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Abstract

In this paper, a quarterly growth-accounting data set is built for the Canadian business sector with the top-down approach of Diewert and Yu (2012). Inputs and outputs are measured and used to estimate the quarterly total factor productivity (TFP). In addition, the estimates of annual TFP growth by Diewert and Yu (2012) are revised and updated to reflect changes in the new national economic accounts and national balance-sheet accounts. The quarterly series also provide suitable data for studying short-run dynamics. To demonstrate, a simple vector autoregressive model is estimated to study the responses of hours worked and investment to TFP shocks. Hours worked drop and investment rises in reaction to a positive TFP shock.

JEL classification: O47, D24, F43
Bank classification: Productivity

Résumé

Les auteurs se servent de l'approche descendante de Diewert et Yu (2012) pour constituer un ensemble de données trimestrielles visant à quantifier les facteurs de la croissance du secteur canadien des entreprises. Les intrants et extrants sont mesurés et utilisés pour estimer la productivité totale des facteurs (PTF) trimestrielle. De plus, les estimations de Diewert et Yu (2012) de la croissance annuelle de la PTF sont réexaminées et mises à jour en fonction des changements apportés dans le cadre de la révision des comptes économiques nationaux et des comptes du bilan national. Les séries trimestrielles fournissent par ailleurs des données pertinentes pour analyser les dynamiques de court terme. À titre d'illustration, un modèle vectoriel autorégressif simple est estimé afin d'étudier les répercussions des chocs de productivité sur les heures travaillées et l'investissement. Devant un choc positif sur la PTF, le nombre d'heures travaillées diminue et l'investissement augmente.

Classification JEL : O47, D24, F43
Classification de la Banque : Productivité

Non-Technical Summary

Motivation and Question

Total factor productivity (or multifactor productivity) growth is an important indicator in monitoring the economy. In Canada, Statistics Canada measures annual multifactor productivity growth for the business sector using detailed industry-level data. Diewert and Yu (2012), on the other hand, measure total factor productivity using national accounts and national balance sheets. However, these annual data are not suitable for assessing the impacts of various types of shocks, for example, productivity shocks, on the economy at the quarterly frequency. In this paper, we build a new quarterly data set of total factor productivity for the Canadian business sector, following the approach of Diewert and Yu (2012). At the same time, we revise the original Diewert-Yu annual estimates to reflect revisions to national accounts.

Methodology

We adopt the "top-down" method as in Diewert and Yu (2012). This method is chosen for two reasons. First, the top-down method requires only publicly available aggregate data from the national accounts and national balance sheets. In contrast, the "bottom-up" method, used by Statistics Canada, requires data for highly disaggregated inputs across detailed sectors. Second, the top-down approach enables one to decompose the growth of real gross domestic income into contributions from total factor productivity, primary inputs (capital and labour), and the terms of trade. The bottom-up method, in its current form, cannot isolate the contribution from the terms of trade.

Key Results and Contributions

The new quarterly data set provides estimates of total factor productivity and measured seasonally unadjusted series of prices and quantities of outputs and inputs. Outputs consist of domestic consumption, government spending, business investment, exports and imports. Capital consists of machinery and equipment, non-residential structures, intellectual property products, business inventory, and land. Labour inputs consist of compensation and hours worked for 12 types of workers. In the new data, total factor productivity grew, on average, at a year-over-year rate of 0.5 per cent over the 1982Q1–2013Q4 period, faster than the Statistics Canada estimates. For the period after 2000, the two estimates of productivity displayed much closer growth rates. Improved terms of trade, in the absence of strong productivity growth, contributed more than a quarter of the annual growth of real gross domestic income from 2001 to 2009, and continued to make a significant contribution to the real income growth after the most recent recession. We illustrate the usefulness of our new data with a study of how productivity shocks affect hours worked and investment.

Future Work and Comments

There is a discrepancy between the quarterly and annual estimates of total factor productivity, which is attributed to the discrepancies in capital stocks and investment among Statistics Canada's national accounts, national balance sheets, and its data on the stock and flow of capital. We expect that this issue will be resolved after Statistics Canada harmonizes the measures of capital and investment across different accounts.

1 Introduction

One challenge for policy-makers interested in having a timely assessment of the extent and implications of structural adjustment is data availability. In Canada, annual data sets of total factor productivity (TFP) have been produced by Statistics Canada using a bottom-up industry-based approach, and by Diewert and Yu (2009, 2012) using a top-down approach with National Accounts data. However, two weaknesses of annual data are that they are only available with a considerable lag, and their low frequency limits their usefulness for examining issues where meaningful economic variation and responses may occur within a year. The lack of TFP data at quarterly frequency prevents assessment of impacts of shocks to TFP and the terms of trade on income, wealth, investment, employment and labour productivity at the quarterly level. These impacts can be different within a year than at longer horizons, such as across years, raising the need for quarterly data when assessing policy decisions that are made multiple times and tend to have effects within a year (e.g., monetary policy).

In this paper, a new quarterly data set of Canadian TFP is constructed using the top-down methodology of Diewert and Yu (2009, 2012). The availability of such data will facilitate economic research and timely analysis of structural and cyclical economic developments. The advantage of the Diewert-Yu methodology is that, because it uses national accounts data that include exports and imports, it provides a unified framework for measuring TFP and quantifying the contribution of the terms of trade to real income growth. In contrast, this is not currently possible using an industry-based bottom-up approach for Canada, since exports and imports are not currently separated from other inputs and outputs in the Canadian input-output tables.[1]

The quarterly TFP data are useful for research on business cycles. For example, Basu, Kimball and Fernald (2006) find that, in the short run, labour inputs contract following a technological improvement. The availability of quarterly data for Canada allows one to examine the productivity-labour input relationship for a small open economy, for which the role of the terms of trade should be taken into account. This paper takes a first step in this direction.

The outline of the paper is as follows. Section 2 reviews the methodology of Diewert and Yu (2012). Section 3 performs the quarterly TFP estimation. The robustness of the estimated quarterly series of TFP is assessed according to its sensitivity to data used and assumptions made in the

[1]The top-down approach has disadvantages relative to the bottom-up approach. As implemented in this paper, the top-down methodology uses measures of labour and capital inputs that are subject to aggregation errors, potentially leading to biased estimates of TFP. Diewert and Yu (2012) discussed the sources of aggregation error in the top-down approach. It is unclear whether, overall, the bias is upward or downward. Another challenge is that the top-down approach does not provide the necessary decomposition to examine the role of the reallocation of inputs in aggregate productivity growth. While the two approaches can complement each other, and ideally should be integrated, such analysis is beyond the scope of this paper.

growth accounting exercise. Quarterly and annual data are also compared, suggesting the usefulness of quarterly productivity data. The section ends with a vector autoregressive (VAR) estimation of the effects of shocks to TFP using the quarterly data. Section 4 examines the contribution of the terms of trade to real income growth. Finally, Section 5 concludes.

2 The top-down approach of measuring TFP

This section provides an overview of the methodology of Diewert and Yu (2012), which we adopt to construct the quarterly TFP data. Diewert and Yu (2012) construct annual measures of the growth of TFP in the business sector for the 1961-2011 period. As noted above, their main assumption that real revenue, as a function of output prices and input quantities, has a translog form, making the chained Törnqvist index an exact measure of growth. To measure output and inputs, Diewert and Yu use the data from the national accounts and national balance-sheet accounts. Output is the value added in the business sector that is measured using final demand. Inputs include quality-adjusted labour input and capital services.[2]

To understand the relationship between real income, TFP, input quantities and relative prices, it is convenient to introduce some notation: y^t is the output vector of $m = 1, \cdots, M$ products, y^t_m, with p^t the corresponding vector of p^t_m, the ratio of M output prices to a measure of the aggregate price level, such as the consumption price; x^t is the the input vector of $n = 1, \ldots, N$ inputs, x^t_n, with w^t the corresponding vector of w^t_n, the ratio of N input prices to the same aggregate price level; and $g^t(p, x)$ is real income generated in the business sector. In this notation, the superscript t denotes the time period.

The growth of real income can be decomposed into the contributions from three components: TFP growth $\gamma^t(p, x)$, the rate of change of real output prices $\alpha^s(p^{t-1}, p^t, x)$, and the growth of real inputs $\beta^s(p, x^{t-1}, x^t)$. TFP growth captures the change in real output between two periods, assuming that in both periods the same amount of inputs (capital and labour) are used for production:

$$\gamma^t(p, x) = g^t(p, x)/g^{t-1}(p, x). \tag{1}$$

If inputs remain constant, such a change in output is often interpreted as a change in technology.[3] The real income growth due to changes in output price assumes that inputs are fixed at some

[2]The Diewert-Yu estimates of the growth of historical annual TFP are substantially higher than estimates by Statistics Canada. The difference between the two estimates is largely owing to differences in the period between 1961 and the mid-1990s, as will be seen later in this paper.

[3]Such a change may also reflect other factors, for instance, imperfect competition. See Basu and Fernald (2002) for gaps between measured productivity and technology.

reference level x and production technology at period s is used, as follows:

$$\alpha^s(p^{t-1}, p^t, x) = g^s(p^t, x)/g^s(p^{t-1}, x). \tag{2}$$

Analogous to this concept is the growth of real income due to changes in input quantities:

$$\beta^s(p, x^{t-1}, x^t) = g^s(p, x^t)/g^s(p, x^{t-1}). \tag{3}$$

Using these definitions, it can be shown that the growth rate of real income can be decomposed as follows:

$$g^t(p^t, x^t)/g^{t-1}(p^{t-1}, x^{t-1}) \approx \gamma^t \alpha^t \beta^t. \tag{4}$$

Diewert and Yu (2012) show that, if the real revenue function $g^t(\cdot)$ has the translog form, equation (4) holds exactly. The measures of the three components of growth in real revenue can take the following forms:

$$\ln \alpha^t = \sum_{m=1}^{M} \frac{1}{2}\left[\frac{p_m^{t-1} y_m^{t-1}}{p^{t-1} \cdot y^{t-1}} + \frac{p_m^t y_m^t}{p^t \cdot y^t}\right] \ln\left(\frac{p_m^t}{p_m^{t-1}}\right); \tag{5}$$

$$\ln \beta^t = \sum_{n=1}^{N} \frac{1}{2}\left[\frac{w_n^{t-1} x_n^{t-1}}{w^{t-1} \cdot x^{t-1}} + \frac{w_n^t x_n^t}{w^t \cdot x^t}\right] \ln\left(\frac{x_n^t}{x_n^{t-1}}\right); \tag{6}$$

$$\gamma^t = \frac{g^t(p^t, x^t)/g^{t-1}(p^{t-1}, x^{t-1})}{\alpha^t \beta^t}, \tag{7}$$

where α^t turns out to be the Törnqvist index of the output price, and β^t is the Törnqvist index of input quantity. In this formulation, the contribution of the real output price can be further decomposed as the product of Törnqvist price indexes of individual output components:

$$\alpha^t = \alpha_1^t \alpha_2^t \cdots \alpha_M^t, \tag{8}$$

where

$$\ln \alpha_m^t = \frac{1}{2}\left[\frac{p_m^{t-1} y_m^{t-1}}{p^{t-1} \cdot y^{t-1}} + \frac{p_m^t y_m^t}{p^t \cdot y^t}\right] \ln\left(\frac{p_m^t}{p_m^{t-1}}\right). \tag{9}$$

The contribution from changes in quantities of all inputs has a similar exact decomposition:

$$\beta^t = \beta_1^t \beta_2^t \cdots \beta_N^t, \tag{10}$$

$$\ln \beta_n^t = \frac{1}{2}\left[\frac{w_n^{t-1} x_n^{t-1}}{w^{t-1} \cdot x^{t-1}} + \frac{w_n^t x_n^t}{w^t \cdot x^t}\right] \ln\left(\frac{x_n^t}{x_n^{t-1}}\right). \tag{11}$$

Additional details on the methodology are summarized in Appendix A.

The above method for decomposing real gross domestic income (GDI) is based on Diewert and Morrison (1986) among others, where improvement in the terms of trade is similar to technological progress in the sense that both contribute directly to real GDI growth. There are alternative measures of the income effect of the terms of trade. Similar to the Diewert-Yu method, Kohli (2004, 2006) measures gains in trading as the change in the gross domestic product (GDP) price relative to the price of GDP excluding imports. Reinsdorf (2010) uses the Fisher price index to decompose this effect into two components: contribution from the changes in the terms of trade and contribution from the changes in the relative price of tradables. The latter relative price is the harmonic mean of export and import prices divided by the price of domestic outputs. Reinsdorf's method relies on the Fisher index, however, while in the Diewert-Yu method, the Törnqvist index is used.

3 Measuring quarterly TFP

This section provides an overview of the construction of quarterly productivity data for the Canadian business sector, leaving the details to Appendix B. Some series from the annual estimates of productivity are needed to measure quarterly productivity. The annual Diewert-Yu estimates are therefore updated first, to reflect changes to the national accounts and national balance-sheet accounts, which are reported in Appendix C.[4]

In estimating quarterly TFP, the top-down methodology is applied to the quarterly series of output and inputs from the new Canadian System of National Accounts, CSNA12, and new National Balance Sheet Accounts, NBSA12. One important issue that does not exist with the annual data, but must be addressed in constructing the quarterly estimates, is that of seasonality. The preferred approach would be to use raw seasonally unadjusted data at all stages to construct seasonally unadjusted indexes. These seasonally unadjusted data could then be seasonally adjusted if such a format of data were desired. Unfortunately, data limitations prevented straightforward application of the approach for the annual exercise to quarterly seasonally unadjusted data. The process followed is described below, and robustness analysis of alternatives is discussed.

[4] Appendix C also reviews the revisions to the Canadian System of National Accounts data (CSNA12) and National Balance Sheet Accounts data (NBSA12), and compares the revised annual TFP estimates with the original Diewert-Yu estimates as well as those of Statistics Canada.

