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A Note on Central Counterparties in Repo Markets

by Hajime Tomura

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Abstract

The author introduces a central counterparty (CCP) into a model of a repo market. Without the CCP, there exist multiple equilibria in the model. In one of the equilibria, a repo market emerges as bond dealers and cash investors choose to arrange repos in an over-the-counter bond market. In another equilibrium, the repo market collapses due to aggregate cash shortage for dealers. Introducing a CCP into the repo market blocks the latter equilibrium. This stabilizing effect of a CCP is robust to idiosyncratic default risk of dealers and asymmetric information about the risk.

JEL classification: G24
Bank classification: Payment, clearing, and settlement systems; Financial markets; Financial stability

Résumé

L'auteur introduit une contrepartie centrale dans un modèle du marché des pensions. En l'absence de contrepartie centrale, le modèle admet des équilibres multiples. Dans l'un de ceux-ci, les courtiers en obligations et les investisseurs qui disposent de liquidités se rencontrent sur le marché pour conclure ensemble de gré à gré des opérations de pension. Dans un autre équilibre, le marché des pensions s'écroule en raison d'une pénurie générale de liquidité chez les courtiers. La création d'une contrepartie centrale sur le marché des pensions empêche l'émergence d'un tel équilibre. Cet effet stabilisateur n'est pas sensible au risque de défaut idiosyncrasique des courtiers ni à l'asymétrie de l'information relative à ce risque.

Classification JEL : G24
Classification de la Banque : Systèmes de paiement, de compensation et de règlement; Marchés financiers; Stabilité financière

1 Introduction

The market for repurchase agreements, or repos, showed instability during the recent financial crisis. For example, the Financial Crisis Inquiry Commission (2011) notes the concern expressed by Federal Reserve Board Chairman Ben Bernanke about a collapse of the U.S. tri-party repo market, which is the main repo market in the United States:[1]

> The $2.8 trillion tri-party repo market had "really [begun] to break down," Bernanke said. "As the fear increased," short-term lenders began demanding more collateral, "which was making it more and more difficult for the financial firms to finance themselves and creating more and more liquidity pressure on them. And, it was heading sort of to a black hole." He saw the collapse of Bear Stearns as threatening to freeze the tri-party repo market, leaving the short-term lenders with collateral they would try to "dump on the market. You would have a big crunch in asset prices." (pp. 290−91)

Also, in accordance with Bernanke's comment, Adrian, Burke and McAndrews (2009) at the Federal Reserve Bank of New York note an increase in haircuts before the Bear Stearns collapse:

> Most repos are organized as triparty contracts . . . . In the two weeks prior to the creation of the PDCF on March 16, 2008, liquidity conditions in the repo market grew very strained. Lenders were concerned about both the creditworthiness of borrowers and the riskiness of the collateral pledged — particularly mortgage-backed securities. With lenders worrying that they could lose money on the securities they held as collateral, haircuts increased — doubling for some agency mortgage securities and increasing significantly even for borrowers with

[1] See Copeland, Martin and Walker (2010) for the institutional features of the U.S. tri-party repo market.

high credit ratings and on relatively safe collateral such as Treasury securities. (p. 2)

Following these observations, the Committee on Payment and Settlement Systems (2010) at the Bank for International Settlements has recommended that a central counterparty (CCP) be introduced into the repo market in each country as a safeguard against a collapse of the repo market. A CCP is a clearing house that interposes itself between the two parties in a financial market transaction to be the counterparty for each party. This service is called novation. Whereas novation shields the two parties from direct exposure to each other, a CCP requires that each of its participants submit collateral, called clearing funds, to offset its losses due to default and settlement failures by other participants. Thus, participants in a CCP mutualize losses among themselves.

