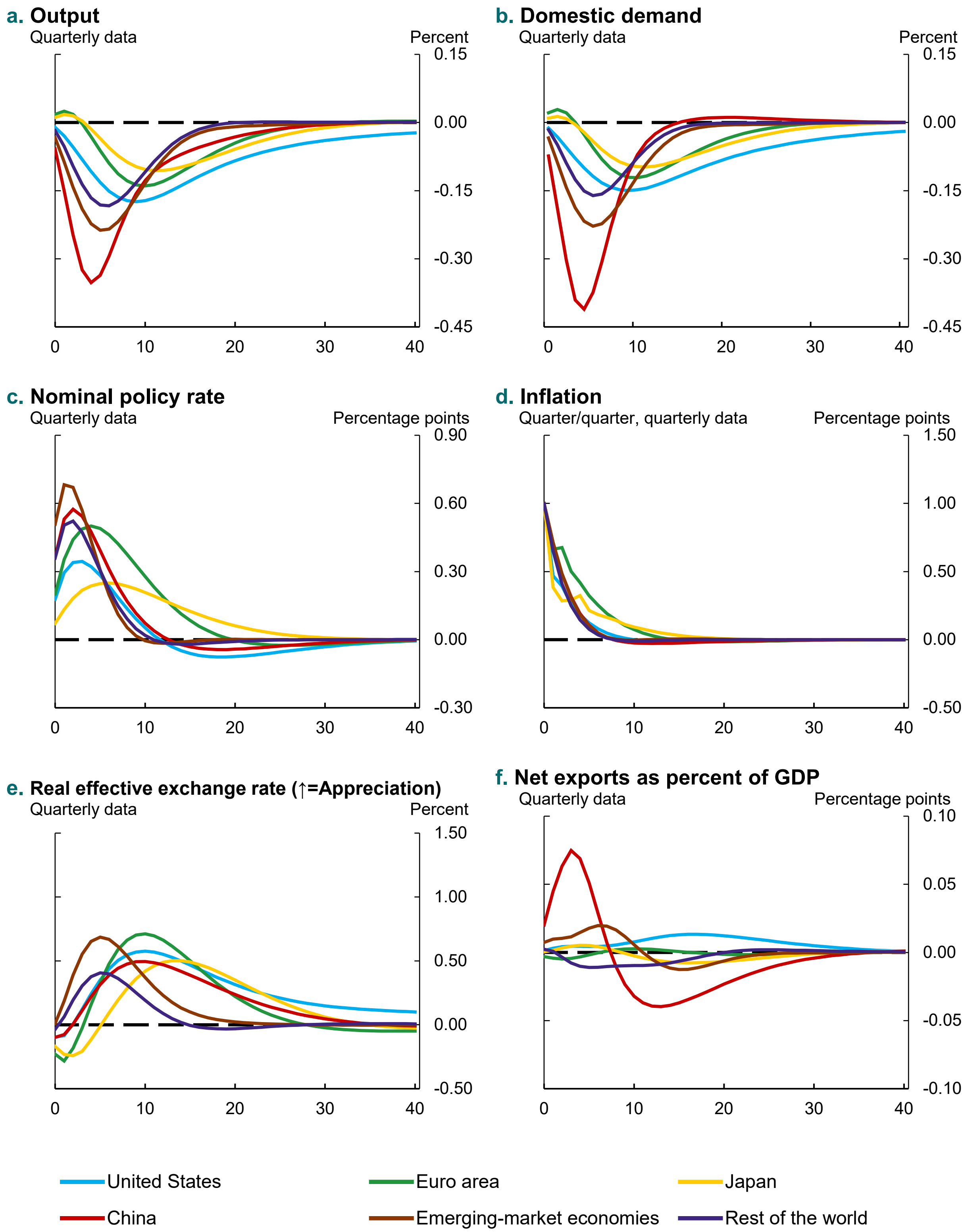


Figure B.3: Response of each region to its own monetary policy shock