3.1 Discussion of the Data

One challenge in the construction of quarterly data is the lack of consistently measured and seasonally unadjusted data.[5] To overcome this limitation, some adjustments were made to the data. First, the existence of price seasonality is checked, by comparing the seasonally adjusted and unadjusted constant-dollar series. For some variables, the implicit price indexes of seasonally unadjusted constant-dollar series are identical or very close to the implicit price indexes of the seasonally adjusted measures, suggesting that there is no seasonality in those price indexes.[6] For these variables, the real quantity used in the growth accounting exercise is the seasonally adjusted chained-dollar quantity. For other variables, the price indexes of seasonally adjusted and unadjusted constant-dollar series differ, suggesting the existence of price seasonality. When this is the case, the price seasonality is calculated as the ratio of two implicit price indexes, one calculated using the seasonally unadjusted current-price values and seasonally unadjusted constant-dollar quantities, and the other using the seasonally adjusted current-price values and seasonally adjusted constant-dollar quantities. This price seasonality ratio is then applied to the implicit price indexes obtained from the seasonally adjusted chained-dollar series, giving measures of implicit price indexes that are then used to obtain seasonally unadjusted chained-dollar quantities.[7]

In the case of output measures, seasonally unadjusted current-price and chained-dollar values of all components can be obtained from CSNA12, except for exports and imports. Price seasonality ratios cannot be calculated for the 12 types of traded goods because the seasonally unadjusted constant-dollar values are unavailable. An examination of the price indexes of merchandise trade indicates that there is no significant price seasonality for these exports and imports except for the import price of energy products. The price indexes of seasonally adjusted chained-dollar series for the exports and imports of goods are therefore used as proxies for the price indexes of seasonally unadjusted series for those with small differences between price indexes of adjusted and unadjusted series. For the rest of exports and imports, the price seasonality of aggregate export and import

[5] In the real world, production is not organized in a seasonally adjusted way and using seasonally adjusted (i.e., smoothed) series may distort true production.

[6] Implicit prices of the chained-dollar series can differ from those of the constant-dollar series because of changes in the composition of the variable over time. One such example is investment in machinery and equipment, with the price index of the seasonally adjusted chained-dollar series being very different from that of the seasonally adjusted constant-dollar series. This arises because of the significant decline in the prices of computers as well as the increased use of computers.

[7] Let $P^{adj,const}$ be the deflator of the seasonally adjusted constant-dollar series, calculated as the current value divided by the constant-dollar quantity, $P^{adj,const} = \frac{V^{adj}}{Q^{adj,const}}$. The deflator of the seasonally unadjusted constant-dollar series, $P^{unadj,const}$, is defined and calculated in the same way. The price seasonality is defined as $\xi = \frac{P^{unadj,const}}{P^{adj,const}}$, which is used to calculate the deflator for seasonally unadjusted chained-dollar quantities as $P^{unadj,chain} = \xi \cdot P^{adj,chain}$. The seasonally unadjusted chained-dollar quantity is then calculated as $Q^{unadj,chain} = \frac{V^{unadj}}{P^{unadj,chain}}$.

prices is applied respectively to the implicit price indexes of the seasonally adjusted chained-dollar exports and imports at the two-digit level of the North American Product Classification System (NAPCS).[8]

The quarterly labour inputs, including hours worked and compensation for 36 types of workers, are constructed from the Labour Force Survey (LFS) public use microdata files.[9] Hours worked are measured as actual hours worked on all jobs.[10] These actual hours worked are adjusted for holidays when calculating the quarterly hours worked for each type of workers. The LFS also provides hourly earnings for usual hours worked on the main job (available starting from 1997Q1), but not for actual hours worked on all jobs. For each of these 36 worker types, total quarterly compensation for all hours worked is estimated, assuming that the compensation rates for usual hours and actual hours are the same. For compensation series, quarterly data before 1997Q1 are obtained by linearly interpolating the annual data, and applying aggregate wage seasonality obtained from the Survey of Employment, Payrolls and Hours (SEPH). Thus, for periods before 1997Q1, the seasonal changes in compensation are the same for all types of workers. Finally, to ensure consistency, the 72 quarterly series of hours worked and compensation are benchmarked to the data used in measuring the annual TFP.

The quarterly capital stocks, used to measure capital services, are constructed using the perpetual inventory method, similar to the annual estimates. This requires data on the initial-period capital stocks, quarterly depreciation rates and quarterly investments. Investments are directly available from CSNA12. The initial-period capital stocks are from the annual data on productivity estimates, because the quarterly capital stock data in NBSA12 are available for only three types of assets (non-residential structures, machinery and equipment, and intellectual property products (IPP)) while nine types of assets are used for the quarterly estimates. For the same reason, quarterly depreciation rates are calculated from annual values, assuming these quarterly rates do not vary within any given year. Using the annual data requires the asset types in the annual and the quarterly source data to be the same. Because the asset types in the annual data do not agree exactly with those of investment in the quarterly data, primarily for machinery and equipment, it is necessary to aggregate assets into nine types: four types of machinery and equipment, two types of non-residential structures, and three types of IPP. Machinery and equipment includes computers,

[8] This may create a bias in the volatility of some exports and imports because their degree of seasonal variation is smaller than that for aggregate exports and imports. For instance, since the seasonality of imported energy products is significant, a large portion of the seasonality of aggregate imports may arise from that of imported energy products. At the time of our estimates, there appear to be no other ways to obtain the price seasonality of individual exports and imports.

[9] The 36 worker types consist of three levels of education achievement, three age groups, two gender groups, and employees versus self-employed.

[10] The LFS includes information on usual and actual weekly hours worked for both main jobs and all jobs.

telecommunications equipment, industrial machinery, and transportation and furniture. Grouping machinery and equipment in this way permits the matching of asset types between annual and quarterly data by current-price investments.[11] This grouping also allows the constructed capital services to capture the faster technological progress made in computers and telecommunications equipment relative to other types of capital.

3.2 Quarterly TFP results

The general trends in the quarterly series for output, TFP, and labour inputs and capital services are the same as in the annual series (Figure 1).[12] First, as in the annual data, although quarterly TFP expanded, on average, over the period from 1981Q1 to 2013Q4 (0.5 per cent year-on-year), it contracted between the early 2000s and 2013Q4. Some positive growth was observed in the early part of the recovery from the most recent recession, but subsequently stalled. Second, the business sector has experienced capital deepening since late 2003, at about the same time that TFP started contracting noticeably. This suggests that the business sector may have experienced a structural transformation over this period.

Some of the basic correlation properties of TFP and other important macroeconomic measures of inputs and relative prices are summarized in Table A. First, TFP growth is contemporaneously positively correlated with growth of real output in the business sector. Second, the growth rates of both hours worked and labour input (quality-adjusted hours worked) are negatively correlated with contemporaneous TFP growth, but positively correlated with its growth lagged by one quarter. These negative correlations are consistent with the findings in Basu, Kimball and Fernald (2006).[13]

[11] Extensive comparison of quarterly investments (CANSIM 380-0068) with annual investments (CANSIM 031-0003) appears to suggest that annual investments used for constructing annual capital stocks are not updated as frequently as the quarterly series. For the same series (with identical names), discrepancy exists between the quarterly and annual data.

[12] The series shown in the charts are seasonally adjusted with the X-13ARIMA-SEATS Seasonal Adjustment Program developed by the United States Census Bureau.

[13] Basu, Kimball and Fernald (2006) find that technology shocks are contractionary in the short run, e.g., hours worked first drop then rise following a positive technology shock. Their findings are consistent with assumptions of nominal rigidity and other frictions in models of business cycles. A more detailed exercise along the lines of that in Basu, Kimball and Fernald (2006) requires separately identifying capacity utilization and is left for future research. By not controlling for changes in capacity utilization, the quarterly TFP measures constructed in this study may confuse technological changes with changes in capacity utilization. While there is no consensus on how the effects of capacity utilization should be separated from measured TFP, a blunt adjustment for capacity utilization can be made by subtracting the change in capacity utilization rates estimated by Statistics Canada from TFP growth. Assuming an additive relationship between technological change and change in capacity utilization, $\Delta \ln TFP = \Delta \ln A + \Delta \ln U$, where A is the true technology and U is the capacity utilization rate. Subtracting capacity utilization from the measured total factor productivity gives the utilization-adjusted total factor productivity (or A). Figure 6 shows that measured TFP and utilization-adjusted TFP do deviate, most notably over the recent recession. The correlation between $\Delta \ln TFP$ and $\Delta \ln U$ is 0.22, while it is -0.23 between $\Delta \ln A$ and $\Delta \ln U$. This suggests that the quarterly estimates of TFP may indeed be influenced by changes in capacity utilization. A more formal study incorporating capacity utilization into growth accounting is left for future research, which requires reliable data on capacity utilization.

Finally, while contemporaneous (or lagged) improvements in the terms of trade are moderately positively correlated with non-residential investment growth, correlations with the other variables are weaker.

Table A: Partial correlations, quarterly data

	ΔTFP	Lagged ΔTFP	ΔToT	Lagged ΔToT
Output (Δy)	0.71	-0.19	0.06	0.10
Labour input (Δl)	-0.18	0.18	0.02	0.16
Hours (Δh)	-0.21	0.23	0.02	0.15
Investment (Δi)	0.05	0.11	0.23	0.40
TFP (ΔTFP)	1.00	-0.62	0.04	0.01
Terms of trade (ΔToT)	0.04	-0.08	1.00	0.28

Just as differences in the measurement of capital services by Diewert and Yu (2012) and by Statistics Canada had implications for the measurement of TFP in the top-down versus bottom-up approaches, the differences in quarterly and annual measures of capital services also have important implications when using the same methodology. The correlation between the growth rate of annual TFP estimated from the quarterly constructions and the growth rate of the directly estimated annual TFP series is 0.91, indicating some gaps between the two—differences that can be traced primarily to the measurement of capital services (Figure 2). The correlation between annual and quarterly growth rates does not increase when quarterly investment is benchmarked to the annual data.

Recognizing that the measurement of capital services may affect any quarterly-annual comparison, it is nonetheless noteworthy that basic correlations among the same macroeconomic variables examined above can be significantly different in the annual and quarterly data. To illustrate this difference, Table B reports the correlation coefficients with the annual data. For example, in the quarterly data, the growth rates of hours and labour input are negatively correlated with TFP growth, but in the annual data, the correlations between these variables are positive.

Table B: Partial correlations, annual data

	ΔTFP	Lagged ΔTFP	ΔToT	Lagged ΔToT
Output (Δy)	0.86	0.20	0.31	0.08
Labour input (Δl)	0.46	0.37	0.46	-0.12
Hours (Δh)	0.46	0.35	0.45	-0.11
Investment (Δi)	0.22	0.22	0.65	0.04
TFP (ΔTFP)	1.0	0.03	0.07	0.11
Terms of trade (ΔToT)	0.07	-0.19	1.0	-0.12

Finally, as a robustness check, labour productivity, or the real value of output per hour worked, calculated from our quarterly series can be compared with the quarterly labour productivity data published by Statistics Canada. In general, real GDP, hours worked and average labour productivity (ALP) are similar, although the measures constructed in this paper are more volatile (Figures 3, 4 and 5).[14] The real output series are fairly close—they differ in that both the paid rent and the rental value of owner-occupied dwellings are excluded in the analysis of this study, whereas only the rental value of owner-occupied dwellings is excluded in the Statistics Canada estimates. The differences in the series of quarterly hours worked between this study and Statistics Canada are greater, since they are drawn from different source data. Hours worked in the Statistics Canada labour productivity program take into account information in both LFS and SEPH data sources, as well as on multiple-job holders, while the analysis in this study relies solely on the LFS data.[15]

3.3 Effects of productivity shocks

The differences in the correlations between annual data and quarterly data underline the fact that such information should not be interpreted as indicative of causality. A simple VAR on quarterly data, however, can provide some insights into the structural drivers of these correlations. Two VARs are estimated using data for TFP, the terms of trade, investment, hours worked, the price of investment and the wage index. In one, all variables are expressed in rates of change (first-difference of the natural logarithm), whereas, in the other, the log-levels of the data are first detrended using a Hodrick-Prescott (HP) filter. In both VARs, four lags are included.

In the VARs, TFP residuals were assumed to correspond with technology shocks. This identification assumes that all other variables in the VAR could respond contemporaneously to technology shocks but that TFP would not respond contemporaneously to other structural shocks. Impulse responses to a technology shock are shown in Figure 7 and Figure 8 for the VAR in growth rates and for the VAR in detrended log-levels, respectively.

In the VAR in growth rates, growth in hours responds negatively on impact, but rebounds strongly one quarter later.[16] While there is some volatility in the responses, the relative sizes of the responses indicate that, after one quarter, the level of hours will remain persistently positive. The wage growth response is positive on impact, but is also quite volatile from one quarter to the next.

[14] This higher volatility may be related to possible differences in implementing the seasonal adjustment in this study and Statistics Canada.

[15] In constructing quarterly hours worked, Statistics Canada uses the geometric mean of hours worked in LFS and SEPH for employees to smooth the measured hours worked, and uses hours worked from LFS for the self-employed. In addition, multiple-job holders in LFS are counted twice to calculate aggregate hours. This paper doesn't use SEPH data, since they have no detailed information on worker types.

[16] Basu, Kimball and Fernald (2006) also find an initial negative impact on hours, although their estimates of a subsequent positive response are insignificant. In addition, their analysis uses annual data.

The response of the investment price is positive on impact and roughly displays a hump shape. In contrast to these outcomes, the response of investment growth is delayed relative to the timing of the shock, with a significant positive response occurring only after four quarters.

The responses to a technology shock in the VAR in detrended log-levels indicate similar dynamics. Of course, as all series are detrended prior to inclusion in the VAR, it is not possible to determine whether a technology shock would have permanent effects on the levels of any of the variables used in this VAR. Nevertheless, VAR results indicate that the responses of hours (after an initial negative response on impact) and investment are positive and quite persistent, with the peak cyclical response of the level of investment occurring somewhat later than the peak cyclical response of hours.