Does the introduction of a CCP ensure the stability of the repo market as intended? I address this question using a dynamic model constructed by Tomura (2012). In the model, a repo market emerges endogenously among bond dealers and cash investors, given that the bond market is an over-the-counter (OTC) market. This result is consistent with the feature of the repo market in practice in normal times. Repos, however, are fragile due to multiple equilibria: there exists another equilibrium in which cash investors stop entering into repos with dealers all at once. In this equilibrium, a disappearance of cash inflows from cash investors to dealers causes aggregate cash shortage for dealers. As a result, dealers fail to obtain enough cash to repurchase bonds from cash investors. In response, cash investors who bought bonds from dealers with repos before and need cash now sell their bonds to other cash investors in search of market liquidity. This bond liquidation justifies the disappearance of cash investors entering into repos, because cash investors holding cash can instead buy the liquidated bonds at a discounted price due to the sellers' imminent need for cash.

In this paper, I show that introducing a CCP into the repo market eliminates the latter equilibrium. Given that a sufficiently large amount of clearing funds is submitted to the CCP, the CCP can guarantee the contracted repurchase price of bonds to cash investors in repos even if dealers default on the repos. Thus, the CCP can prevent the liquidation of repo collateral. This effect of the CCP, in turn, eliminates the reason for cash investors holding cash to stop entering into repos with dealers. Accordingly, dealers do not suffer aggregate cash shortage. Hence, the repo market sustains.

This stabilizing effect of a CCP is robust to idiosyncratic default risk of dealers. Even if a dealer defaults on a repo due to an idiosyncratic shock, the CCP can liquidate the bonds held as repo collateral to other dealers without a loss, given no aggregate cash shortage for dealers. Hence, repo collateral covers the default risk of dealers. This result sustains even if the default risk of each dealer is known only to the dealer and the cash investor entering into a repo with the dealer.

This paper adds to the recent literature on CCPs. In this literature, Koeppl and Monnet (2010) provide a model of a CCP in an OTC derivatives market. While they show that a CCP improves risk sharing among risk-averse investors, both dealers and cash investors are risk neutral in this paper. Also, Duffie and Zhu (2010) analyze the effects of multilateral nettings by CCPs. This paper abstracts from this feature of a CCP. See Tomura (2012) for other related literature.

2 The Set-Up of the Baseline Model

The analysis of CCPs in this paper is built upon the baseline model constructed by Tomura (2012). See the appendix for an analytical definition of the model. Here I briefly describe the structure of the baseline model.

There are cash investors and bond dealers in the model. In each period, a group of cash

investors enters the model with an endowment of cash. Their objective is to maximize the expected rate of return on their cash in the next period, in which they need to pay out their cash. I call investors in their first period "young" and those in their second period "old." For simplicity, investors exit from the model after paying out their cash in their second period.

Cash investors have two instruments for cash investments. One is treasury bills, on which cash investors can put any amount of cash at a fixed rate of return. The other is bonds, which pay a fixed amount of dividends every period. A cash investor can store cash by buying bonds when young and reselling them when old. When buying or selling bonds, a cash investor can trade with either a bond dealer in the dealer market or another cash investor in the brokered market. While dealers are always available for bond trade, meeting with another cash investor is possible only if there are other cash investors entering into the brokered market. In each bond transaction, the trading parties negotiate the terms of trade bilaterally. Dealers try to maximize the profits from bond transactions with cash investors in each period. The total supply of bonds is fixed.

In addition, dealers can trade bonds among themselves in an interdealer bond market. The market is so competitive that dealers take the interdealer bond price as given. Similarly, there is a competitive interdealer market for overnight loans. Dealers can settle transactions with cash investors in each period by bonds and cash obtained in the interdealer markets in the same period.

3 The Main Results of the Baseline Model

Before introducing a CCP, I briefly summarize the main results of the baseline model. There exists a symmetric stationary equilibrium in which no investor enters the brokered market because no entry of investors into the brokered market leads to a zero matching probability in the brokered market. Accordingly, investors buy bonds from dealers when young, and

resell the bonds to dealers when old. In this equilibrium, each dealer can negotiate down the bond price when an old investor resells bonds to the dealer, because the old investor needs to obtain cash to pay out within the same period.[2] This ex-post price discount on cash investors' bonds induces dealers to sell bonds with repos to cash investors at a reduced price ex ante, so that cash investors can earn a competitive rate of return on their bonds. Repos make these transactions viable for dealers because dealers' ex-ante losses from selling bonds at a reduced price are offset by their ex-post gains from repurchasing the bonds at a discounted price later.