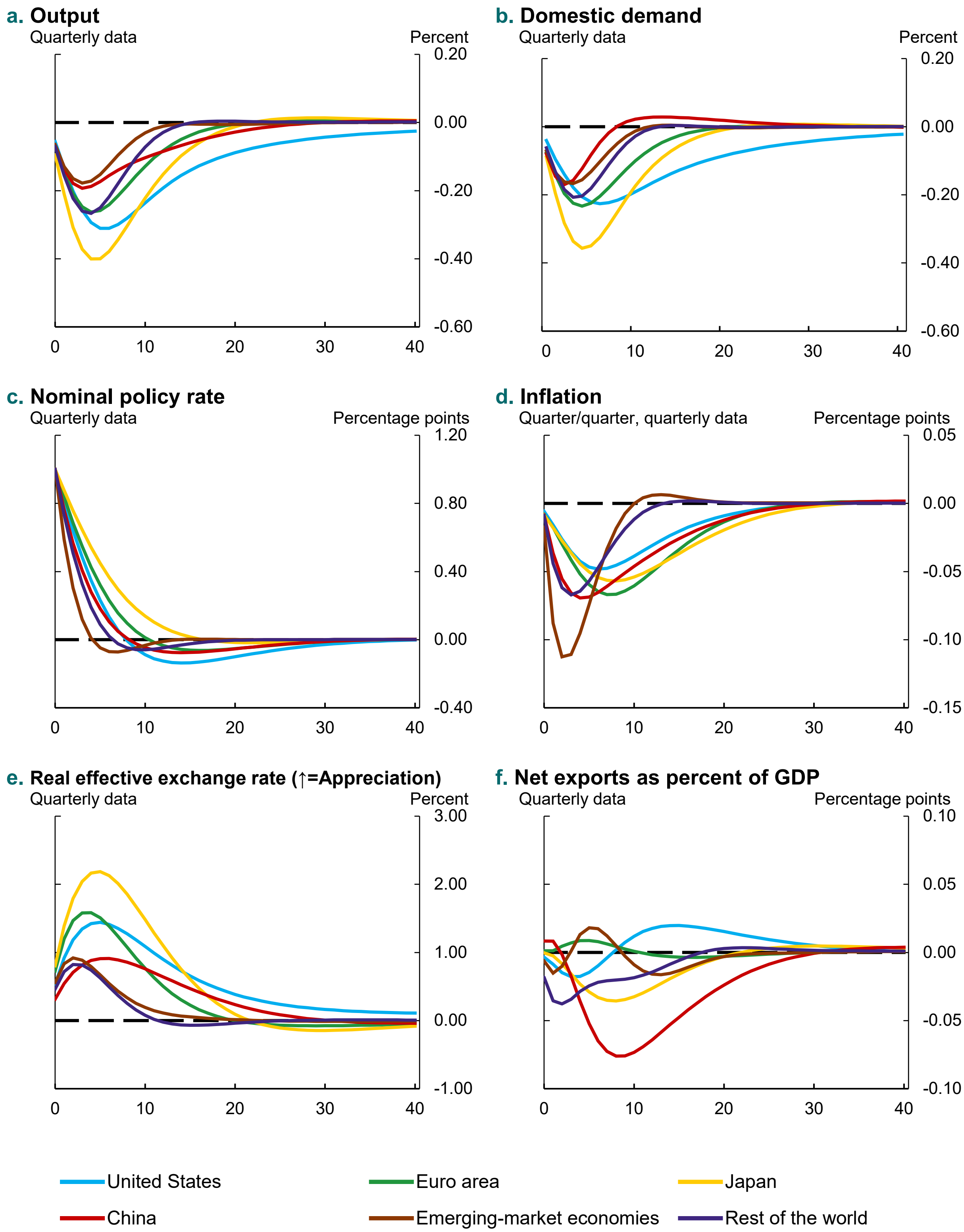
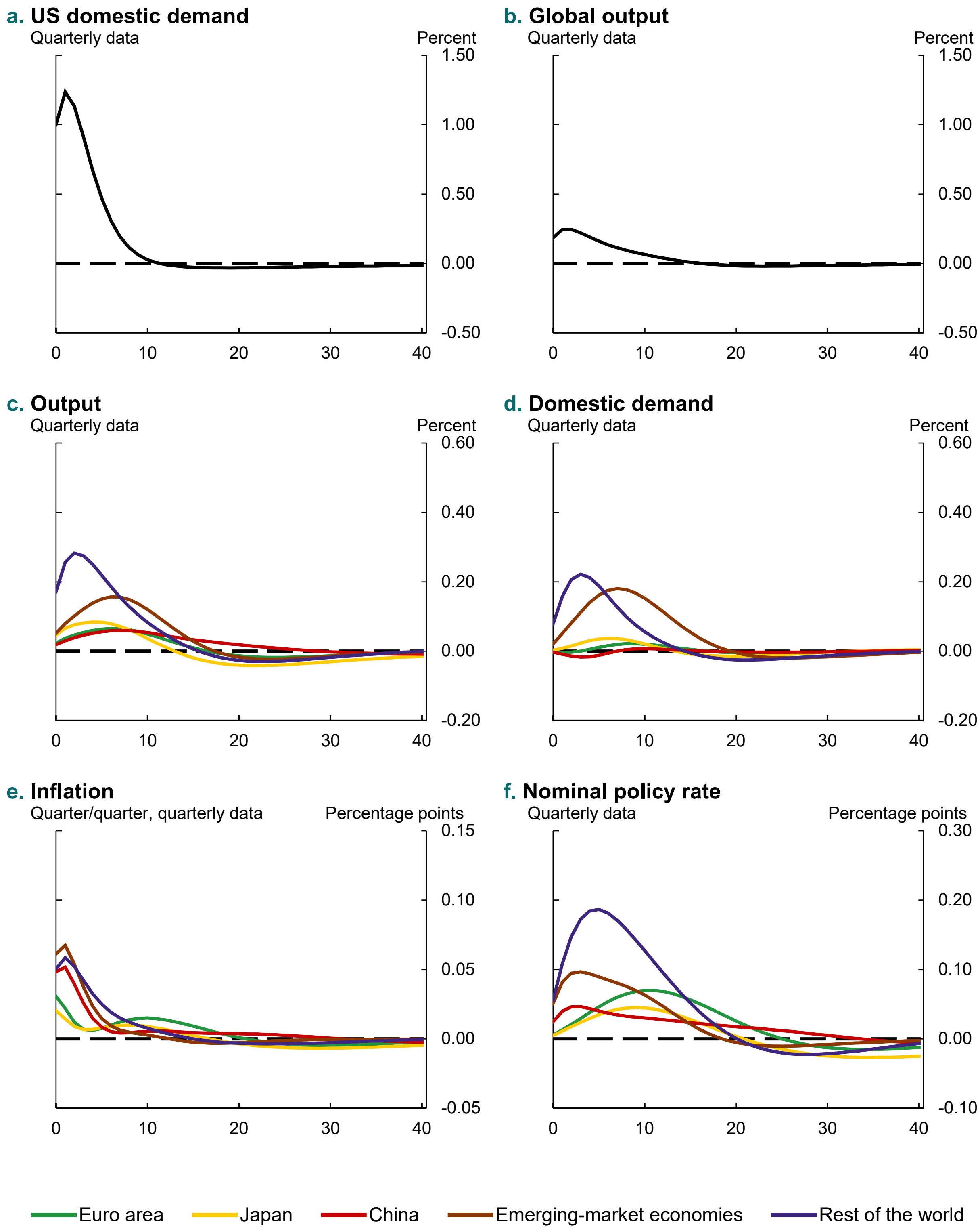


