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Abstract

This paper develops and estimates a dynamic general equilibrium model that realistically accounts for an input-output linkage between firms operating at different stages of processing. Firms face technological change which is specific to their processing stage and charge new prices according to stage-specific Calvo-probabilities. Only a fixed fraction of households have an opportunity to adjust nominal wages to new information each period. Intermediate-stage technology shocks account for the bulk of output variability at business cycle frequencies, while final-stage technology shocks do not explain much. Although technology shocks drive the business cycle, the model predicts weakly procyclical real wages, and a near-zero correlation between return to working and hours worked. Furthermore, the model has rich implications for the dynamics of business cycles.

JEL classification: E32
Bank classification: Business fluctuations and cycles; Economic models

Résumé

Les auteurs élaborent et estiment un modèle dynamique d'équilibre général où est représentée avec réalisme la relation qui lie, par le biais des intrants et des extrants, les entreprises intervenant aux diverses étapes du processus de production. Les chocs technologiques subis diffèrent selon que l'entreprise se charge de l'étape intermédiaire ou de l'étape finale de la production, tout comme la probabilité qu'elle révise ses prix à chaque période (dans un cadre à la Calvo). À chaque période également, une proportion fixe des ménages voient leur salaire nominal modifié à la lumière des nouvelles données reçues. Les chocs qui touchent les techniques de fabrication à l'étape intermédiaire expliquent le gros de la variabilité de la production au cours du cycle économique, alors que ceux qui concernent l'étape de production finale en expliquent fort peu. Même si les chocs technologiques sont à l'origine de la plupart des fluctuations du cycle économique dans le modèle, l'évolution du salaire réel se révèle faiblement procyclique, et la corrélation entre la productivité et le nombre d'heures travaillées s'avère presque nulle. Enfin, le modèle parvient fort bien à décrire la dynamique des cycles économiques.

Classification JEL : E32
Classification de la Banque : Cycles et fluctuations économiques; Modèles économiques

1 Introduction

This paper develops a dynamic general equilibrium (DGE) framework that sheds new light on the cyclical effects of technological change while offering new evidence of the importance of technology shocks for business cycles. Most macroeconomic models, from standard real business cycle (RBC) models to DGE models featuring nominal rigidities and/or real frictions, assume that firms operate at the final stage of production, hence facing technological change only at this single processing stage. However, in reality the production of several finished goods typically goes through more than one stage of processing. This simple fact forces some potentially important questions about the role of technological change as a source of business cycles. Can technology shocks at other than the final stage have an impact on aggregate fluctuations? If so, is this effect large quantitatively? If technology shocks are found to contribute substantially to business cycles in a model with multiple processing stages, does the same model help remedy the anomalies that have plagued a large class of DGE models in which technological change is the dominant source of aggregate fluctuations?

Our paper provides affirmative answers to these questions using an estimated DGE model with five main features. First, our model embeds a two-stage production and pricing structure. The production of finished goods goes through two stages of processing: a stage of intermediate goods and a stage of finished goods. Unlike most existing models, our framework assumes that firms face productivity shocks which are specific to their stage of processing. Firms at the intermediate stage use capital and labor to produce intermediate goods, while firms at the final stage utilize a composite of goods fabricated at the intermediate stage as intermediate inputs, in addition to capital and labor, to produce finished goods. Recent contributions by Basu (1995) and Huang, Liu and Phaneuf (2004) suggest that combining nominal rigidities with an input-output structure helps understand some important aspects of reality. However, instead of assuming that firms are linked through a *horizontal* roundabout input-output structure within a *single* stage of production as these authors do, we focus on an input-output linkage between firms operating at *different* stages of processing. Our framework is thus closer in spirit to the model of Huang and Liu (2005) that incorporates an input-output linkage between sectors to analyze the design of optimal monetary policy with several sources of nominal price rigidities.[1]

Second, nominal prices at both stages of processing are determined by staggered Calvo-contracts, producers at each stage responding to a stage-specific probability of reoptimizing their prices in each period. Hence, unlike the class of models featuring sticky prices only at the final stage of pro-

[1]Also, Huang and Liu (2001) propose a DGE model of the transmission of monetary shocks that stresses production chains and Taylor's (1980) staggered price contracts.

cessing, our model accounts for nominal price rigidities at different stages of production. Therefore, a key characteristic of our framework compared to standard sticky-price models is that changes in the relative price of intermediate goods to final goods can have an allocative role.