However, there also exists another equilibrium as part of multiple equilibria. Suppose that the economy was in the symmetric stationary equilibrium with repos in period $t-1$. Under certain conditions described by Tomura (2012), there exists such an equilibrium that every investor enters the brokered market unexpectedly in period t and the economy returns to the stationary equilibrium with repos from period $t+1$ onward. In this equilibrium, a disappearance of cash inflows from young investors to dealers causes aggregate cash shortage for dealers in period t. As a result, dealers fail to obtain enough cash to repurchase bonds from old investors in the period. In response, old investors enter the brokered market in search of market liquidity for their bonds.[3] This bond liquidation by old investors justifies the disappearance of young investors from the dealer market, because they can instead buy the liquidated bonds at a discounted price in the brokered market due to old investors' imminent need for cash.[4] Thus, the repo market collapses in period t.

[2]This result holds regardless of the existence of a repo, because a repo can be renegotiated in the model.

[3]Thus, old investors cause settlement failures in this case.

[4]Regarding the interpretation of this result, note that money market funds (MMFs), a main group of cash lenders in the U.S. tri-party repo market, cannot hold long-term securities by regulation. Implicitly, a repo-market collapse, described in section 4, can be interpreted as occurring when ultimate cash investors, such as corporate treasuries, stop putting their cash on MMFs and instead buy bonds directly from other cash investors who liquidate repo collateral.

4 A Central Counterparty in the Repo Market

In this section, I show that introducing a CCP into the repo market eliminates the equilibrium with a repo-market collapse.

As noted earlier, a CCP is a clearing house that interposes itself between the two parties in a financial market transaction to be the counterparty for each party. Whereas such novation shields the two parties from direct exposure to each other, a CCP requires that each of its participants submit collateral called clearing funds to offset its losses due to default and settlement failures by other participants. Thus, participants in a CCP mutualize losses among themselves.

To incorporate the novation and the loss mutualization by a CCP, I establish a simple set of rules for the behaviour of the CCP in the model. The CCP can accept to novate a repo unless it expects to incur a loss from the first or the second leg of the repo.[5] If the CCP novates a repo, then it commits to the contracted repurchase price in the second leg of the repo and does not accept any renegotiation.[6] Old investors and dealers can still default on repos against the CCP without renegotiations in the second leg of the repos. If default on a repo occurs, then the CCP buys or sells bonds in the interdealer bond market to deliver bonds or cash to the other contracting party to the repo.[7] The CCP takes the competitive interdealer bond market as given.

Young investors in each period must submit a certain amount of cash as clearing funds to the CCP if they want to have novation of their repos by the CCP. The CCP uses the clearing

[5]The first leg of a repo is the initial sale of bonds from a dealer to a cash investor. The second leg of the repo is the repurchase of the bonds by the dealer from the investor. In each leg, the CCP novates the transaction.

[6]Here, I assume that the public nature of the CCP prevents it from renegotiating contracts flexibly like a private company. For example, the Canadian Derivatives Clearing Corporation will be the CCP in the Canadian repo market. It will be supervised by the Bank of Canada under the Payment Clearing and Settlement Act because of its systemic importance.

[7]Alternatively, I can assume that the CCP sends an agent to the OTC dealer market for bond buyers or bond sellers. In this case, however, I need to specify some utility function for the CCP to define the Nash bargaining problem in the market, which complicates the analysis.

funds to offset its loss from default in the next period. Thus, young investors participating in the CCP mutualize losses among themselves. The CCP invests clearing funds in treasury bills in the current period and returns unused clearing funds, including interest earned, to the contributors of the funds in the next period.

If the clearing funds are not sufficient to cover all of the losses, then the CCP defaults on all of the repos it novates and returns all of the clearing funds to the contributors of the funds. In this case, the CCP cannot take loans in the interdealer loan market. Because the CCP does not need interdealer loans otherwise, there is no occasion in which the CCP takes interdealer loans.