Overall, the impulse responses help resolve the diverging correlations among the variables in the quarterly versus the annual data. It is worth noting that the analysis here is only meant to be illustrative of the sorts of questions for which quarterly data might provide some insights not available from annual data.[17]

4 Real GDI growth, TFP and trading gains

In measuring TFP, one advantage of the top-down approach relative to the bottom-up approach is that the former allows one to examine the contribution of trading gains to the growth of real GDI. Such gains arise from the changes in the terms of trade and trade imbalances. This is particularly useful for studying the improved terms of trade in the 2000s. In this section, the trading gains due to changes in relative trade prices are examined.[18]

In the national accounts, current-price GDI is the sum of nominal labour income and nominal capital income, which equals the current-price GDP. If the implicit price of GDP is used as the deflator, real GDI and real GDP are the same. But if other deflators are used to deflate the current-price GDI, real GDI and real GDP can then be different. In studying the contribution of relative price changes to growth in real income, the implicit price of domestic goods (consisting of consumption, government spending and investment) is used as the deflator to obtain real GDI. The possible deviation of real domestic income from real domestic product is then attributed to the deviation of prices of exports and imports from the prices of domestic goods.

[17]As was noted earlier, the data are constructed without controlling for changes in utilization. In this sense, using the identification described above, the TFP residuals could be a combination of technology and utilization shocks.

[18]The use of the term "trading gains" is to differentiate from welfare and productivity gains from trade, which have been studied extensively in the literature on international trade.

Growth of real income can be decomposed into five factors as follows:[19]

$$g^t/g^{t-1} = \gamma^t \alpha_X^t \alpha_M^t \beta_L^t \beta_K^t,$$

where g^t is aggregate real income growth, γ^t is the growth of TFP, α_X^t is the rate of change in the real export price, and α_M^t is the rate of change in the real import price. If $\alpha_X^t > 1$, Canadian export prices grow faster than the price of domestic output, hence, α_X^t measures the contribution of rising export prices to the growth of real income generated by the business sector. Conversely, if $\alpha_M^t > 1$, it represents the contribution of falling real import prices to the growth of real gross output.[20] The larger the value of α_M^t, the larger the decline in the real import price (relative the price of domestic output). The export price and import price can be pooled together to form the trading gains, $\alpha_{XM}^t = \alpha_X^t \alpha_M^t$. Finally, the last two terms in the above equation represent the growth of quantities in labour inputs (β_L^t) and capital services (β_K^t). The cumulative counterparts of the above decomposition equation can be obtained for each term. For example, cumulative growth of the TFP is $\Gamma^t = \Gamma^{t-1} \cdot \gamma^t$ with $\Gamma^0 = 1$.

The trading gains can be further decomposed into two effects on real income growth: the terms-of-trade effect and the relative-price effect, as shown in Kohli (2004, 2006) and Reinsdorf (2010). The terms-of-trade effect reflects the contribution of changes in the export-import price ratio to real income growth. The relative-price effect captures the contribution resulting from trade imbalances, as well as from deviation in the price of tradables from the price of non-tradables. Let s_X^t and s_M^t be the two-period average share of current-price exports and imports, respectively, in nominal GDI in period t. Note that s_M^t is negative, since imports make a negative contribution to domestic output. Further let P_X^t/P_M^t be the export-import price ratio (i.e., terms of trade), define $P_T^t = (P_X^t \cdot P_M^t)^{\frac{1}{2}}$ as the price of tradables, and let P_D^t be the price of domestic output. The trading gains can be re-formulated as follows:

$$\begin{aligned}
\alpha_{XM}^t &= \left(\frac{P_X^t}{P_D^t} \Big/ \frac{P_X^{t-1}}{P_D^{t-1}} \right)^{s_X^t} \cdot \left(\frac{P_M^t}{P_D^t} \Big/ \frac{P_M^{t-1}}{P_D^{t-1}} \right)^{s_M^t} \\
&= \left(\frac{P_X^t}{P_M^t} \Big/ \frac{P_X^{t-1}}{P_M^{t-1}} \right)^{\frac{s_X^t - s_M^t}{2}} \cdot \left(\frac{P_T^t}{P_D^t} \Big/ \frac{P_T^{t-1}}{P_D^{t-1}} \right)^{s_X^t + s_M^t} .
\end{aligned}$$

The first term in the second line is the terms-of-trade effect on real income growth, and the second

[19] To be consistent with Diewert and Yu (2012), growth is the ratio of levels between period t and period $t-1$.

[20] Note that imports make a negative contribution to GDP. The greater the amount of imports, the lower the GDP. In equation (25) of Appendix A, the term in the square brackets is negative because imports add to the GDP as a negative value. When the real import price falls, α_M^t rises, leading to growth in the real income.

term is the relative-price effect. The relative-price effect is one if either the trade is balanced (i.e., $s_X^t + s_M^t = 0$) or the ratio of tradable to non-tradable prices does not change from one period to the next.

The above decomposition of real income growth can be implemented with both the annual and quarterly data. The results reported here use the year-over-year growth from the quarterly data. Using the annual data gives similar results.

Before the decomposition of real income growth is reported, it is noted that the trading gains over the sample period came primarily from the terms-of-trade effect. The relative-price effect is small, because the trade imbalance is, on average, small relative to the level of nominal GDP, and overall, the price of tradables relative to that of non-tradables does not vary significantly. Over the period of 2001 to 2007, the trading gains contributed 1.0 percentage point to the annual growth of real GDI, of which the improvement in the terms of trade contributed 1.1 percentage points per year, while the relative-price effect was -0.1 percentage points.

Growth of real GDI in Canada has been supported by TFP growth and improvements in the terms of trade, although their relative importance has changed over time. Real GDI and real GDP tracked each other relatively closely prior to 2000, but over the past decade, real gross income in Canada has exceeded and grown faster than real output, owing to an improvement in the terms of trade. From 2001 to 2013, real GDI in the business sector increased by 26.3 per cent, while real gross output grew by 18.2 per cent and the trading gains improved by 8.5 per cent.[21] Thus, about one-third of real income growth over the period of 2001 to 2013 is accounted for by the improvement in the terms of trade (again, the relative-price effect is small).

The trading gains contributed significantly to the real income in the early 2000s at about the same time that TFP growth started to slow. Contributions to average annual growth rates of real GDI are provided for different sub-periods in Figure 9. Over the 1982-2000 period, the trading gains made only a small contribution to annual real income growth, whereas close to one third of real income growth came from the contribution of TFP growth. In contrast, over the period from 2001 to 2007, productivity growth slowed considerably and the trading gains began to contribute significantly to the annual growth of real GDI. In this period, one third of real income growth is attributed to trading gains, a sufficiently sizable boost that the standard of living measured by real income growth did not worsen even though TFP growth slowed. In the recession of 2009, aggregate real income in the business sector dropped by 9.6 per cent, and real GDP dropped by 5.1 per cent. Close to half of the decline in real income was due to the deterioration in the terms of trade. TFP declined by 2.5 per cent and accounted for half of the drop in real GDP and more than a quarter of

[21] Again, growth rates are log-differences of values in 2001 and 2013, for the purpose of decomposition.

the drop in real income. Finally, in the recovery period following the recession, the terms of trade continued to be important for real income growth, and TFP started to show modest growth.

The significant trading gains over the period 1981 to 2013 reflect the fact that the export price had been rising at a faster rate than the import price. Over this period, both the export price and the import price increased (Figure 10). In particular, since 2000, the terms of trade improved, primarily because the export price experienced a sharp increase, at an average of 4.0 per cent per year, while the import price index rose at a rate of 2.8 per cent per year.[22]

Investigating the sources and causes of improvement in the terms of trade, as well as its impact on output and inputs, are beyond the scope of this paper and are left for future research.

5 Concluding remarks

In this paper, quarterly data on total factor productivity for the Canadian business sector have been constructed for the period from 1981 to 2013, and the data on annual TFP used by Diewert and Yu (2012) have been revised to reflect changes in the new national accounts and national balance-sheet accounts. The quarterly data are more timely for monitoring productivity developments. To illustrate their usefulness, a simple VAR model is estimated to study the effects of productivity shocks. The main findings are that hours worked respond negatively to technology shocks in the short run. In future research, equilibrium models could be used to improve understanding of issues of causality, such as the dynamic effects of structural drivers of improvements in the terms of trade.

The current quarterly estimates of TFP growth are slightly different from the annual estimates, mainly because they use different sources of investment data to estimate capital services. Harmonization of the quarterly and annual investment data used to construct real capital stocks would reduce the differences between quarterly and annual TFP estimates.

One important advantage of the top-down approach of Diewert and Yu (2012) over the bottom-up approach is that the contribution of the terms of trade to real income growth can be measured. The terms of trade, in the absence of strong productivity growth, have contributed to real income growth since 2001, helping to improve the standard of living in Canada. Looking forward, boosting productivity growth will be important for growth of output and real income, in particular, if the terms of trade stabilize.

[22] Export and import prices are used as they are from the new national accounts; any measured bias in these prices is carried in measuring the terms of trade.

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A Appendix A: Diewert-Yu methodology for measuring TFP

The growth accounting approach to productivity growth decomposes the growth of real income in the business sector into the contribution from productivity growth, growth in primary inputs and changes in real prices. Given data for real income growth, growth in primary inputs and changes in real prices, productivity growth can be constructed. Here we briefly summarize the methodology; additional details are available in Diewert and Yu (2012).

Notation. In most cases, the same notation as in Diewert and Yu (2012) is used. Let $y^t = (y_1^t, y_2^t, \cdots, y_M^t)$ be the **output** vector of M products, $x^t = (x_1^t, x_2^t, \cdots, x_N^t)$ be the **input** vector of N inputs, $P^t = (P_1^t, P_2^t, \cdots, P_M^t)$ be the **output price** vector, and $W^t = (W_1^t, W_2^t, \cdots, W_N^t)$ be the **input price** vector.

The production technology of a firm producing y^t is summarized by f^t. Then, for example, the production function for output m is represented as $y_m^t = f^t(y_1^t, \cdots, y_{m-1}^t, y_{m+1}^t, \cdots, y_M^t; x^t)$. The cost function of producing all M outputs is $c^t(y_1^t, y_2^t, \cdots, y_M^t; W_1^t, W_2^t, \cdots, W_N^t)$ given the above production technology $f^t(\cdot)$. The aggregate revenue function is $g^t(P^t, x^t)$ if firms produce the optimal amount of outputs y^t. If the economy is in perfect competition and the production function displays constant returns to scale, then $g^t(P^t, x^t) = P^t \cdot y^t = W^t \cdot x^t$.

The real income generated in the business sector would be $g^t(p^t, x^t) = p^t \cdot y^t = w^t \cdot x^t$, where $p^t = P^t/P_c^t$ and $w^t = W^t/P_c^t$. P_c^t is the price of consumption.

Decomposing growth in real income. The growth of real income can be decomposed into three components: growth of technology or total factor productivity (TFP), growth of real output prices, and growth of inputs.

TFP growth is the ratio of the real income that can be produced using the period t versus $t-1$ technology with input volumes held constant at some reference level x and with prices held constant at some reference unit price p: $\gamma^t(p, x) = \frac{g^t(p,x)}{g^{t-1}(p,x)}$. It is natural to use the geometric mean of a Laspeyres measure and a Paasche measure of productivity growth as follows:

$$\gamma^t = \left[\frac{g^t(p^{t-1}, x^{t-1})}{g^{t-1}(p^{t-1}, x^{t-1})} \cdot \frac{g^t(p^t, x^t)}{g^{t-1}(p^t, x^t)} \right]^{\frac{1}{2}}. \tag{12}$$

It should be noted that $g^t(p^{t-1}, x^{t-1})$ and $g^{t-1}(p^t, x^t)$ are theoretical terms; no data are available to measure them.

The **growth of real income** due to changes in prices is defined as $\alpha(p^{t-1}, p^t, x, s) = \frac{g^s(p^t,x)}{g^s(p^{t-1},x)}$. It measures the change in real income produced by the market sector that arises from changes in output prices between period $t-1$ and period t, where output is produced using technology in period s and using some reference input level x. This growth can be expressed as the geometric mean of a Laspeyres measure and a Paasche measure, given by

$$\alpha^t = \left[\frac{g^{t-1}(p^t, x^{t-1})}{g^{t-1}(p^{t-1}, x^{t-1})} \cdot \frac{g^t(p^t, x^t)}{g^t(p^{t-1}, x^t)}\right]^{\frac{1}{2}}. \tag{13}$$

Analogous to the growth of real income due to changes in output prices, the theoretical index of **growth of real income** due to changes in input quantities is defined as $\beta(x^{t-1}, x^t, p, s) = g^s(p, x^t)/g^s(p, x^{t-1})$. It measures the growth of real income, where output is produced using technology in reference period s and using the reference real output prices p. It can be expressed as

$$\beta^t = \left[\frac{g^{t-1}(p^{t-1}, x^t)}{g^{t-1}(p^{t-1}, x^{t-1})} \cdot \frac{g^t(p^t, x^t)}{g^t(p^t, x^{t-1})}\right]^{\frac{1}{2}}. \tag{14}$$

The **growth of real income** is defined as $gr^t = \frac{g^t(p^t, x^t)}{g^{t-1}(p^{t-1}, x^{t-1})}$, which can be written as

$$gr_1^t = \frac{g^t(p^t, x^t)}{g^{t-1}(p^t, x^t)} \cdot \frac{g^{t-1}(p^t, x^t)}{g^{t-1}(p^{t-1}, x^t)} \cdot \frac{g^{t-1}(p^{t-1}, x^t)}{g^{t-1}(p^{t-1}, x^{t-1})}. \tag{15}$$

The growth of real income can also be written as

$$gr_2^t = \frac{g^t(p^{t-1}, x^{t-1})}{g^{t-1}(p^{t-1}, x^{t-1})} \cdot \frac{g^t(p^t, x^{t-1})}{g^t(p^{t-1}, x^{t-1})} \cdot \frac{g^t(p^t, x^t)}{g^t(p^t, x^{t-1})}. \tag{16}$$

Taking the geometric mean of equation (15) and equation (16), we obtain the following:

$$gr^t = \gamma^t \cdot \left[\frac{g^{t-1}(p^t, x^t)}{g^{t-1}(p^{t-1}, x^t)} \cdot \frac{g^t(p^t, x^{t-1})}{g^t(p^{t-1}, x^{t-1})}\right]^{\frac{1}{2}} \cdot \beta^t, \tag{17}$$

or

$$gr^t = \gamma^t \cdot \alpha^t \cdot \left[\frac{g^{t-1}(p^t, x^t)}{g^t(p^{t-1}, x^{t-1})} \cdot \frac{g^{t-1}(p^{t-1}, x^t)}{g^t(p^t, x^{t-1})}\right]^{\frac{1}{2}}. \tag{18}$$

Diewert and Yu claim that, in equation (17), the square root term in the middle is approximately equal to α^t; and in equation (18), the square root term is approximately equal to β^t. The **growth of real income** is then decomposed as follows:

$$gr^t \approx \gamma^t \alpha^t \beta^t, \tag{19}$$

where γ^t is TFP growth, α^t is growth in real output prices, and β^t is growth in primary inputs.