Third, households are imperfectly competitive with respect to labor skills. The nominal wages of different skills are also determined by Calvo-contracts.[2] The preferences of households are subject to a shock. Preference shocks have sometimes been identified as the main source of cyclical movements in output and hours worked during the postwar period [e.g., Shapiro and Watson (1988), Hall (1997) and Galí and Rabanal (2004)].[3]

Fourth, our model includes real rigidities in the form of costs of adjusting the stock of physical capital and hours worked. Kim (2000) provides evidence showing that capital adjustment costs had a significant impact on business cycle fluctuations, while Ambler, Guay and Phaneuf (2006) show that labor adjustment costs and sticky nominal wages have been first-order elements in shaping U.S. postwar business cycle dynamics.

The fifth ingredient in our model is a monetary policy rule that sets short-term interest rates in response to variations in finished-good inflation and final output measured in deviations from their steady-state values. Following Rotemberg and Woodford (1999) and Clarida, Galí and Gertler (2000), our specification allows for interest-rate smoothing, and therefore includes the lagged interest rate. However, some authors have questioned whether the lagged rate is a fundamental component of the policy rule. They argue that the apparent significance of the lagged rate in estimated rules could be attributed to serially correlated errors or Fed's response to factors not included in the policy rule [e.g., Rudebusch (2002) and English, Nelson and Sack (2003)]. Hence, the policy rule incorporates both the lagged interest rate and serially correlated policy errors.

We estimate the structural parameters of our models and various second moments of the data using postwar U.S. quarterly data and a maximum likelihood procedure similar to the one implemented in Ireland (2003, 2004a, 2004b). Our main findings can be summarized as follows. First, according to our estimates, the two-stage production and pricing structure is strongly supported by the data. The key structural parameters of the model, including the share of intermediate goods entering the fabrication of finished goods and the parameters governing nominal contracts and real frictions, are economically meaningful and statistically significant. The price contracts of finished

[2]Thus, our model features both nominal wage and price rigidities. Other examples of DGE models with both types of nominal rigidity include Huang, Liu and Phaneuf (2004) and Christiano, Eichenbaum and Evans (2005), among others. However, these models have only one stage of production.

[3]Ireland (2004a) also finds that preference shocks had a strong impact on the variability of inflation during the postwar period.

goods are shorter than the price contracts of intermediate goods, the former lasting on average 2.9 quarters compared to 3.3 quarters for the later. Wage contracts last longer on average than price contracts. Also, we find no evidence of interest-rate smoothing or highly serially correlated policy shocks.

Second, intermediate-stage technology shocks account for the bulk of output variability at business cycle frequencies (contributing for example to 31, 52 and 62% of the one, four and eight-quarters ahead forecast errors of final output). The strong impact of intermediate-stage productivity shocks on final output can be explained as follows. An intermediate-stage technology improvement is followed, by a persistent, hump-shaped decline in the relative price of intermediate goods to final goods and by a persistent, hump-shaped increase in the production of intermediate goods. With intermediate-stage output strongly increasing, finished-good firms use more intermediate inputs, and thus final output rises. Also, following a positive shock to intermediate-stage technology, real wages initially *decline*. The fall in real wages further boosts hours worked and final output.

Third, intermediate-stage technology shocks generate rich business-cycle dynamics, producing persistent, hump-shaped responses of final output, consumption, investment and total hours worked. Hence, the model meets the challenge posed by King, Plosser and Rebelo (1988) who have shown that the standard neoclassical growth model does not account for the dynamics of output, investment and hours as it fails to produce hump-shaped impulse responses following technology shocks.[4]

Meanwhile, final-stage technology shocks explain only a small percentage of the cyclical variance of final output (less than 10% at business cycle frequencies). A positive shock to final-stage technology is followed by a relatively small increase in the relative price of intermediate goods to final goods. Furthermore, it is followed by a persistent, hump-shaped increase in real wages. Therefore, intermediate-stage hours and output decline. Final-stage hours also fall, and with finished-good firms using less intermediate inputs, the increase in final output is relatively small and lasts only for a few periods. Thus, a unique feature of the estimated two-stage model lies in its ability to predict that total hours worked may either *rise* or *fall* following a technology improvement depending on the source of technological change.