4.1 Participation in the CCP in an equilibrium with repos

I first show that investors and dealers participate in the CCP in a symmetric stationary equilibrium with repos. In this equilibrium, no investor enters the brokered market, because of a self-fulfilling zero matching probability for each entrant in the market, as described in section 3. Accordingly, young investors enter the dealer market for young investors and then arrange repos with dealers.

Do young investors and dealers choose to have novation of their repos by the CCP? To answer this question, I derive the terms of repos that the CCP can novate. On the one hand, a dealer in a repo novated by the CCP will not default against the CCP in the second leg of the repo, if and only if the contracted repurchase price, denoted by $E_t p_{RP,t+1}$, is expected to be equal to, or less than, the interdealer bond price in the next period, denoted by $p_{ID,t+1}$.[8] This result holds because, if $E_t p_{RP,t+1} > p_{ID,t+1}$, then dealers would not be able to earn profits from reselling the repurchased bonds in the interdealer market.

On the other hand, an old investor in a repo novated by the CCP will not default if and

[8]Here, $p_{RP,t+1}$ denotes the repurchase price of bonds in period $t+1$. Thus, the contracted repurchase price equals its expected value in the previous period.

only if $E_t p_{RP,t+1}$ is equal to, or greater than, the bond price in the dealer market for old investors, which is denoted by $p_{O,t+1}$. I can show that $p_{O,t+1} = 0.5 p_{ID,t+1}$, because a dealer can negotiate down the price of bonds sold by an old investor. Overall, the CCP novates a repo if $E_t p_{RP,t+1} \in [0.5 E_t p_{ID,t+1},\ E_t p_{ID,t+1}]$.

The young investor and the dealer in each repo choose to set $E_t p_{RP,t+1} = E_t p_{ID,t+1}$ by having novation by the CCP. To see this result, note that, for a given rate of return on a repo for a young investor, the sale price of bonds with the repo can be higher as the contracted repurchase price is higher. Dealers prefer a higher sale price of bonds because of an assumption that they prefer current profit to future profit.[9] Note that if they do not have novation of their repos by the CCP, then the dealer can commit only to the renegotiation-proof repurchase price. This price equals the expected spot bond price in the dealer market, $E_t p_{O,t+1}$, and hence $0.5 E_t p_{ID,t+1}$, because otherwise a renegotiation of a repo would make the renegotiated repurchase price equal to the spot price.[10]

In the equilibrium, there is no opportunity cost for young investors to submit clearing funds, because the CCP returns the funds with interest at the treasury bill rate when the investors become old. (Remember that $E_t p_{RP,t+1} = E_t p_{ID,t+1}$ ensures no default against the CCP.) Thus, young investors can submit all of their residual cash to the CCP as clearing funds. Hereafter, I assume that the CCP requires young investors to submit all of their cash as clearing funds.[11]

[9] Precisely speaking, the time-discount factor for dealers, β, is assumed to be so small that $\beta(1+r) < 1$, where r is the exogenous treasury bill rate.

[10] Considering the possibility of a renegotiation is consistent with the fact that repos are regarded as secured loans in practice. See Tomura (2012) for more details.

[11] For dealers, submitting clearing funds is costly because of the impatience of dealers: their intertemporal marginal rate of substitution, denoted by β^{-1}, is assumed to be less than the rate of return on treasury bills, denoted by $1 + r$.

4.2 The stabilizing effect of the CCP

The presence of the CCP blocks the equilibrium with a repo-market collapse described above by preventing liquidation of repo collateral by old investors. Suppose that the economy is in the symmetric stationary equilibrium with repos and the CCP in period $t-1$. Accordingly, old investors and dealers have novation of their repos by the CCP at the beginning of period t. In this case, even if every young investor in period t unexpectedly entered the brokered market, the CCP would be able to guarantee the contracted repurchase price, $E_{t-1}p_{ID,t}$, for each old investor by using submitted clearing funds.[12] As a result, every old investor resells bonds to the CCP.