Assumptions for exact index number. Diewert and Yu (2012) impose assumptions on functional forms and parameters for revenue so that the decomposition (19) holds exactly.

Assumption 1. *Translog revenue function*

$$\begin{aligned}\ln g^t(p^t, x^t) &= a_0^t + \sum_{m=1}^{M} a_m^t \ln p_m^t + \frac{1}{2}\sum_{m=1}^{M}\sum_{j=1}^{M} a_{mj} \ln p_m^t \ln p_j^t + \sum_{n=1}^{N} b_n^t \ln x_n^t \\ &\quad + \frac{1}{2}\sum_{n=1}^{N}\sum_{j=1}^{N} b_{nj} \ln x_n^t \ln x_j^t + \sum_{m=1}^{M}\sum_{n=1}^{N} c_{mn} \ln p_m^t \ln x_n^t. \end{aligned} \tag{20}$$

The translog form of the revenue function enables the decomposition equation to hold exactly. Translog form is a second-order approximation of a general twice-differentiable revenue function.

Assumption 2. *The following restrictions on parameters hold:*

(i) $\sum_{m=1}^{M} a_m^t = 1$ *for any* t*;*

(ii) $\sum_{n=1}^{N} b_n^t = 1$ *for any* t*;*

(iii) $a_{mj} = a_{jm}$ *for all* m, j*;*

(iv) $b_{nj} = b_{jn}$ *for all* n, j*;*

(v) $\sum_{j=1}^{M} a_{mj} = 0$ *for* $m = 1, \cdots, M$*;*

(vi) $\sum_{j=1}^{N} b_{nj} = 0$ *for* $n = 1, \cdots, N$*;*

(vii) $\sum_{n=1}^{N} c_{mn} = 0$ *for* $m = 1, \cdots, M$*;*

(viii) $\sum_{m=1}^{M} c_{mn} = 0$ *for* $n = 1, \cdots, N$*.*

Diewert (1974) shows that the revenue function is homogeneous of degree one in p^t, if assumptions (i), (iii), (v) and (viii) hold. Similarly, the revenue function is homogeneous of degree one in x^t, if assumptions(ii), (iv), (vi) and (vii) hold. These assumptions are necessary for the revenue function to be well-defined and to have the properties of a revenue function seen in the production theory.

Decomposition with translog function. Under Assumptions 1 and 2, the three components of growth in real revenue are given by

$$\ln \alpha^t = \sum_{m=1}^{M} \frac{1}{2}\left[\frac{p_m^{t-1} y_m^{t-1}}{p^{t-1}\cdot y^{t-1}} + \frac{p_m^t y_m^t}{p^t \cdot y^t}\right] \ln\left(\frac{p_m^t}{p_m^{t-1}}\right); \tag{21}$$

$$\ln \beta^t = \sum_{n=1}^{N} \frac{1}{2}\left[\frac{w_n^{t-1} x_n^{t-1}}{w^{t-1}\cdot x^{t-1}} + \frac{w_n^t x_n^t}{w^t \cdot x^t}\right] \ln\left(\frac{x_n^t}{x_n^{t-1}}\right); \tag{22}$$

$$\gamma^t = \frac{gr^t}{\alpha^t \beta^t}, \tag{23}$$

where α^t is the Törnqvist index of output prices, and β^t is the Törnqvist index of input quantity.

Contribution of output prices. Diewert and Yu (2012) show that the above decomposition can take into account changes in relative prices of GDP components. Suppose the mth output price p_m changes in period $t-1$ to t. The overall measure of the effects on real income from changes in the real output price p_m is given by the geometric mean of a Laspeyres measure and a Paasche measure, as follows:

$$\alpha_m^t = \left[\frac{g^{t-1}(p_1^{t-1},\cdots,p_{m-1}^{t-1},p_m^t,p_{m+1}^{t-1},\cdots,p_M^{t-1},x^{t-1})}{g^{t-1}(p^{t-1},x^{t-1})} \cdot \frac{g^t(p^t,x^t)}{g^t(p_1^t,\cdots,p_{m-1}^t,p_m^{t-1},p_{m+1}^t,\cdots,p_M^t,x^t)}\right]^{\frac{1}{2}}. \tag{24}$$

Under Assumptions 1 and 2,

$$\ln \alpha_m^t = \frac{1}{2}\left[\frac{p_m^{t-1} y_m^{t-1}}{p^{t-1}\cdot y^{t-1}} + \frac{p_m^t y_m^t}{p^t \cdot y^t}\right] \ln\left(\frac{p_m^t}{p_m^{t-1}}\right). \tag{25}$$

This implies that the aggregate real output price contribution in period t is exactly decomposed into the product of separate price contributions, as follows:

$$\alpha^t = \alpha_1^t \alpha_2^t \cdots \alpha_M^t. \tag{26}$$

In the case of the **terms of trade** (export price relative to import price), let the real export price be the export price deflated by the domestic output price, and let the real import price be the import price deflated by the domestic output price. Note that, at this aggregate level, there are three output components: domestic output, exports and imports. The contribution of changes in real export and import prices are α_{EX}^t and α_{IM}^t, respectively, both defined as in equation (25). The trading gains are then

$$\alpha_{XM}^t = \alpha_{EX}^t \cdot \alpha_{IM}^t.$$

Contribution of primary inputs. In the same fashion, the contribution of changes in primary inputs β^t can be further decomposed. Suppose that the nth input x_n changes. Its contribution to real income is β_n^t, measured as the geometric mean of a Laspeyres measure and a Paache measure, given by

$$\beta_n^t = \left[\frac{g^{t-1}(p^{t-1}, x_1^{t-1}, \cdots, x_{n-1}^{t-1}, x_n^t, x_{n+1}^{t-1}, \cdots, x_N^{t-1})}{g^{t-1}(p^{t-1}, x^{t-1})} \cdot \frac{g^t(p^t, x^t)}{g^t(p^t, x_1^t, \cdots, x_{n-1}^t, x_n^{t-1}, x_{n+1}^t, \cdots, x_N^t)} \right]^{\frac{1}{2}}. \tag{27}$$

With the translog form of real GDP function, β_n^t can be written as

$$\ln \beta_n^t = \frac{1}{2} \left[\frac{w_n^{t-1} x_n^{t-1}}{w^{t-1} \cdot x^{t-1}} + \frac{w_n^t x_n^t}{w^t \cdot x^t} \right] \ln \left(\frac{x_n^t}{x_n^{t-1}} \right). \tag{28}$$

The contribution from changes in quantities of all inputs has the following exact decomposition:

$$\beta^t = \beta_1^t \beta_2^t \cdots \beta_N^t. \tag{29}$$

B Appendix B: Data for quarterly TFP estimates

This section describes in detail how the outputs and inputs in the Canadian business sector are measured. The focus is on quarterly data series, since quarterly growth accounting is new. CANSIM tables used for quarterly estimates are listed in Table F at the end of this appendix.

In this paper, the business sector includes sectors except the public administration sector and non-profit institutions serving households (NPISH), as well as the non-business parts of the education sector and the health care sector. The quarterly data start from 1981Q1. While data in NBSA12 start in 1990Q1, the required series are extended back to 1981Q1 using data in the old national balance-sheet accounts.

CSNA12 is a major revision to the national accounts. In addition to revisions to many tables, several changes are relevant for measuring productivity. First, CSNA12 introduced a new economic sector, NPISH. This sector is treated as part of the general government. Second, CSNA12 introduced intellectual property products (IPP), a new asset type consisting of software, mineral exploration and evaluation, and research and development. Prior CSNA12, software was a component of machinery and equipment; mineral exploration and evaluation was a component of non-residential structures; and research and development was treated as an intermediate input for businesses and as a final consumption expenditure for government. Third, exports and imports are re-categorized using the North American Product Classification System (NAPCS) Canada 2007. In addition, the measurement of prices of exports and imports is improved by using micro price surveys.

In constructing each variable, two of three values are needed: current-price value, real value (or quantity) and the implicit price index. In most cases, current-price values and real quantities are available. Often, aggregating different series is necessary. A chained Fisher index is used when aggregating multiple components of the same type. For example, a Fisher index is used to construct the aggregate consumption expenditure by removing housing services from the total household expenditure. A chained Törnqvist index is used when aggregating all output (input) components into the aggregate output (input). Nevertheless, the results are not sensitive to the choice of price indexes.

To construct the seasonally unadjusted series of quantities and prices, two issues need to be addressed. First, some variables have no quarterly data and it is impossible to estimate them. Since no quarterly series exist for current-price GDP at basic prices, quarterly data are obtained by interpolating annual series at the quarterly frequency when needed. Second, seasonally unadjusted chained-dollar series are not available for many variables. In CSNA12 tables, seasonally unadjusted series are often available only in constant dollars, not in chained dollars. In such a case, seasonality is detected by checking whether there is a significant difference between seasonally unadjusted and adjusted constant-dollar values. If the underlying variable displays no seasonal variation, seasonally adjusted chained-dollar values are used. If the underlying variable displays an obvious seasonal variation, seasonally unadjusted chained-dollar values are obtained by applying a seasonality ratio to the implicit price indexes of the seasonally adjusted chained-dollar values. This seasonality ratio is the ratio of two implicit price indexes, calculated from the seasonally unadjusted and adjusted series for both current-price values and constant-dollar values.[23]

B.1 Quarterly output

Output is the final demand expenditure, which is a measure of value added because it excludes intermediate inputs. It consists of 5 categories of domestic output, 12 categories of exports and 12 categories of imports. Domestic output includes household final consumption expenditure, sales of businesses to non-businesses, business investment, government spending and changes in business inventories. Most output components in the business sector are from the expenditure-based GDP tables in CSNA12.

Household final consumption expenditure. Final consumption expenditure is the total final household consumption spending on goods and services, excluding paid rent and the imputed rent for owner-occupied dwellings.[24] Obtaining the imputed rent for owner-occupied dwellings in the business sector requires GDP at basic prices in the business sector. No current-price quarterly GDP at basic prices is available in CSNA12. First, the implicit price of the imputed rent is constructed from the seasonality of the imputed rent from the table Detailed household final consumption expenditure, using seasonally unadjusted and adjusted constant-dollar prices.[25] It is then applied to the linearly interpolated annual prices of the imputed rent, resulting in the seasonally unadjusted chained-dollar price of the imputed rent. For real values of the imputed rent in the business sector, series from the tables of GDP at basic prices according to the North American Industry Classification System (NAICS) are used. The quantity and price of the paid rent are from the table

[23]The differences between seasonally adjusted chained-dollar values and seasonally adjusted constant-dollar values arise from changes in the composition of the underlying outputs and inputs. For example, the constant-dollar value and chained-dollar value of investment in machinery and equipment are very different in 1980s and 1990s, owing to the significant decline in prices of computer equipment and significant increase in the use of computers. For some other variables, the chained-dollar price and the constant-dollar price are virtually the same.

[24]In the growth accounting data at Statistics Canada, only the imputed rent from owner-occupied dwellings is excluded from the output measure.

[25]Here, the term "seasonally adjusted constant-dollar price" refers to the implicit price index (deflator) calculated from dividing seasonally adjusted current values by seasonally adjusted real values at constant dollars. Other similar terms are defined in the same way.

of detailed household final consumption expenditure. The seasonally adjusted prices in constant dollars and in chained dollars are identical for paid rent; seasonally unadjusted prices (in constant dollars) of paid rent are used without further adjustment.

For the aggregate household consumption expenditure, the seasonally adjusted chained-dollar implicit price and the seasonally adjusted constant-dollar implicit price diverge over time, thus requiring adjustment. The price seasonality is obtained in the table of GDP at 2007 constant prices (expenditure-based). The seasonally adjusted chained-dollar price is obtained from the expenditure-based GDP table. The seasonally unadjusted chained-dollar price is then calculated as the seasonally adjusted chained-dollar price multiplied by the price seasonality.

The final consumption expenditure net of residential rents is obtained using the Fisher price and quantity indexes. Consumption taxes are then removed from the final net consumption expenditure. The effective consumption tax rate is calculated and used to adjust the seasonally unadjusted chained-dollar price, as well as the real value of net final consumption expenditure.

Net sales of businesses to the non-business sector. Calculating the net sales of businesses to the non-business sector (NSBN) is described in detail in Diewert and Yu (2012). According to their theory, the current value of NSBN equals the current value of the government's general final consumption expenditure (at market prices), subtracting the current value of GDP at basic prices in the non-business sector. The government's general consumption expenditure equals the sum of government consumption expenditure and the NPISH's final consumption expenditure, which is obtained from the expenditure-based GDP table.