Figure B.4: Spillovers to other regions from a US domestic demand shock



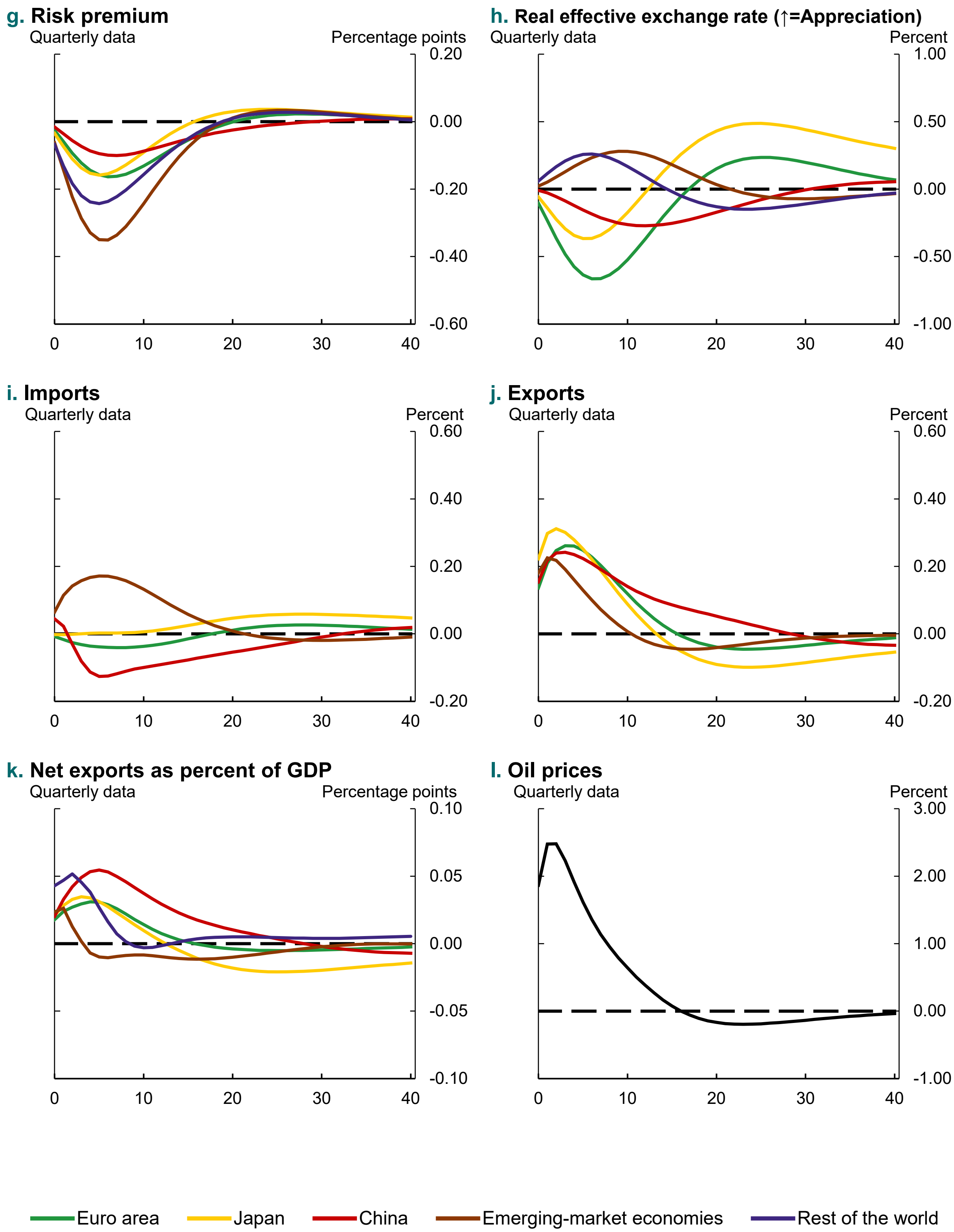
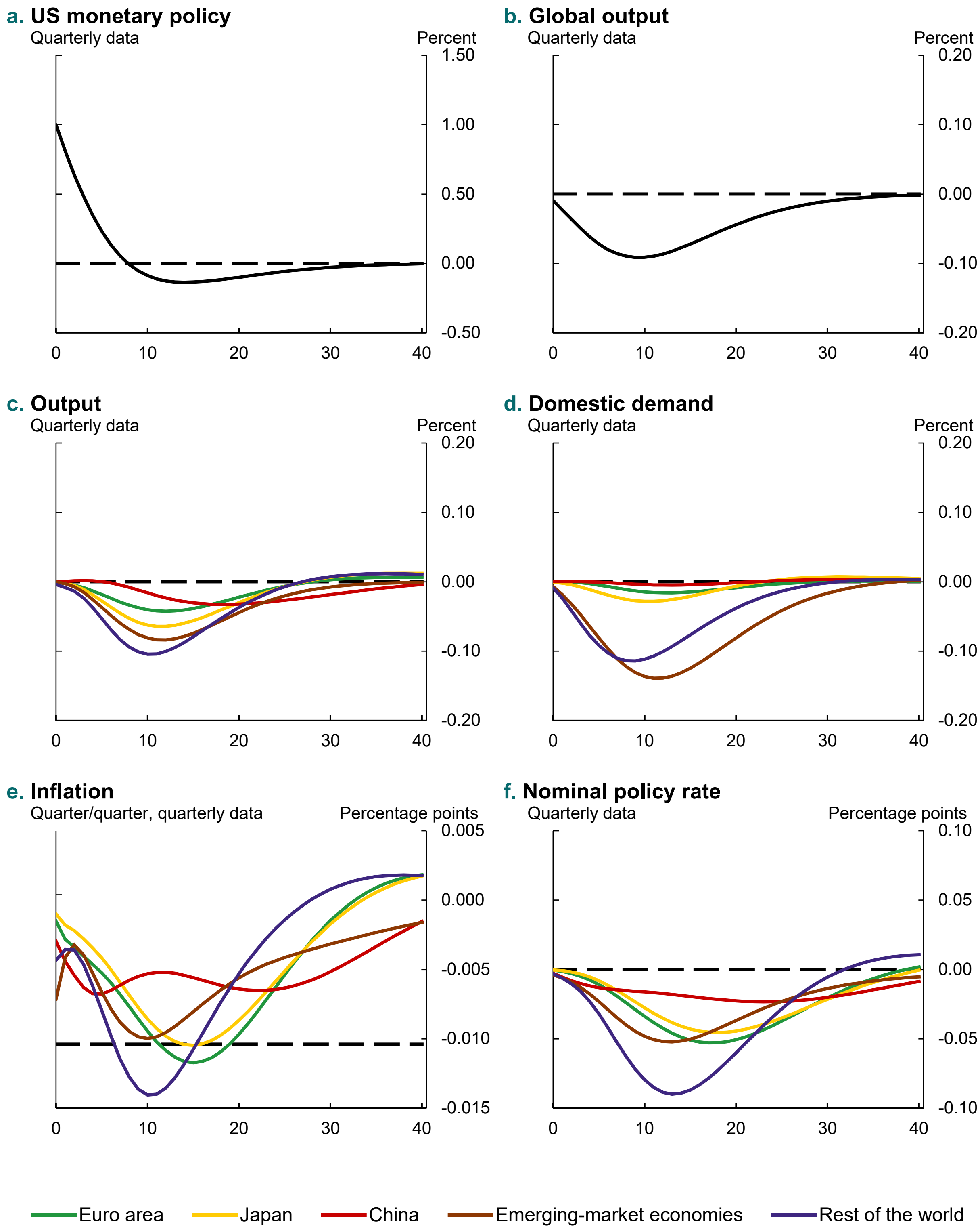
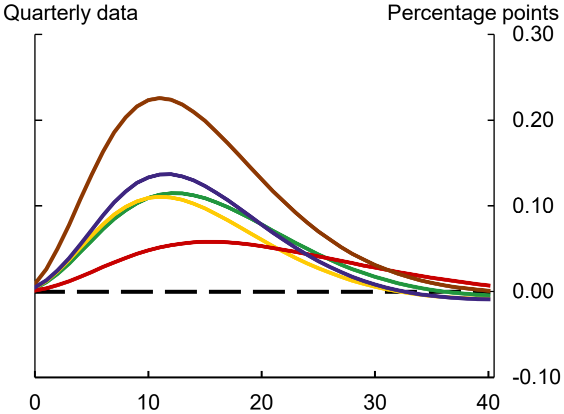