Because no old investor enters the broker market, young investors cannot buy bonds at a discounted price in the brokered market. They can buy bonds only in the dealer market for young investors, and thus arrange repos with dealers in the market. Hence, the presence of the CCP blocks the equilibrium with a repo-market collapse.

4.3 Robustness check: counterparty risk

Next, I introduce idiosyncratic default risk of dealers into the model, because one of the functions of a CCP is to shield the contracting parties to a repo from counterparty risk by novation. I assume that each dealer exits from the economy at the beginning of each period with probability γ ($\in (0,1)$).[13] If a dealer exits, then the investor who entered into a repo with the dealer in the previous period loses the counterparty. Thus, a random exit of dealers causes counterparty risk. To keep the population of dealers constant, exiting dealers are

[12] Tomura (2012) shows that the equilibrium with a repo-market collapse exists only if the cash endowment for each young investor is not too small or too large. I can show that young investors can submit a sufficient amount of clearing funds in period $t-1$ for the CCP to guarantee the contracted repurchase price if dealers are sufficiently impatient; that is, their time-discount factor, β, is sufficiently small.

[13] At the beginning of each period in the stationary equilibrium with repos, dealers own only repos arranged with investors in the previous period. Thus, exiting dealers just default on their repos and consume nothing.

replaced with new dealers.[14]

The stability effect of a CCP is robust in this case. Suppose that cash investors and dealers participate in the CCP. The presence of the CCP ensures that dealers receive cash inflows from young investors through repos, as described above. Thus, the interdealer bond market does not suffer aggregate cash shortage. Accordingly, even if some dealers default against the CCP due to idiosyncratic default risk, the CCP can obtain enough cash to repurchase bonds from cash investors by liquidating bonds in the interdealer bond market. Because the CCP does not incur any loss from idiosyncratic default of dealers, the CCP can return the entire clearing funds to old investors in each period.[15] Without any opportunity cost of submitting clearing funds, cash investors and dealers participate in the CCP.

In summary, repo collateral covers the default risk of dealers, because the presence of the CCP maintains the market liquidity of bonds held as repo collateral. This effect of the CCP in turn ensures the participation of investors and dealers into the CCP.

4.4 Robustness check: asymmetric information about counterparty risk

Finally, I consider the case in which the dealer and the young investor in each repo have private information about whether the dealer exits from the economy in the next period. Thus, the CCP must novate repos with asymmetric information. The motivation for this analysis is to investigate whether moral hazard impairs the stabilizing effect of the CCP. Here, I do not consider asymmetric information between the contracting parties to each repo, because the bargaining problem would become intractable.

The stabilizing effect of the CCP is robust to the asymmetric information. On the one hand, dealers who will not exit in the next period arrange repos with the same terms of trade

[14] New dealers enter the economy without any endowment.

[15] Because the contracted repurchase price, $E_{t-1}p_{RP,t}$, equals the interdealer bond price, $p_{ID,t}$, the CCP can liquidate repo collateral in the interdealer bond market without any loss or gain if a dealer defaults on a repo that it novates.

as in the stationary equilibrium with repos and the CCP described above. On the other hand, dealers who will exit in the next period would like to increase the contracted repurchase price, $E_t p_{RP,t+1}$, as much as possible, because they are only interested in maximizing the price of bonds that they receive from young investors in the current period. However, dealers can set $E_t p_{RP,t+1}$ only up to the expected interdealer bond price, $E_t p_{ID,t+1}$, because otherwise the CCP would not novate the dealers' repos by expecting their default in the next period. Hence, dealers who will exit in the next period arrange the same terms of repos as those who will not exit. As a result, the stabilization effect of the CCP sustains, as in the case without asymmetric information.

5 Conclusions

In this paper, I have analyzed the effect of a CCP on the stability of the repo market. The model I use yields three main results:

- The repo market collapses if cash investors holding cash stop entering into repos and instead buy repo collateral liquidated by other cash investors at a fire-sale price. The presence of a CCP prevents a repo-market collapse by stopping the liquidation of repo collateral.