For GDP at basic prices in the non-business sector, CSNA12 has no quarterly current-price GDP at basic prices by sector. The real value of GDP at basic prices in the non-business sector is found in the NAICS table for GDP at basic prices. The implicit price of GDP at basic prices for the non-business sector is obtained by interpolating its annual data.

The net sales of businesses to non-businesses are then calculated as in the Fisher quantity index from general government spending and non-business GDP at basic prices.

Government investment. Government investment consists of investment by government and by NPISH. Current and real values of these investments are from the table for GDP (expenditure-based) and the table for GDP at 2007 constant-dollar prices. The seasonality of investment prices is obtained from the GDP table at 2007 constant-dollar prices.

Business investment. Business investment includes investment in residential structures, non-residential structures, machinery and equipment, and intellectual property products. The current values and seasonally adjusted chained-dollar prices of these variables are from the table of expenditure-based GDP. Since these variables display various degrees of seasonality, the table for GDP at 2007 constant-dollar prices is used to obtain the price seasonality needed to obtain seasonally unadjusted chained-dollar prices.

Business inventories. The real values of changes in business inventories are obtained from the quarter-over-quarter changes in real stocks of inventories, instead of values from the expenditure-based GDP table. Real stocks of inventories are, in turn, measured using the current values of business inventories in NBSA12 and implicit prices. However, since the appropriate implicit price is unavailable in CSNA12, it is borrowed from Diewert and Yu, who constructed it based on prices provided by Statistics Canada. This price series is interpolated and then extended to the latest quarter using the Industrial Product Price Index (IPPI).

Exports and imports. There are 12 types of goods and one type of service in both exports and imports, at the two-digit NAPCS level. Seasonally unadjusted real values of exports and imports are not available in CSNA12, except for the aggregate goods and services. It turns out that most exports and imports of goods at the two-digit NAPCS level do not exhibit significant seasonality, and there is evidence that the seasonally adjusted prices are fairly close to unadjusted prices.[26] For exports and imports of goods exhibiting seasonality, the price seasonality for the aggregate good is applied.

For exports of services, seasonally adjusted prices are fairly different from unadjusted prices, suggesting price seasonality.

At the aggregate level, the prices of exports of goods do not exhibit strong seasonality. The seasonally adjusted and unadjusted prices of aggregate exports of goods, based on 2007 constant-dollar quantities, are fairly close to each other, with a deviation of 0.28 per cent on average. The largest deviation is 0.76 per cent, in 2007Q4. At the two-digit NAPCS level, data for both seasonally adjusted and unadjusted prices are available for the period from 1997Q1 to 2013Q1. These prices are compared, and it is found that the average deviation of the two price series among all 11 types of exports of goods is 0.3 per cent, suggesting that the price seasonality of exports is small. The export of energy products observes the largest deviation, at 1.7 per cent on average, followed by the export of farm and food products, at 1.2 per cent on average. The mean deviation for the export of metal and non-metallic mineral products is 1.0 per cent. The deviations for the rest of exports are all very small, significantly below 1.0 per cent. This evidence suggests that the prices of exported goods

[26] Seasonally unadjusted prices of exports and imports (Laspeyres and Paache indexes) are available in tables of international merchandise trade.

do not exhibit strong seasonality. Therefore, the following exported goods are assumed to have no price seasonality, including farm and food products, energy products, metal and non-metallic mineral products, motor vehicles and parts, aircraft and other transportation equipment and parts, and consumer goods. For other exports with larger differences between seasonally adjusted and unadjusted prices, the seasonality of the aggregate export of goods is applied to the chained-dollar seasonally adjusted prices.

The import of services does not display strong seasonality, since the seasonally adjusted and unadjusted prices (at 2007 constant dollars) are close to each other, with a deviation of 0.25 per cent on average. In order to be consistent with the quantities and prices of exports of services, price seasonality is applied to the seasonally adjusted prices, obtaining the seasonally unadjusted chained-dollar import prices of services.

The price seasonality of imports of goods is largely similar to that of exports of goods. The deviation between seasonally unadjusted and adjusted prices of the aggregate imports of goods is on average 0.33 per cent. For prices of imports at the two-digit NAPCS level, the average deviation among 11 types of imported goods is 0.44 per cent. In contrast to export prices, the import prices of energy products observe large seasonality, with an average deviation of around 3.3 per cent. The largest deviation is in 2009Q1 at 6.5 per cent. The mean deviation of prices is 1.6 per cent for imports of farm and food products, and 1.2 per cent for metal ores and non-metallic minerals. All other imported goods have very small differences between seasonally adjusted and unadjusted prices. For imports with small differences between adjusted and unadjusted prices, the seasonally adjusted chained-dollar implicit prices are used. These imports include metal and non-metallic mineral products, basic and industrial chemical and plastic and rubber products, and forestry products and building and packaging materials. For the remaining imports, price seasonality of import prices of aggregate imports in goods is applied to the chained-dollar seasonally adjusted import prices.

Finally, import custom duties are removed from the import prices.

B.2 Quarterly labour Inputs

Labour inputs are the aggregate of quality-adjusted hours worked by 12 types of workers. In Diewert and Yu (2012), annual data series are available for labour compensation and hours worked for each of the 36 types of workers (including two gender groups, three age groups, three education groups, and paid versus self-employed workers). These 36 types of workers are aggregated to 12 types (two gender groups, three education groups, and paid versus self-employed workers).

There are no readily available data to measure quarterly labour inputs. The Labour Force

Survey (LFS) public user micro files are used to construct hours worked and compensation for the 36 types of workers.[27] These quarterly series are then benchmarked to the Diewert-Yu annual series using the Denton method. After being benchmarked, the labour inputs display quarterly variation according to the LFS data, and adding quarterly values in each year will replicate the annual values.

Hours worked and earnings in LFS. The LFS is a primary survey of the labour market. It covers employment status, hours worked and compensation. To measure the quarterly labour inputs in the business sector, workers who report that they work in the business sector (all sectors except education, health care and public administration) are selected. Individuals are also included if they are private sector employees working in education services (NAICS 61) or health care and social assistance (NAICS 62). The fraction of private sector workers in education services is very small, while it is large in health care (close to 40 per cent in recent years). All self-employed workers are in the business sector, since they are labelled as private sector workers in the LFS.

The LFS has information on both usual and actual weekly hours for both the main job and all jobs. We use the actual weekly hours worked in all jobs. The quarterly total hours worked for each of the 36 types of workers are obtained by aggregating individual weekly hours, in which monthly hours worked are adjusted for holidays. The total quarterly compensation of each type of worker is obtained as the average hourly earnings (for main jobs) multiplied by total quarterly hours for each type of the worker.

Benchmarking. The Denton method is used to benchmark the 72 series of hours worked and compensation to the annual data as in CANSIM 383-0024.[28] Compensation is not available in the LFS before 1997Q1 for all types of workers. For the period of 1981Q1 to 1996Q4, the annual series of compensation are interpolated where the quarterly variation is the same as the quarterly wages and salaries and supplementary labour income in the business sector from SEPH.[29] Therefore, before 1997Q1, the quarterly variation in compensation is common for all types of workers.

The quarterly data for quarters after 2010Q4 cannot be benchmarked, because the last period of the annual data is 2010. All benchmarked series are extended to the latest quarter using the

[27] Another microdata source, the Survey of Employment, Payrolls and Hours (SEPH), also measures quarterly employment, compensation and hours. But SEPH does not have labour inputs by education and age. In Statistics Canada's Labour Productivity Measures, the employment and hours of employees in the business sector are obtained as the geometric mean of the series from LFS and SEPH, then benchmarked to annual data.

[28] In the Stata software, this method is developed by Baum and Hristakeva (2011).

[29] In SEPH, CANSIM table 382-0021 is used for aggregate quarterly compensation. In this CANSIM table, it is not possible to separate the business sector part of labour income from education services and health care services, so these two sectors are excluded from the business sector (which is inconsistent with the definition of business sector used for quarterly growth accounting).

LFS quarterly series.

B.3 Quarterly capital services.

Capital services are the user cost of capital multiplied by real capital stocks. The challenging part is measuring real capital stocks. The standard practice is to construct capital stocks using investment and depreciation rates by the perpetual inventory method, where the book value of capital stocks in the initial period is also needed. Let K_{it+1} be the capital stock of asset i at the beginning of year $t+1$, I_{it} be the real values of investment, and δ_{it} be capital depreciation rates. The perpetual inventory method to obtain the real capital stocks is given by

$$K_{it+1} = (1 - \delta_{it})K_{it} + I_{it}.$$

Asset categories should be at a reasonably disaggregated level to overcome the aggregation error (for example, due to the heterogeneity of assets). Presumably one can construct flows and stocks of physical assets using the national balance-sheet accounts NBSA12 (for stocks) and the national accounts CSNA12 (for quantities and prices of investment). In fact, this is possible only at the aggregate level, where there are six types of assets: residential structures, non-residential structures, machinery and equipment, inventories, land, and IPP (IPP is new in NBSA12). Capital stocks at the detailed level are unavailable in NBSA12.

We construct the real capital stocks at the disaggregated level, using investment data from CSNA12 and capital stocks (for initial-period stocks and depreciation rates) from the annual data on capital flows and stocks. But the new (quarterly) and annual data do not match exactly. On the investment side, in CSNA12, there are nine types of machinery and equipment: industrial machinery and equipment; computers and computer peripheral equipment; communications and audio and video equipment; other electrical and electronic machinery and equipment; other machinery and equipment; furniture, fixtures and prefabricated structures; passenger cars; trucks, buses and other motor vehicles; and aircraft and other transportation equipment. In the annual data (of capital flows and stocks), there are also nine types of machinery and equipment, but they do not align with those in CSNA12. For example, there is no stand-alone investment in trucks and investment in agricultural machinery in CSNA12. These differences make it difficult to combine the new investment data with the annual capital stocks data (for the perpetual inventory method).

Table C lists the types of investment in machinery and equipment. Judging by names and also using the concordance table provided by Statistics Canada, it appears that the types of quarterly investment and annual investment (capital flows) do not match.

Table C: Types of machinery and equipment in quarterly and annual series of investment

Quarterly (CANSIM 380-0068)	Annual (CANSIM 031-0003)
Industrial machinery and equipment	Industrial machinery
Computers and computer peripheral equipment	Computers
Communications and audio and video equipment	Telecommunication equipment
Other machinery and equipment	Other machinery and equipment
Furniture, fixtures and prefabricated structures	Furniture
Passenger cars	Automobiles
Trucks, buses and other motor vehicles	Trucks
Aircraft and other transportation equipment	Other transportation
Other electrical and electronic machinery and equipment	Agricultural machinery

Moreover, even for what appear to be the same types of investment, differences between new quarterly data and the annual data are large, in both current values and real values, and these differences appear to be random over time. To obtain the annual investments from new quarterly data (CANSIM 380-0068), we use the annual averages of quarterly investments that are seasonally adjusted at annual rates. Table D shows the differences for the same or very similar capital types between CANSIM 380-0068 and CANSIM 031-0003. Investment in engineering structures is different between quarterly and annual data in almost all years. For some other asset types, the large differences in investment in current values occur mostly in more recent years, suggesting that CANSIM 031-0003 may not have the most recent revised values. In particular, investment in IPP is new in both CANSIM 031-0003 and in CSNA12, the differences between quarterly and annual values have been large since 2007. Investment in mineral exploration and evaluation in 2011 and 2013 is much lower in CANSIM 380-0068 than in CANSIM 031-0003. For investment in software, the differences start in 2000, with quarterly values becoming significantly larger than annual values from 2009 onward. Investment in machinery and equipment has been increasingly lower in CANSIM 380-0068 than in CANSIM 031-0003 since 2000. In summary, except for investment in IPP and its components, it appears that quarterly-annual differences in investment do not have a clear pattern, suggesting that investment data are not consistent across different CANSIM tables.

Capital stocks. To overcome the above differences in investment series between quarterly and annual data, assets are aggregated into nine types in the quarterly estimates: four types of machinery and equipment, two types of non-residential structures, and three types of IPP. Non-residential structures consist of non-residential buildings and engineering structures. IPP comprises mineral exploration and evaluation, research and development, and software. Machinery and equipment consists of computers, communications equipment, industrial machinery and equipment, and trans-

Table D: Deviation of investment between CANSIM 031-0003 and 380-0068, in percentages

Capital type	Current value		Chained-dollar value	
	Mean	Maximum	Mean	Maximum
Non-residential structures	2.2	4.7	3.3	6.0
Buildings	5.2	20.9	4.9	20.3
Engineering	3.6	12.1	3.8	11.0
Intellectual property products	1.3	9.9	13.6	28.3
Mineral exploration and evaluation	0.9	21.6	3.5	18.7
Research and development	1.4	5.0	18.8	44.2
Software	2.9	20.4	18.1	35.3
Machinery and equipment	1.8	7.9	1.9	7.8
Computers	5.3	14.9	2.5	11.5
Telecommunication equipment	4.2	23.8	5.3	23.9
Furniture	15.6	55.7	20.3	79.1

portation and furniture. This last asset is the aggregate of transportation equipment and furniture. Grouping machinery and equipment in this way brings the quarterly and annual values of investments the closest to each other. Industrial machinery and equipment from CANSIM 380-0068 consists of industrial machinery and equipment and other machinery and equipment, and from CANSIM 031-0003 it consists of industrial machinery, agricultural machinery and other machinery. Computers and communications equipment are the same assets by definition in CANSIM 380-0068 and CANSIM 031-0003, respectively. Investments in transportation and furniture are obtained by subtracting investments in the first three types of machinery and equipment from total investments, calculated using the Fisher quantity index.

To calculate capital stocks with the perpetual inventory method, capital depreciation rates are first calculated from annual data of flows and stocks of capital (CANSIM 031-0003).[30] It is assumed that depreciation rates are constant in all four quarters of a year. Capital depreciation rates are then smoothed using the Lowess non-parametric method. The 1980 capital stocks are used to start the recursion, and seasonally unadjusted investments are used from quarterly investment data in CSNA12 (CANSIM 380-0068).