g. Risk premium
Quarterly data
Percentage points
0.20
0.00
-0.20
-0.40
-0.60
0
10
20
30
40
h. Real effective exchange rate (↑=Appreciation)
Quarterly data
Percent
1.00
0.50
0.00
-0.50
-1.00
0
10
20
30
40
i. Imports
Quarterly data
Percent
0.60
0.40
0.20
0.00
-0.20
0
10
20
30
40
j. Exports
Quarterly data
Percent
0.60
0.40
0.20
0.00
-0.20
0
10
20
30
40
k. Net exports as percent of GDP
Quarterly data
Percentage points
0.10
0.05
0.00
-0.05
-0.10
0
10
20
30
40
l. Oil prices
Quarterly data
Percent
3.00
2.00
1.00
0.00
-1.00
0
10
20
30
40
Euro area
Japan
China
Emerging-market economies
Rest of the world

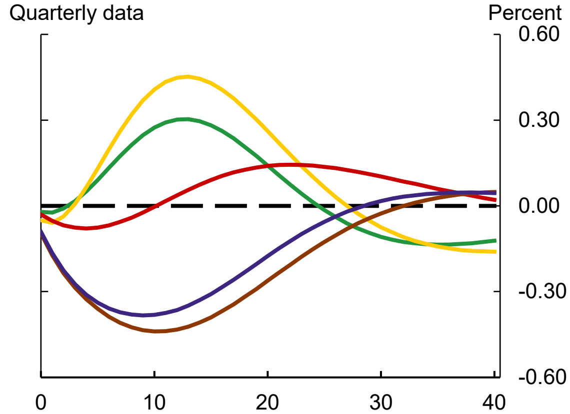
Figure B.5: Spillovers to other regions from a US monetary policy shock



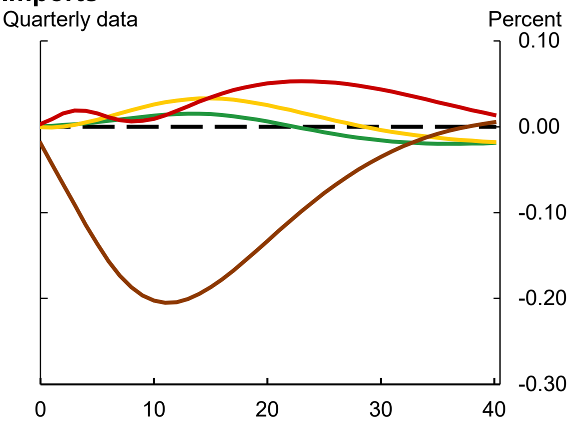
g. Risk premium



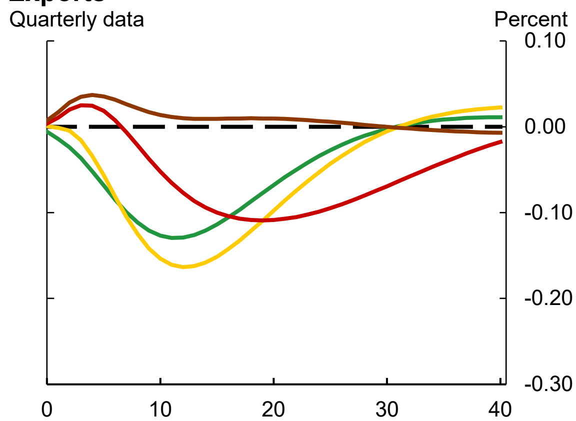
h. Real effective exchange rate (↑=Appreciation)