- The stabilization effect of a CCP is robust to idiosyncratic default risk of dealers. Because the presence of a CCP maintains the market liquidity of repo collateral, repo collateral covers the default risk of dealers for the CCP.

- Asymmetric information about the default risk of dealers between a CCP and repo contracting parties does not impair the stabilization effect of the CCP. The CCP can protect itself from moral hazard by taking sufficient repo collateral.

The important assumption for these results is that the interdealer bond market is competitive. Even though this is a standard assumption in the literature (e.g., Duffie, Gârleanu and Pedersen 2005), a question remains regarding the sensitivity of the results to different degrees of the market powers of dealers and a CCP in the interdealer bond market. Also, while I analyze the implication of asymmetric information about the default risk of dealers, it is important to analyze the implication of asymmetric information about the quality of repo collateral, such as non-agency mortgage-backed securities. These issues are left for future research.

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Appendix

A The baseline model (Tomura 2012)

Time is discrete and its horizon is infinite. In each period, a $[0, 1]$ continuum of risk-neutral investors are born with a fixed amount e_I (> 0) of a cash endowment for each. They live for two periods and consume cash in the last period of their lives. This fixed timing of consumption by each investor represents the fact that cash investors, in practice, cannot change the timing of their cash payments easily when they need to pay out cash. Each investor is indexed by $i \in \mathbb{Z} \times [0, 1]$, which is the pair of the period when the investor is born and the real number assigned to the investor on the unit continuum of the cohort. I call investors in their first period "young" and those in their last period "old."

Young investors can invest their cash in two instruments. One is safe short-term bills that return a fixed amount $1 + r$ (≥ 1) of cash in the next period for each unit of cash invested. I call this instrument "treasury bills." The other instrument is safe long-term bonds, which generate a fixed amount d (> 0) of cash dividends for the holders of bonds at the beginning of every period. I call this instrument "bonds." Bonds are divisible and their supply is fixed to unity. Thus, bonds are Lucas trees. An investor can store cash by buying bonds when young and reselling them when old.

A.1 Direct trade between cash investors in a brokered market

Investors can trade bonds in a brokered market. Bond buyers (the young) and sellers (the old) in the market can meet through pairwise random matching.[16] The matching probabilities

[16]This matching can be interpreted implicitly as arranged by brokers.

are:

$$\mu_{BR,Y,t} \equiv \min\left\{1,\ \frac{\theta_{BR,O,t}}{\theta_{BR,Y,t}}\right\}, \quad \mu_{BR,O,t} \equiv \min\left\{1,\ \frac{\theta_{BR,Y,t}}{\theta_{BR,O,t}}\right\}, \tag{1}$$

where t denotes the time period; $\mu_{BR,Y,t}$ and $\mu_{BR,O,t}$ denote the matching probabilities for a young and an old investor in the market, respectively; and $\theta_{BR,Y,t}$ and $\theta_{BR,O,t}$ are the fractions of young and old investors entering the market, respectively. Thus, the short side of the market can match for sure. If $\theta_{BR,Y,t} = \theta_{BR,O,t} = 0$, then $\mu_{BR,Y,t} = \mu_{BR,O,t} = 0$. Investors take the matching probabilities as given. The outcome of each match is determined by Nash bargaining, with equal bargaining powers for both parties in the match.

A.2 Over-the-counter dealer markets for investors

Also, there exists a $[0, 1]$ continuum of infinite-lived risk-neutral dealers maximizing the expected discounted consumption of cash:

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} c_{j,s}, \tag{2}$$

where β ($\in (0, 1)$) is the time discount factor for dealers, j ($\in [0, 1]$) is the index for each dealer, and $c_{j,t}$ is the consumption of cash. With the presence of dealers, each investor chooses either to trade bonds with a dealer or to enter the brokered market to meet with another investor in each period. An investor cannot choose both options, because it takes time to find a trading counterparty.