Seasonally unadjusted real values of capital are available only for the aggregate capital of three types of assets: non-residential structures, machinery and equipment, and IPP. It is assumed that assets belonging to the same aggregate asset exhibit the same seasonality as the aggregate asset.

[30] The annual capital depreciation rate for asset i is given by

$$\delta_{it} = \frac{QK_{it+1} - QK_{it} + I_{it}}{QK_{it}}$$

where QK_{it}, is the annual book value of capital stock from CANSIM 031-0003. This follows Diewert and Yu (2012).

The price seasonality for the three aggregate capital types is obtained first, using current values from CANSIM 380-0068 and real values from CANSIM 380-0084. Seasonally adjusted prices for the nine types of assets are obtained from CANSIM 380-0068. Applying the price seasonality to the seasonally adjusted prices gives the seasonally unadjusted prices of investments. With the seasonally unadjusted current values and prices, we can obtain the seasonally unadjusted real values of investments.

Table E reports the quarterly depreciation rates of capital. Depreciation rates for computers are again much larger than all other assets.

Table E: Quarterly capital depreciation rates

Asset	Average depreciation rate (%)
Non-residential structures	
Buildings	1.31
Engineering	2.36
Machinery and equipment	
Computers	17.17
Communications equipment	6.79
Industrial machinery	5.17
Transportation and furniture	7.40
Intellectual property products	
Software	10.33
Mineral exploration and evaluation	3.93
Research and development	6.66

Land. Quarterly quantities of land are obtained by interpolating the annual series for both agricultural land and business (non-agricultural) land, as well as residential land (for calculating the property tax rate).

Property tax rate. It is assumed that there are property taxes for five types of property: residential land, residential structures, agricultural land, non-agricultural business land and non-residential structures. Of these properties, residential land and residential structures are not inputs of production. The effective property tax rate is the total property tax revenue divided by the total value of the five types of properties.

The general business tax applied to the use of capital stocks is also needed, which is obtained as in Diewert and Yu (2012).

Real rate of return. The real rate of return is obtained by setting total revenue equal to total input costs, implied by the assumption of a constant return to scale in the production function. On the output side, there are 10 output components; on the input side, there is the cost of labour for 12 types of workers and the cost of capital for 14 types of assets.

The user cost of capital for each asset can be calculated using the obtained series of tax rates, the real rate of return and prices of capital.

Table F: List of CANSIM tables used for quarterly TFP data

Variable	Table	Comments
Aggregate consumption expenditure	380-0064, 380-0084	
Consumption tax	380-0080, 380-0081	
Imputed rent	379-0027, 379-0031	Price is from annual estimates
Paid rent	380-0085	
Government spending	380-0064, 380-0084	
Non-business GDP at basic price	379-0027,379-0031	Price is from annual estimates
Government investment	380-0064, 380-0084	
Business investments	380-0064, 380-0084	
Business inventories	378-0121, 329-0056	Price is from annual estimates
Exports/imports	380-0064, 380-0070, 380-0080, 380-0084	
Labour inputs	383-0024,382-0021, LFS	
Capital services	380-0068, 031-0003	
property tax	378-0121, 380-0080	

C Appendix C. Annual TFP: Revision to Diewert-Yu estimates

In the annual TFP estimates, business sector output is disaggregated into $M = 21$ components: household final consumption expenditure, sales of businesses to non-businesses, government investment, business investments, changes in business inventories, exports and imports. Business investment consists of investment in each of residential structures, non-residential structures, machinery and equipment, and intellectual property products (IPP). Exports and imports are both disaggregated into 13 products and services: farm, fishing and intermediate food products; energy products; metal ores and non-metallic minerals; metal and non-metallic mineral products; basic and industrial chemical, plastic and rubber products; forestry products and building and packaging materials; industrial machinery, equipment and parts; electronic and electrical equipment and parts; motor vehicles and parts; aircraft and other transportation equipment and parts; consumer goods; and services.[31]

[31] Special transactions and other balance of payments adjustments are excluded from measuring the export and import aggregates.

Labour input is aggregated from hours worked and compensation of 36 types of workers (including three levels of education achievement, three age groups, two gender groups, and employees versus self-employed). The same data are used for the labour input series as in Diewert and Yu (2012) up to 2010. These data are extended to 2013 using series of labour inputs constructed from the Labour Force Survey (LFS) public use microdata files.

The capital service input is aggregated from stocks of 19 types of assets, weighted by the cost of capital. Capital stocks are constructed using the perpetual inventory method. The constructed capital consists of nine types of machinery and equipment, three types of IPP, four types of non-residential structures, and two types of land. To minimize the potential for aggregation error caused by, for example, the heterogeneity of assets, asset categories should be disaggregated to the maximum possible extent.[32] Table G shows the average depreciation rates of 16 types of assets. Some of these rates are similar to those estimated by Statistics Canada. Of special interest, the depreciation rate of research and development (R&D) is on average 24.1 per cent. The measured average depreciation rate implies that 40 per cent of investment in R&D is depreciated after two years, and 90 per cent is depreciated after eight years of investment. The depreciation rates of R&D stocks are in line with estimates in other papers. For instance, of the estimated depreciation rates of R&D for selected sectors in Huang and Diewert (2011), the largest estimate is 29 per cent for manufacturing.

While annual estimates of productivity are already available from Diewert and Yu (2012), Statistics Canada has since made major revisions to national economic data for Canada. This section briefly reviews these revisions and provides revised estimates of TFP using the new data.

Revisions to national accounts: In October 2012, Statistics Canada published a major revision to the Canadian System of National Accounts and to the National Balance Sheet Accounts, referred to as CSNA12 and NBSA12, respectively. Major changes include the following:

[32] In theory, flows and stocks of physical assets can be constructed using the national balance sheets (for stocks) and the national accounts (for quantities and prices of investment). However, at the time the analysis in this paper was conducted, because of data limitations in CSNA12, such constructions were possible only at an aggregate level with six types of assets (residential structures, non-residential structures, machinery and equipment, inventories, land, and IPP). Due to the limited availability of capital stock data at the disaggregated level in NBSA12, in this study, the same data as in Diewert and Yu are used for stocks of fixed non-residential capital (CANSIM 031-0003). While CSNA12 has investment data at the disaggregated level, the categories of investments in machinery and equipment in CSNA12 do not align with those in the the table of capital flows and stocks, leaving series from the latter table the only choice for constructing real capital stocks and depreciation rates. The exception is that investment in IPP and its components are from CSNA12. An additional challenge with data series of flows and stocks of fixed non-residential capital is that investment values differ from values of the same investment in CSNA12. For example, although investment in IPP was added to CANSIM 031-0003, the reported data differ from the "same" investment in CSNA12.

Table G: Annual capital depreciation rates

Machinery and equipment		Non-residential structures	
Furniture	0.23	Industrial buildings	0.06
Agricultural machinery	0.33	Commercial buildings	0.05
Industrial machinery	0.20	Institutional buildings	0.03
Automobiles	0.40	Engineering	0.09
Trucks	0.34	Intellectual property products	
Other transportation	0.19	Software	0.35
Other machinery and equipment	0.14	Mineral exploration and evaluation	0.15
Computers	0.53	Research and development	0.24
Communication equipment	0.24		

- **Research and development was reclassified as an investment.** Prior to CSNA12, business expenditures on R&D were treated as an intermediate expenditure, and expenditures on R&D by government were treated as the final consumption expenditure of government. In CSNA12, R&D expenditures are treated as an addition to the fixed capital formation. This change led to an upward revision to real GDP.

- **A new type of asset, intellectual property products (IPP), was introduced.** IPP consists of research and development, mineral exploration and evaluation, and software. Prior to CSNA12, mineral exploration and evaluation was a component of non-residential structures, and software was treated as a type of machinery and equipment.

- **A new sector, non-profit institutions serving households (NPISH), was created.**

- **Revisions were made to exports and imports of goods and services.** New price indexes for exports and imports (based on price surveys) are used in CSNA12, reflecting more accurately changes in export and import prices. In addition, exports and imports are reclassified with more detailed categories, in accordance with the North American Product Classification System (NAPCS) Canada 2007.

- **A new method is applied to evaluate the stocks of non-financial assets.** One important change in NBSA12 is that machinery and equipment in the corporate sector is now valued at the replacement cost, instead of the historical cost (price at the time of purchase). This change led to a downward revision to the value of fixed assets in the corporate sector.

Annual TFP estimates on revised data: The constructed real GDI index and the cumulative contribution from real GDP, real labour, capital services inputs and TFP are shown in Figure 11.[33] A few observations in this figure are noteworthy. First, although real GDP and real GDI, tracked each other quite closely for the first 20 years of the sample, a wedge between the two opened up in the 2000s. As will be discussed later, this wedge reflects improvements in the terms of trade that provided additional positive contributions to real GDI. Second, TFP has stagnated since the early 2000s. While there is some evidence of cyclicality in the constructed TFP series, this cannot explain the weakness in TFP through the mid-2000s. Third, this stagnation in TFP implies that between 2002 and 2013, the growth of inputs–capital services and labour–were the driving forces behind real GDP growth. Fourth, capital services is the least cyclical of the contributors to real GDP growth. For instance, TFP and labour input accounted for more than 90 per cent of the 4.8 per cent decline in real GDP in the business sector observed in the 2009 recession. Likewise, the recovery from the recession was mainly driven by TFP and labour input, with TFP expanding in 2011 for the first time since 2005.

The re-classification in CSNA12 of R&D as a final investment expenditure increased real GDP and also capital services. The share of investment in R&D in GDP in current values is about 10.9 per cent per year, over the period from 1981 to 2013, although it has been generally increasing, particular since 2000.[34] To assess the implications of R&D for the estimates of TFP, alternative series were constructed excluding R&D from the growth accounting. Both TFP series (normalized to be one in 1981) are shown in Figure 12. On average, including R&D as a final investment expenditure results in estimates of annual TFP growth that are 0.04 percentage points lower.

The estimates of TFP growth based on CSNA12 and NBSA12 are slightly lower than those by Diewert and Yu, but the evolution over time of the two index levels is quite similar (Figure 13). Over the period from 1981 to 2011, TFP grows 0.50 per cent per year on average in the revised estimates, weaker than the 0.66 per cent growth rate of the Diewert and Yu series. Since labour inputs in the two estimates coincide, weaker average TFP growth in the new estimates can be traced to faster growth in capital services in the pre-2005 period. On average, the annual growth of the revised capital service was 0.25 percentage points faster than the Diewert-Yu estimates. The inclusion of investment in R&D did not lead to faster growth in real GDP in the revised estimates.

Given that the revised data have only relatively modest implications for the estimates of TFP following the Diewert-Yu methodology, the differences between the new constructions in this paper and the Statistics Canada productivity series are similar to those between the Diewert-Yu estimates

[33] All indexes are normalized to one in 1981. This figure shows the contribution of each component to the cumulative growth of real GDI. It is different from Figure 1, which plots the cumulative growth of real GDP and its components.

[34] However, in 2010-2013, investment in R&D stayed at around 0.9 per cent of GDP.

and the Statistics Canada productivity series. Statistics Canada's annual multifactor productivity program constructs measures of the the annual growth rate of productivity in the business sector.[35] As with the Diewert-Yu estimates, the top-down TFP estimates constructed in this paper grow at a much faster rate than the bottom-up TFP measure in the Statistics Canada data (Figure 14).[36] Annual TFP growth in the Statistics Canada estimation is only 0.11 per cent over the period from 1982 to 2012. While the average growth rates are considerably different, the cyclical behaviour of the two estimates is similar and, in the most recent decade, the two TFP estimates are fairly close to each other.

It should be noted that labour productivity is quite close between the top-down approach and the Statistics Canada approach, because the output and the labour inputs in both approaches are close. Differences of measured TFP between the top-down approach and the Statistics Canada approach affect only the relative contribution of TFP and capital deepening to labour productivity growth, which does not lead to any difference in labour productivity growth.

The theoretical assumptions in the methodology used by Diewert and Yu (2012) are largely the same as those used by Statistics Canada (see Baldwin, Gu and Yan, 2007). Both estimates of TFP assume perfect competition and constant returns to scale of the aggregate production function. The main difference is that Diewert and Yu (2012) use a real revenue function of the translog form, while Statistics Canada does not assume any particular form of production function. The translog revenue function is the second-order approximation of a flexible production (or revenue) function. It makes exact the Törnqvist index and the decomposition of growth in real revenue. For an arbitrary production function with constant returns to scale, the Törnqvist index is an approximation. Theoretically, these differences between the translog form and the general form of the production function are unlikely to cause any significant difference in TFP estimates.

Differences in estimated TFP growth between our estimates and those of Statistics Canada arise primarily from the measurement of capital services and labour inputs. Statistics Canada uses the input-output table to estimate the outputs and inputs in the business sector. In this bottom-up approach, outputs and inputs in many industries are measured and aggregated to a top level. Diewert and Yu (2012) instead take a top-down approach, by using the national accounts and national balance sheets for most components. In principle, these two approaches should lead to similar estimates of TFP growth. In practice, the bottom-up approach of Statistics Canada uses many more disaggregated input and output types, suggesting that there may be potential aggregation errors in the top-down approach that is applied to a less-detailed disaggregation. The

[35] Multifactor productivity estimates by Statistics Canada are in CANSIM Table 383-0021.

[36] A detailed comparison of the Statistics Canada estimates and Diewert-Yu measures is provided in Appendix 4 of Diewert and Yu (2009).

differences in data inputs between the two approaches result in substantially faster growth in capital services in Diewert-Yu estimates for the entire sample period. In addition, in Diewert and Yu, labour inputs grow more slowly in the 1980s.