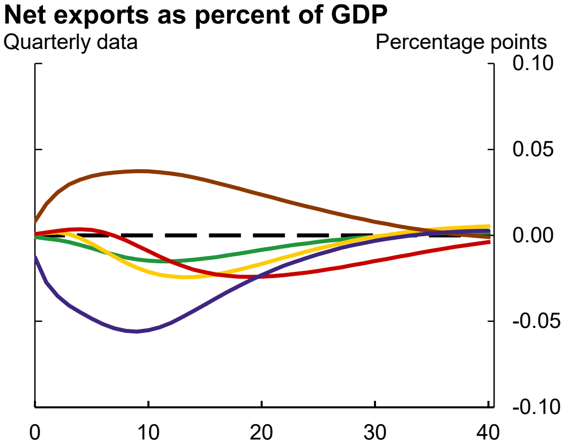
i. Imports



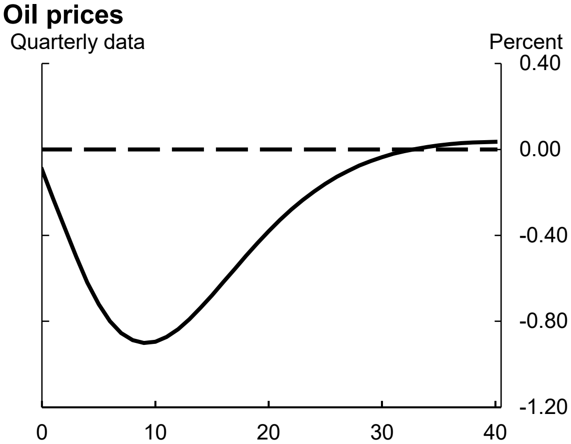
j. Exports



k. Net exports as percent of GDP

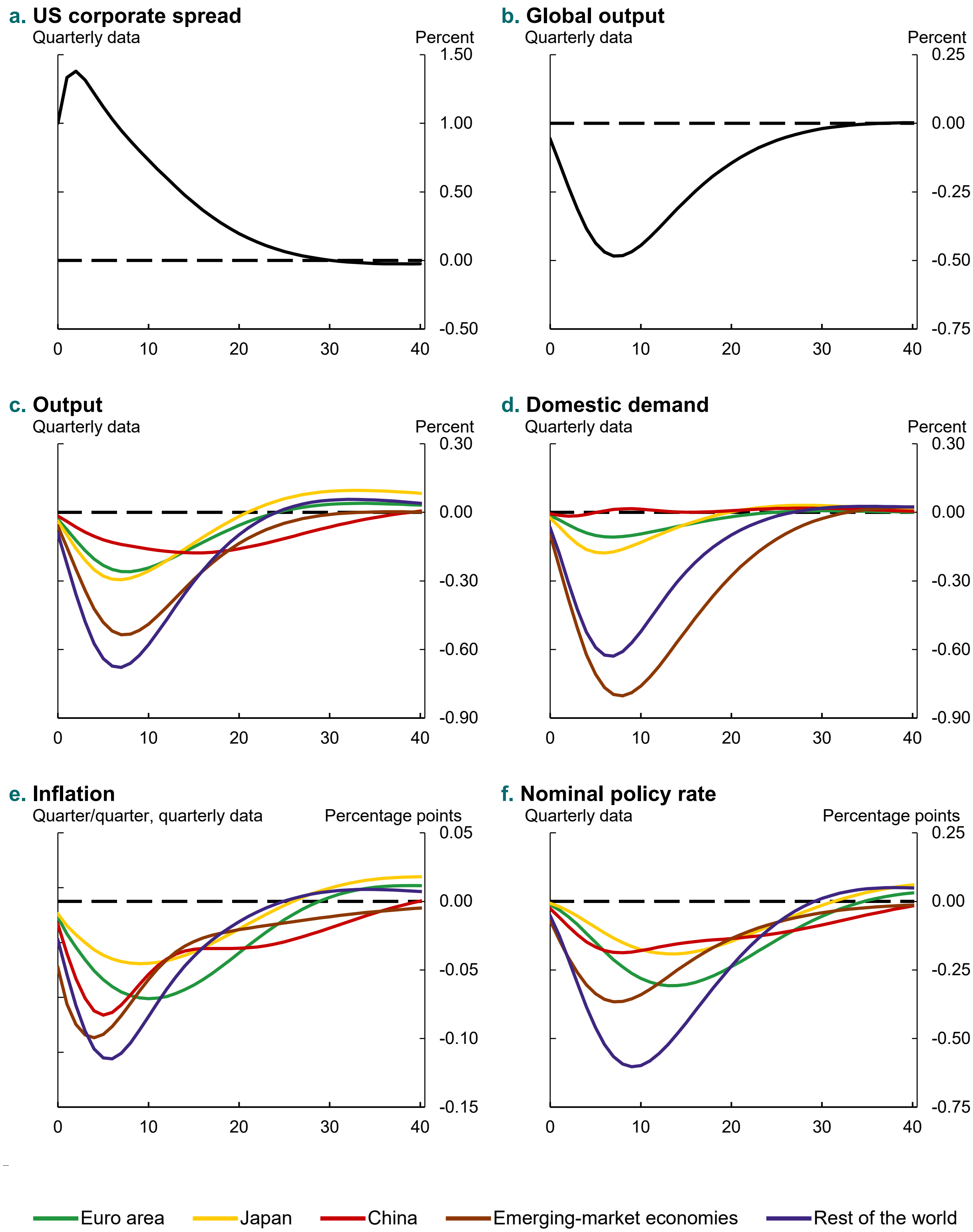


l. Oil prices



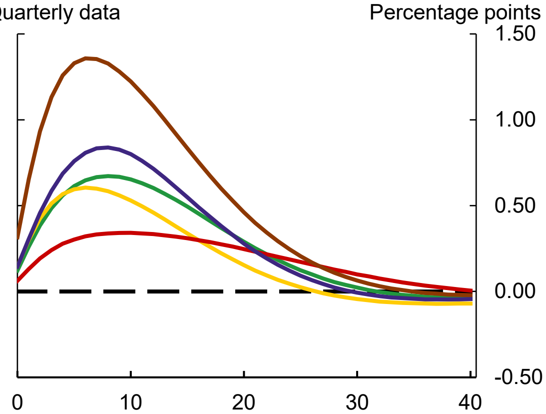
Euro area Japan China Emerging-market economies Rest of the world

Figure B.6: Spillovers to other regions from a US financial shock



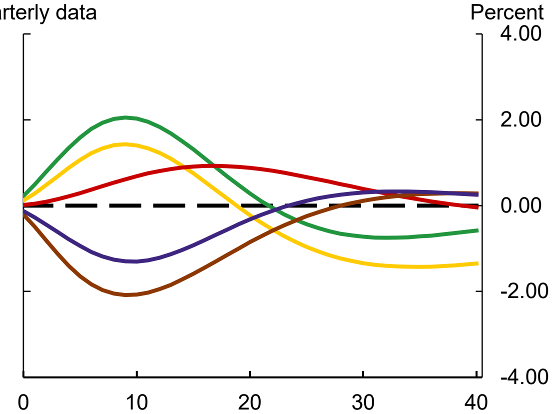
g. Risk premium

Quarterly data | Percentage points



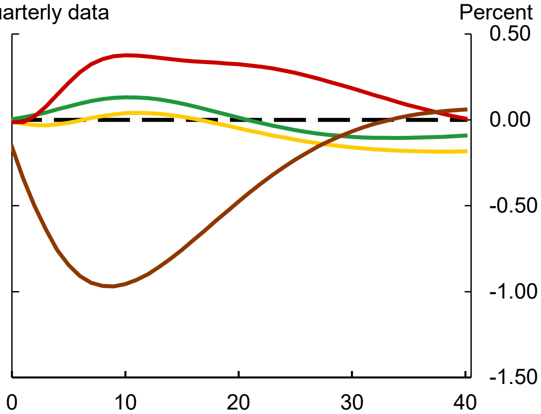
h. Real effective exchange rate (↑=Appreciation)