Investors can meet with dealers in dealer markets. Dealers open markets for young investors, who buy bonds, and old investors, who sell bonds, separately. In each dealer market, investors entering the market are matched with dealers through pairwise random matching. I assume that every dealer participates in each market and that the short side of

each market can match for sure.[17] Accordingly, in each market, the matching probability for an investor is unity and the matching probability for a dealer equals the fraction of investors entering the market. All dealers and investors take the matching probabilities as given.

The terms of trade in each match between a dealer and an investor are determined by Nash bargaining, with equal bargaining powers for both parties in the match, as in the brokered market. In the dealer market for old investors, the term of trade is the price of bonds sold by the old investor in each match. Old investors need to sell all of their bonds because they can consume cash only in the current period.

In the dealer market for young investors, the terms of trade are the price and the quantity of bonds as well as whether to arrange a repo. If a young investor buys bonds from a dealer without a repo, then that investor needs to enter the dealer market for old investors in the next period in order to resell the bonds to a dealer.

In contrast, if a young investor buys bonds with a repo from a dealer, then that investor can meet with the same dealer again without entering the dealer market for old investors in the next period. Either party to a repo can initiate a renegotiation of the repurchase price when they meet again. The outcome of the renegotiation is determined by Nash bargaining with equal bargaining powers for both parties. Thus, dealers can commit only to a renegotiation-proof repurchase price consistent with the expected outcome of ex-post renegotiations.[18]

I assume that an old investor with a repo does not enter the dealer market for old investors if dealers offer the same terms of trade for old investors regardless of the existence of repos. In this case, an old investor with a repo chooses either to meet with the dealer for the repo or to enter the brokered market. Otherwise, an old investor with a repo can also choose

[17] The definition of matching probabilities is similar to equation (1) for the brokered market.

[18] For this assumption, note that if a dealer could commit to an arbitrary repurchase price, then a repo would be just a non-secured loan. This result would contradict the fact that repos are regarded as secured loans, in practice.

to enter the dealer market for old investors. The other investors choose between the dealer market for each investor's type (i.e., young or old) and the brokered market for their bond trade.

The dealer market for young investors opens first, then dealers meet with old investors with repos, and finally the dealer market for old investors opens.[19]

A.3 The interdealer markets and settlements

After meeting with investors, dealers can trade bonds with other dealers in a competitive interdealer bond market. This assumption is based on the feature of the interdealer market for U.S. Treasury securities in practice, in which interdealer brokers allow dealers to trade in size anonymously and distribute the best bid and offer prices to dealers. See Huang, Cai and Wang (2002) and Fleming and Mizrach (2009) for more details.

In addition, dealers can borrow cash from other dealers overnight at a competitive interest rate in an interdealer loan market. The settlement of all transactions takes place only at the end of each period, so dealers can settle transactions with investors after trading in the interdealer markets in the same period. After the settlement, young investors can invest the residual of their cash in treasury bills, treasury bills return cash to old investors, and dealers and old investors can consume cash. See Tables A1 and A2 for a summary of the bond market structure.

All dealers and investors take the competitive interdealer bond price and interest rate as given. An equilibrium is such that these two competitive interdealer prices clear the interdealer markets, given the outcome of each match for bilateral bond trade and the utility-maximizing behaviour of each dealer in the interdealer markets in each period.[20]

[19]The result of the model is insensitive to the order of these markets. See Tomura (2012, Appendix A).

[20]See Tomura (2012, Appendix A) for an analytical definition of the market-clearing conditions for the interdealer markets.