D Appendix D: Figures

Figure 1: Real quarterly GDP and inputs, seasonally adjusted, normalized

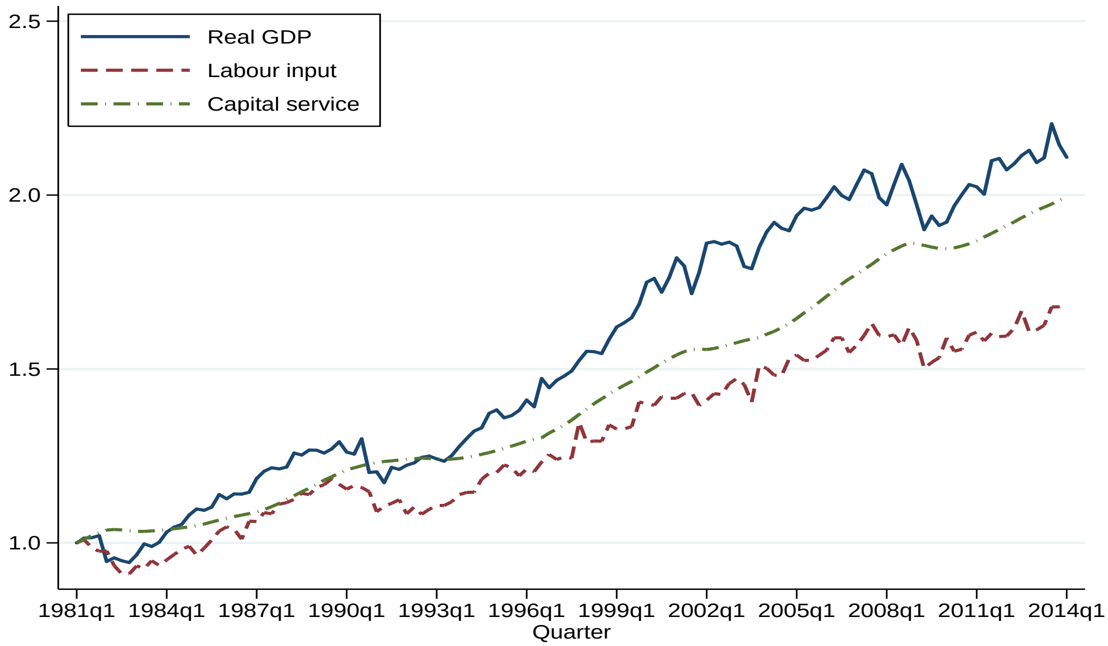


Figure 2: Comparing annual and quarterly TFP growth

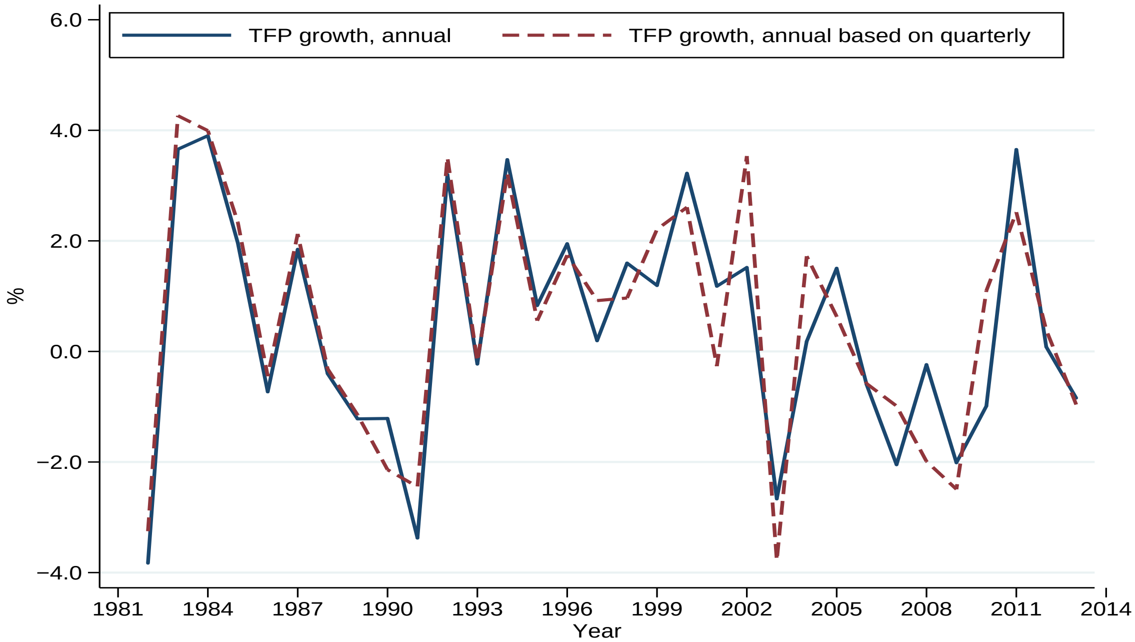


Figure 3: Real GDP index in business sector

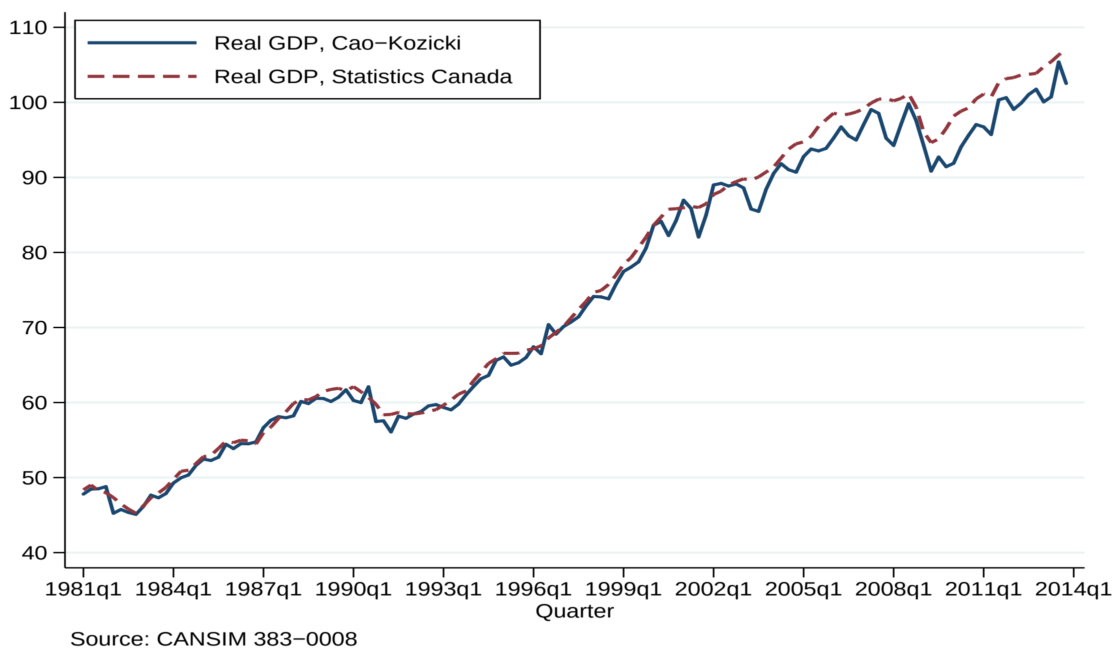


Figure 4: Index of hours worked in business sector

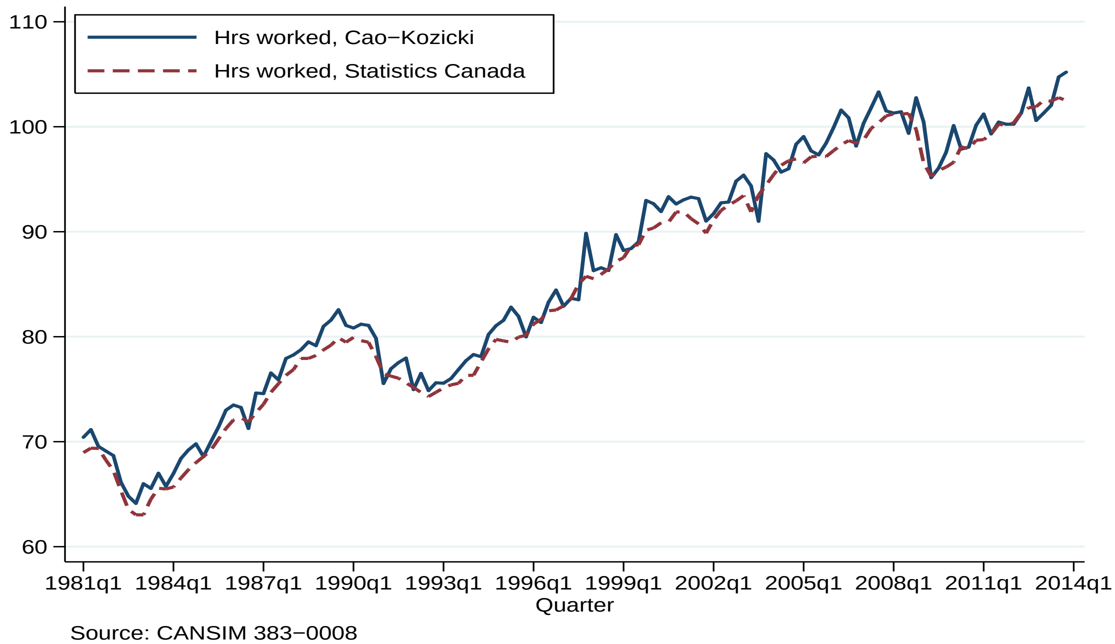


Figure 5: Index of average labour productivity (ALP) in business sector

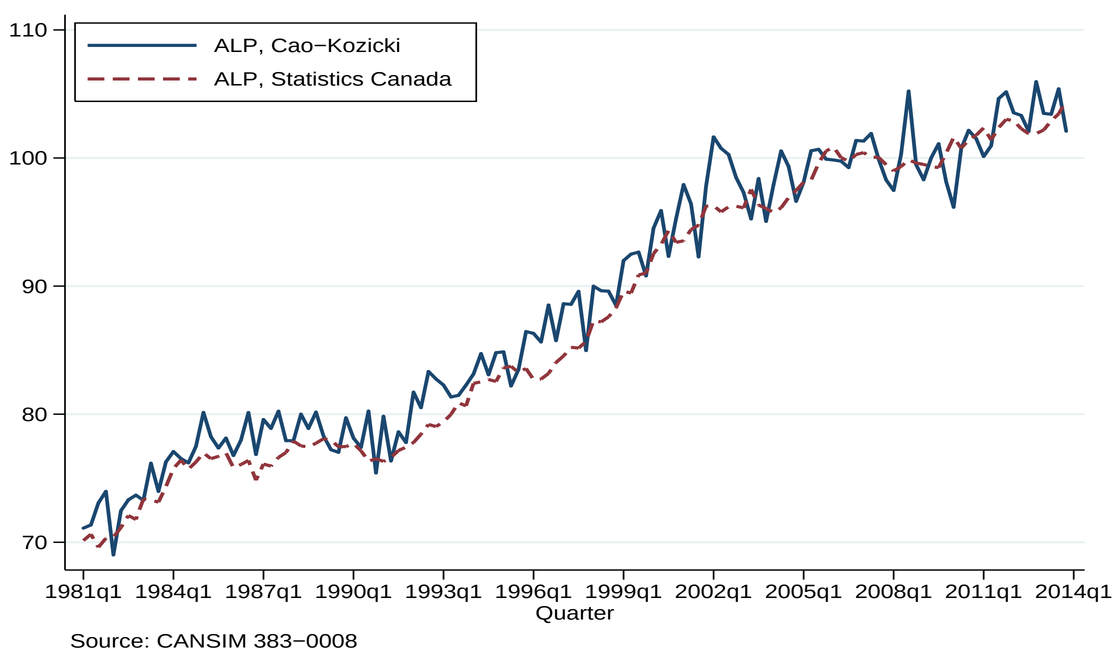


Figure 6: Growth rates of TFP and TFP adjusted for capacity utilization (CU)