Quarterly data | Percent



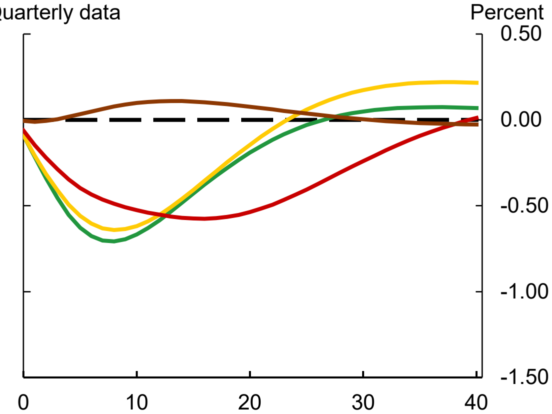
i. Imports

Quarterly data | Percent



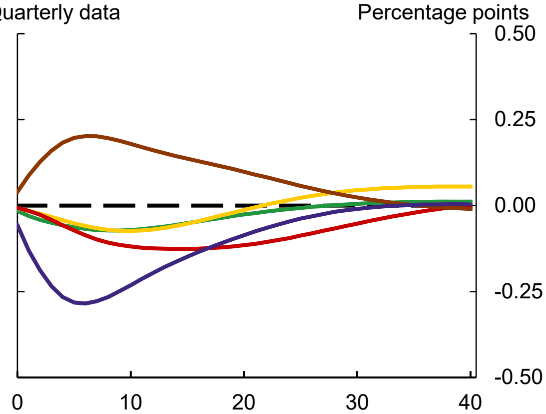
j. Exports

Quarterly data | Percent



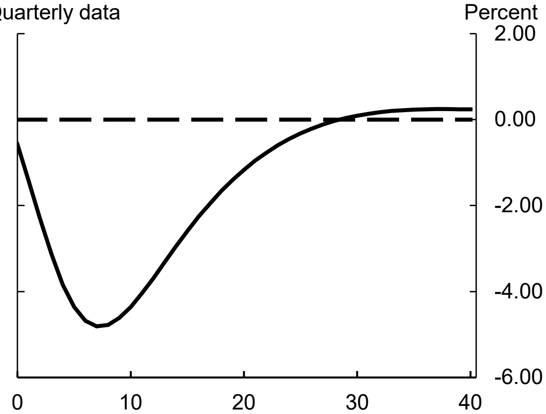
k. Net exports as percent of GDP

Quarterly data | Percentage points



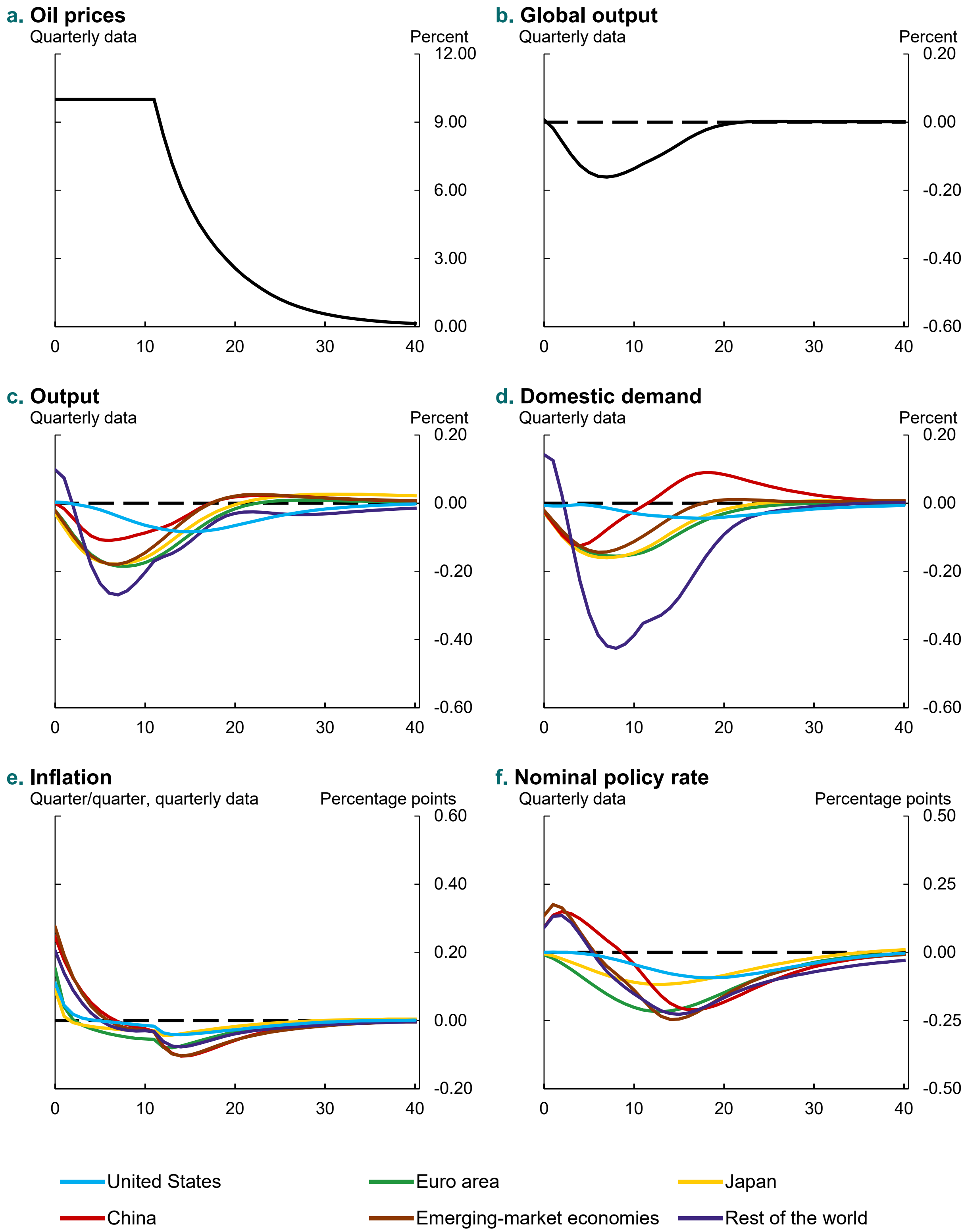
l. Oil prices

Quarterly data | Percent

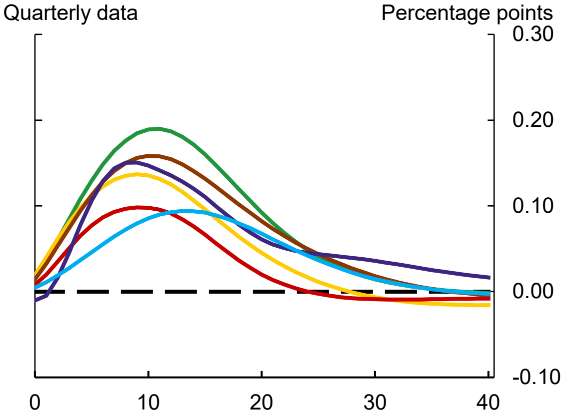


Euro area | Japan | China | Emerging-market economies | Rest of the world

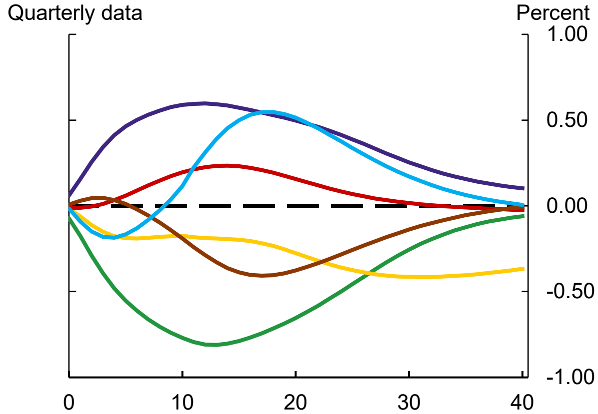
Figure B.7: Spillovers to other regions from an oil supply shock



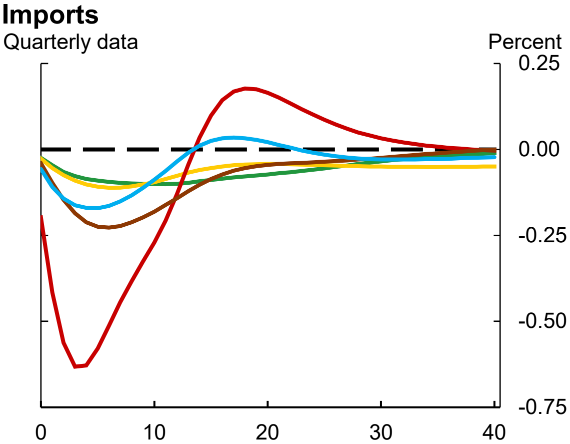
g. Risk premium



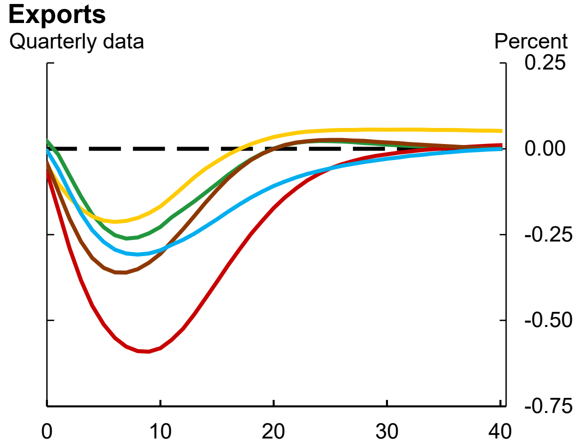
h. Real effective exchange rate (↑=Appreciation)



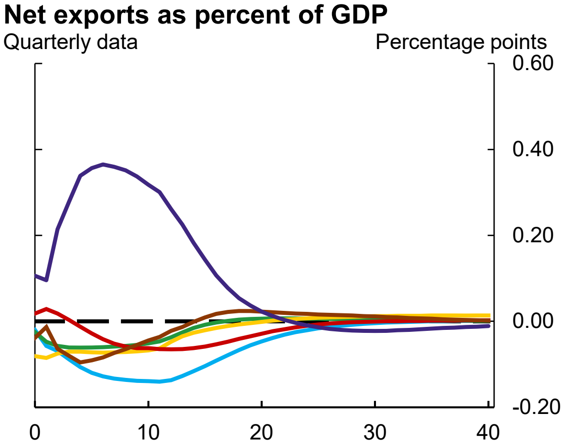
i. Imports



j. Exports



k. Net exports as percent of GDP



United States | Euro area | Japan

China | Emerging-market economies | Rest of the world

Appendix C: US domestic demand in IMPACT

In contrast to the other blocks, the US block in IMPACT explicitly models private consumption, business and residential investment.[43]

Private consumption

The target level of consumption (c^*) is principally modelled as a function of both contemporaneous (yd_t) and permanent disposable income (wh_t) as in Aron et al. (2012).[44] In addition, target consumption is affected by the real effective short- and long-term interest rates (r_{cp} and r_m).[45] We include interest rates in the desired equation to proxy for the user cost of durable goods to determine the desired level of durable good consumption. Finally, real financial (*wfin*) and housing wealth (*wi*) also play a role.