Table A1: The Bond Market Structure in Tomura's (2012) Model

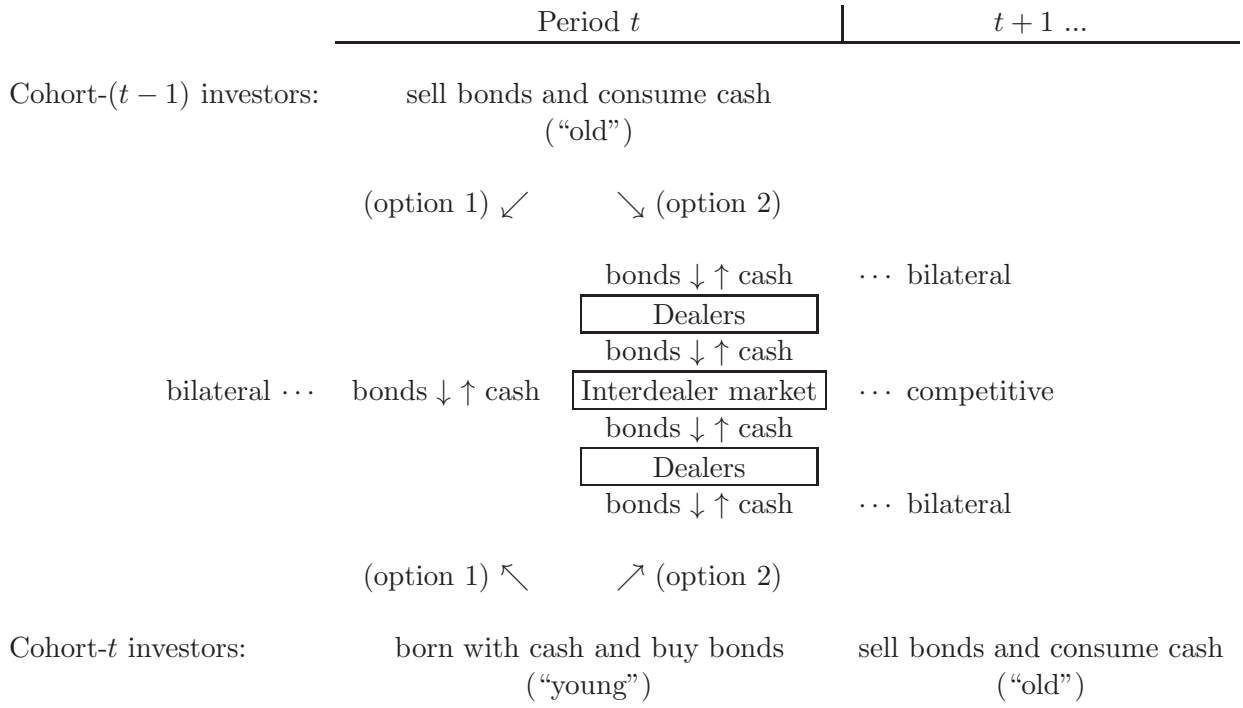
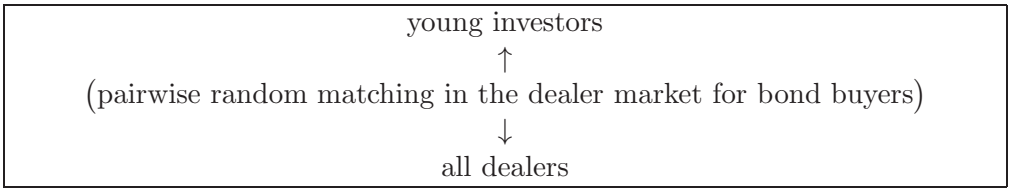


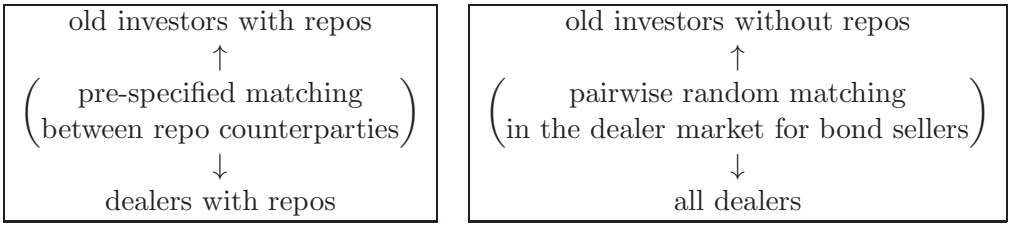
Table A2: Details on Bilateral Matches between Dealers and Investors in Table A1

Young investors choosing option 2



Terms of trade in each match: the price and the quantity of bonds, and whether to arrange a repo.

Old investors choosing option 2



Term of trade in each match: the price of the old investor's bonds.