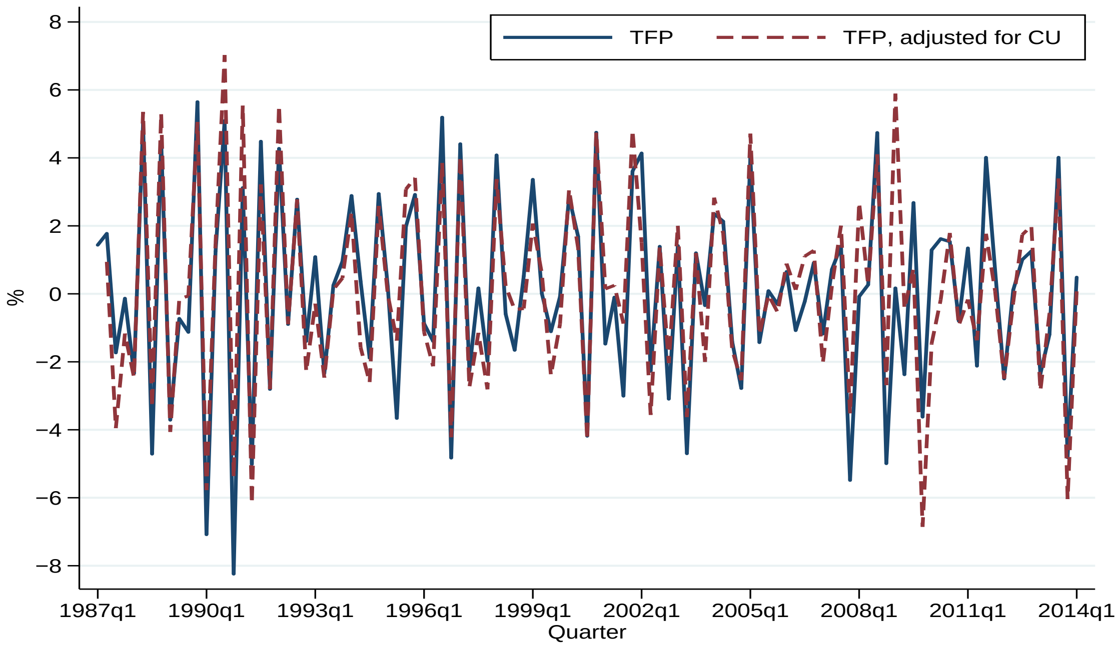
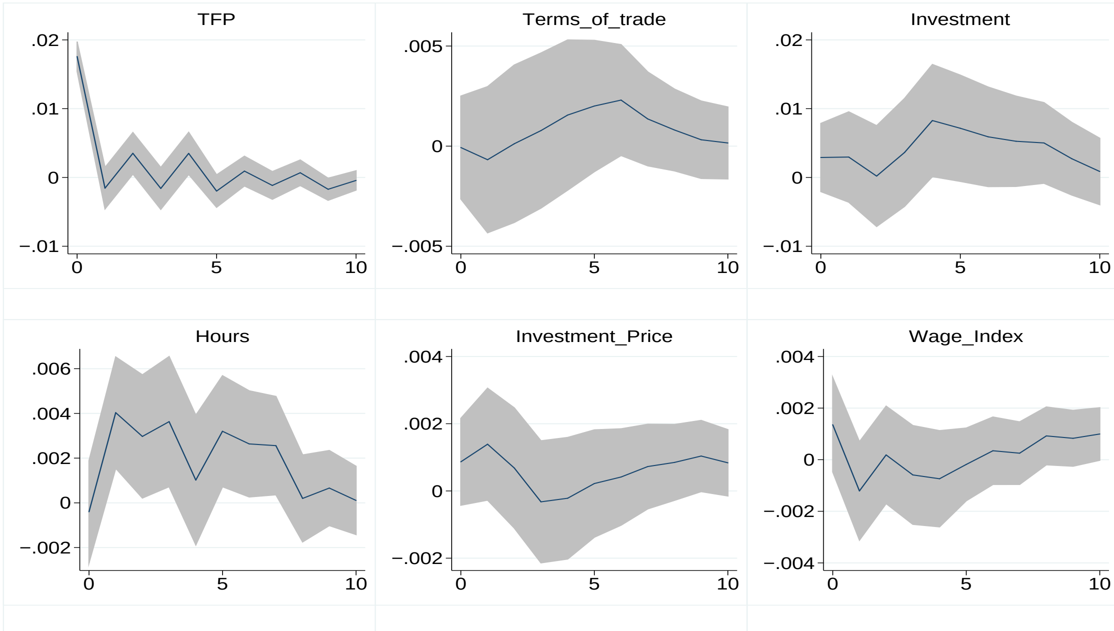


Figure 7: Impulse responses to a technology shock, based on VAR using growth rates

Figure 8: Impulse responses to a technology shock, based on VAR using de-trended data



Note: shaded area is the 95% Confidence Interval.

Figure 9: Decomposition of real income growth

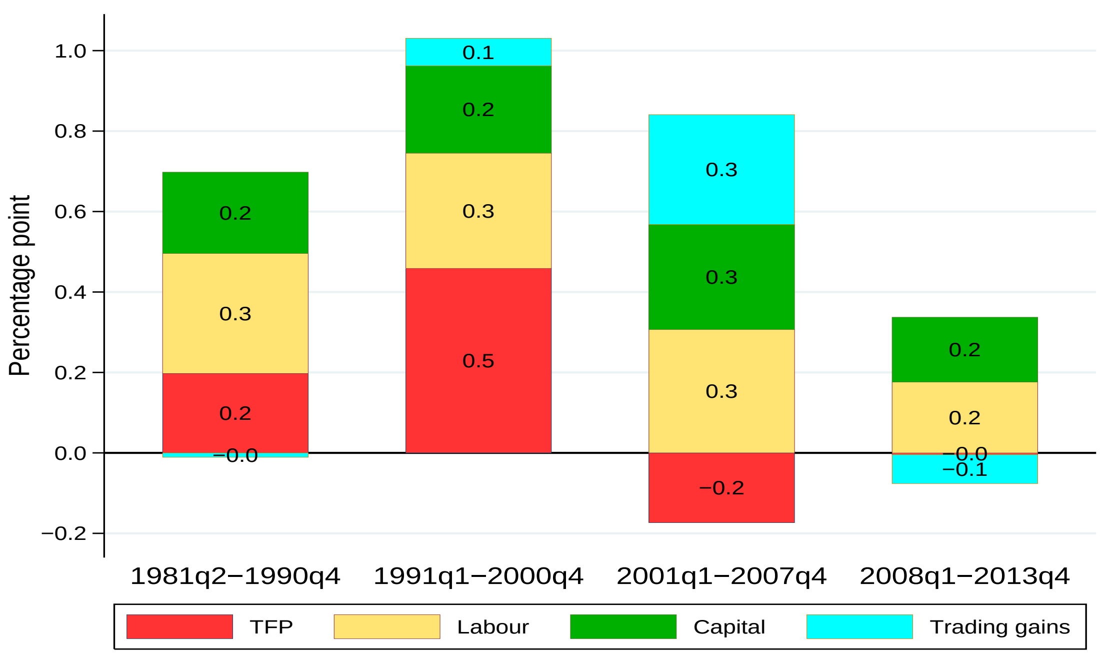


Figure 10: Price indexes of exports and imports, 1981-2013

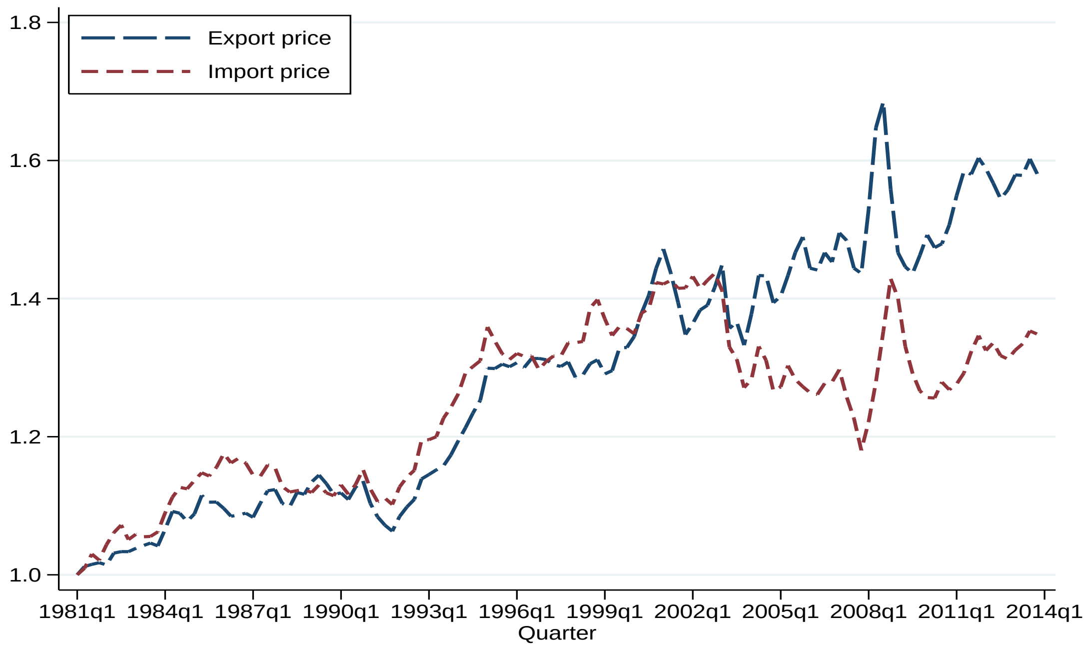


Figure 11: Cumulative growth factors of real GDI and its components, annual, 1981-2013

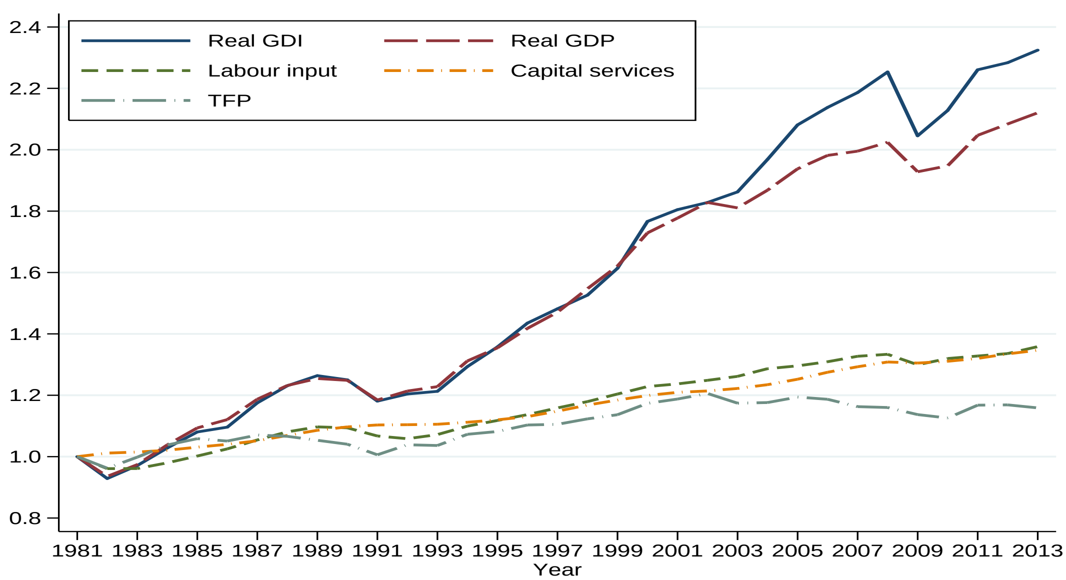


Figure 12: TFP indexes with and without research and development (R&D)

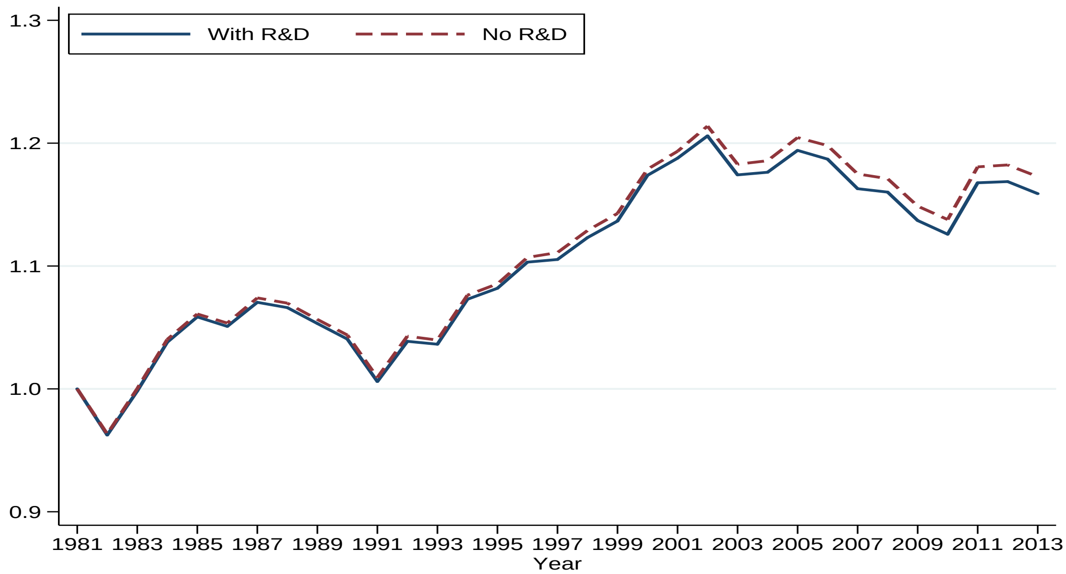


Figure 13: Comparison of TFP index with Diewert-Yu estimates

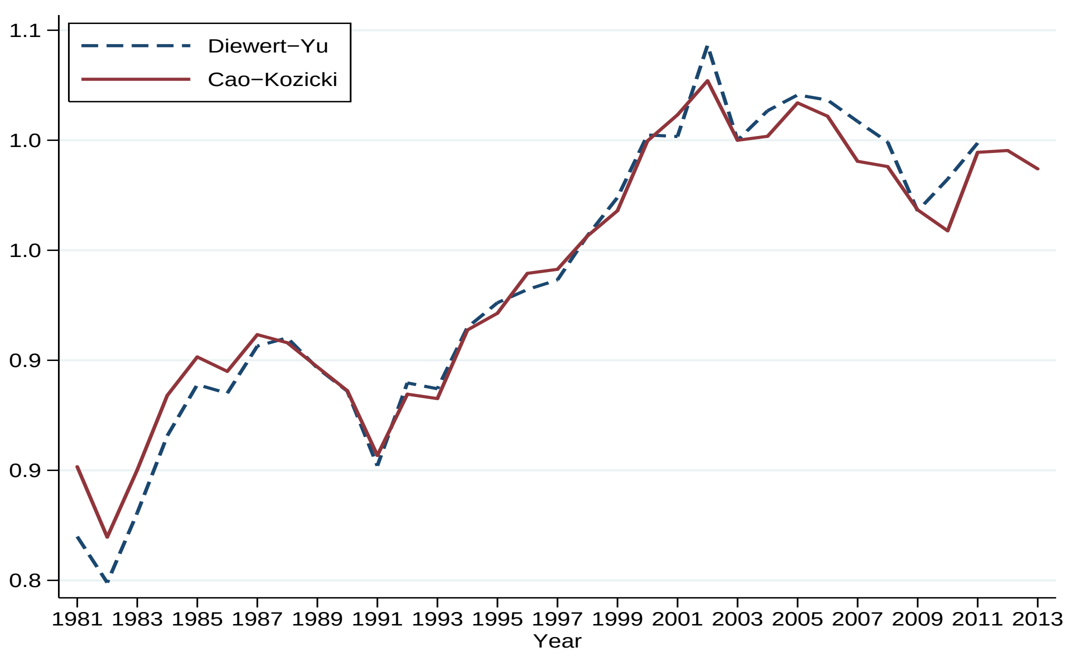


Figure 14: Comparison of TFP index with Statistics Canada estimates