$$\begin{aligned} log\ c_t^* = {} & -1.12 - 0.66\big(0.3r_{cp,t} + 0.7r_{m,t}\big) + 0.32(log\ wh_t - log\ yd_t) + 0.86\ log\ yd_t \\ & + 0.04\ log\ wi_t + 0.10\ log\ wfin_t \end{aligned} \tag{44}$$

The consumption dynamics are then determined by the standard set of REC parameters augmented with the growth of disposable income to proxy for liquidity-constrained consumers:

$$\begin{aligned} \Delta\ log\ c_t = {} & -0.15(log\ c_{t-1} - log\ c_{t-1}^*) + 0.36\Delta\ log\ c_{t-1} + 0.46 \textstyle\sum_{k=0}^{4} f_i \Delta\ log\ c_{t+i}^* + \\ & 0.176\Delta\ log\ yd_t + \varepsilon_{c,t}. \end{aligned} \tag{45}$$

Residential investment

The target level of residential investment follows a neoclassical approach in which investment is a function of the user cost of residential capital (UC_t^{res}).[46] The equation is augmented by a demographic demand variable to proxy for exogenous demand factors (hhf_t):[47]

$$log\ ires_t^* = -7.37 + log\ yd_t + 0.55\ log\ hhf_t - 0.11\ log\ UC_t^{res} \tag{46}$$

$$log\ UC_t^{res} = log\big(r_{mn,t} - r_{mn}^*\big) + log\ P_{res,t}, \tag{47}$$

[43] In MUSE, private consumption and residential investment are aggregated, and their dynamics are explained by a single set of behavioural equations. See Gosselin and Lalonde (2005) for more details.

[44] As in the rest of IMPACT, the consumption, income and wealth variables are expressed as a ratio to GDP, although not explicitly labeled here for simplifying the notation.

[45] The 90-day financial commercial paper rate is used as a proxy for the short rate, and the 30-year mortgage rate is used as the proxy for long-term rates. Using household debt data from the 2016 US Survey of Consumer Finances, we calibrate a 70 percent weight on the long-term rate.

[46] A large portion of literature models the housing market using an error-correction framework. Most of these papers have residential investment responding either to demographic fundamentals (e.g., Demers 2005) or the user cost of capital (e.g., McCarthy et al. 2002).

[47] hhf_t is a trend extracted from volatile household formation data that proxies for exogenous demand factors such as demographic developments. Household formation is a measure of the change in the overall number of households in the United States. We forecast the number of households for different age groups using external projections for both population and headship rates (average number of members in a household).

where $r_{mn,t}$ is the real mortgage rate, (r^*_{mn}) is the steady state of the real mortgage rate, and $P_{res,t}$ is the relative price of residential investment (ratio of residential investment deflator to GDP deflator).[48]

Residential investment dynamics are determined by the error correction term, one lag and the expected changes in the target level of residential investment.

$$\Delta\, log\, ires_t = -0.12(log\, ires_{t-1} - log\, ires^*_{t-1}) + 0.58\Delta\, log\, ires_{t-1} + 0.42\sum_{k=0}^{16} f_i \Delta\, log\, ires^*_{t+i} + \varepsilon_{ires,t} \tag{48}$$

Business investment

The behavioural equations for US business investment in IMPACT closely follow Gosselin and Lalonde (2005), with several simplifications and additions. Whereas MUSE had three investment subcategories, IMPACT features only two subcategories: (i) structures (*str*); and (ii) equipment and intellectual property products (eip). Desired capital for these two subcategories is specified as a share of total capital cost, following Kiley (2001). Firms optimally choose the share of capital of a particular type taking into consideration the user costs (UC) from all types of capital (K). This provides the following estimated translog cost system of equations to determine the desired stock of capital:[49]

$$\frac{UC^{str}K^{str}}{UC^{str}K^{str} + UC^{eip}K^{eip}} = 0.74 + 0.10\, log\, UC^{str} + 0.12\, log\, UC^{eip} \tag{49}$$

$$\frac{UC^{eip}K^{eip}}{UC^{str}K^{str} + UC^{eip}K^{eip}} = (1 - 0.74) + 0.10\, log\, UC^{str} + 0.12\, log\, UC^{eip} \tag{50}$$

Both user costs for the subcomponents of business investment are a function of the long-run corporate bond rate ($r_{b,t}$) and the relative price of investment (ratio of investment deflator to GDP deflator) net of depreciation:

$$log\, UC^{str}_t = log\big[r_{b,t} + 4\delta_{str,t} - 4(1 - 4\delta_{str,t})(\Delta\, log\, P^{80qma}_{str,t})\big] + log\, P_{str,t} \tag{51}$$

$$log\, UC^{eip}_t = log\big[r_{b,t} + 4\delta_{eip,t} - 4(1 - 4\delta_{eip,t})(\Delta\, log\, P^{60qma}_{eip,t})\big] + log\, P_{eip,t}. \tag{52}$$

Investment is modelled following the REC framework of Brayton, Davis and Tulip (2000) and Tinsley (2002). The desired level of investment (I^*) is function of the growth rate of the desired capital, the rate of depreciation and the level of desired capital in the previous period.[50] More specifically, the desired level of investment is given by:

$$I^{*i}_t = 4(\delta_i + \Delta\, log\, K^{*i}_t)K^{*i}_{t-1}, \forall\, i \in \{str, eip\}. \tag{53}$$

The dynamic equations for both types of investment are given by:

[48] The relative price of residential investment is itself modelled as a function of real 30-year mortgage rates to capture the asset component of residential real estate.

[49] The regularity in the equation reflects the constraints implied by a symmetric and linear homogenous cost function derived for a constant-returns-to-scale production technology. Kiley (2001) shows that this modelling framework can be particularly useful in explaining a secular shift away from structures share of total capital and toward EIP share of total capital due to a secular decline in the relative price of EIP investment in history.

[50] Due to the volatility of the desired capital, the growth rate of the desired capital is smoothed over history and is exogenous over the projection.

$$\Delta\, log\, I_t^{str} = -0.05(log\, I_{t-2}^{str} - log\, I_{t-2}^{*str}) + 0.39\Delta\, log\, I_{t-1}^{str} + 0.27\sum_{k=0}^{16} f_i \Delta\, log\, I_{t+i}^{*str} + 0.33\Delta\, log\, Y_t - 0.25\left(r_{b,t-2} - r_{b,t-2}^{12qma}\right) + 0.05\Delta logOIL_t^{4qma} + \varepsilon_{str,t} \tag{54}$$

$$\Delta\, log\, I_t^{eip} = -0.072\left(log\, I_{t-2}^{eip} - log\, I_{t-2}^{*eip}\right) + 0.39\Delta\, log\, I_{t-1}^{eip} + 0.32\sum_{k=0}^{14} f_i \Delta\, log\, I_{t+i}^{*eip} + 0.29\Delta\, log\, Y_t - 0.25\left(r_{b,t-1} - r_{b,t-1}^{12qma}\right) + \varepsilon_{eip,t}. \tag{55}$$

In these equations, growth is negatively related to the disequilibrium between actual and desired investment. Adjustment costs imply the presence of leads of the desired investment and lags of the actual investment in these equations. Contemporaneous GDP growth proxies cash flow effects on firms whereas the contemporaneous interest rate gap allows for a different short-term elasticity to interest rates. The first difference of the price of oil also enters the structures investment dynamic equation to reflect the capital intensity of the oil and gas sector. We find that actual and desired investments are co-integrated at the 5 percent level for both categories.