Fourth, we find that monetary policy shocks have a significant impact on economic fluctuations at very short horizons, but that their real effects rapidly decline at longer horizons. This finding is broadly consistent with the evidence across different branches of the literature using VARs or

[4]Focusing on the dynamics of output, Cogley and Nason (1995) reach a a similar conclusion for a wide range of RBC models.

estimated DGE models. However, in contrast to the empirical findings of previous estimated DGE models, monetary policy shocks explain a high percentage (more than 70%) of the variability of finished-good inflation at all horizons. Still, technology shocks account for a non negligible fraction of the variability of finished-good inflation (about one fourth). Furthermore, unlike what other researchers have found using single-stage models, preference shocks have a very small impact on fluctuations of hours and output, and almost no effect on the variability of inflation.

Next, the paper turns its attention to the model's ability to account for a fairly comprehensive set of business-cycle statistics. We first look at the size of fluctuations implied by the two-stage model. It accounts very well for the volatility of hours worked relative to volatility of output in the postwar period. Also, the model correctly predicts that output is about twice as volatile as return to working. Several business cycle models have not been able to simultaneously explain these two facts [see Hansen (1985) and Hansen and Wright (1992)]. Furthermore, if the price index of finished goods is approximated by the consumer price index (or GDP deflator) while the price index of intermediate goods roughly corresponds to the producer price index, the model predicts that rate of inflation of finished goods is only about half as volatile as the rate of inflation of intermediate goods, just as in the data.

Extending our analysis to some key comovements, we show that the correlation between total hours and final output is both positive and high. The intermediate-stage technology shock is the key source giving rise to this comovement. The two-stage model also accounts for the so-called Dunlop-Tarshis observation or absence of a strong countercyclical pattern in real wages. Recently, Christiano and Eichenbaum (1992) have reinterpreted the Dunlop-Tarhsis observation in the context of modern business cycle analysis as the near-zero correlation between hours worked and return to working. Our model predicts that real wages are weakly procyclical and that the correlation between hours and productivity is close to zero, as we find in the data. Most business cycle models where technology shocks are a driving force counterfactually predict that real wages are highly procyclical and that the correlation between hours and productivity is both positive and high.

The model's success in explaining these two critical comovements mostly reflects the fact that hours respond very differently to technology shocks depending on the specific source of technological change. Taking, for example, the correlation between return to working and hours, a positive shock to final-stage technology induces a negative comovement between these variables, while the intermediate-stage technology shock generates a positive correlation. Thus, the correlation between hours and productivity produced by the two technology shocks is close to zero, which stands in stark contrast with the predictions of typical RBC models.

Finally, to better understand our model's driving mechanism, we estimate two models that are special cases of our general framework. The first model variant assumes that all firms operate at the final stage, and thus features only one technology shock and a single source of nominal price rigidity. This variant resembles new keynesian models. The second model incorporates the two-stage processing structure, while assuming that wages and prices are perfectly flexible. This variant is closer in spirit to RBC models. As the general framework nests the two model variants, a formal likelihood ratio test can be performed to determine which model is preferred by the data. The general framework is easily preferred to the model variants.

The paper is organized as follows. Section 2 provides a description of our two-stage production and pricing model. Section 3 discusses some econometric issues, data and calibration. Section 4 presents and analyzes our main findings. Section 5 offers concluding remarks.

2 The Model

The economy is inhabited by a large number of infinitely lived households endowed with differentiated labor skills. Households have preferences defined over expected streams of consumption goods, real balances and leisure. In each period, they face a constant probability of adjusting nominal wages. Firms at the intermediate and final stages produce differentiated goods. Finished-good firms utilize a composite of intermediate-stage products as intermediate inputs. Firms experience exogenous technological change which is specific to their processing stage. Producers in each period adjust nominal prices in response to a stage-specific Calvo-probability. The stock of physical capital and hours worked are both costly to adjust.

2.1 The Households

We assume a continuum of households indexed by i, with $i \in (0,1)$ denoting a particular type of labor skill. Household $i's$ preferences are described by the following expected utility function:

$$U(i)_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{\gamma}{\gamma - 1} \kappa_t \log \left(C_t(i)^{\frac{\gamma-1}{\gamma}} + b^{\frac{1}{\gamma}} \left(\frac{M_t(i)}{P_{y,t}} \right)^{\frac{\gamma-1}{\gamma}} \right) - \mu \frac{N_t(i)^{1+\eta}}{1+\eta} \right], \tag{1}$$

where β is a discount factor, $C_t(i)$ is real consumption, $M_t(i)/P_{y,t}$ stands for real money balances, $M_t(i)$ is the nominal money stock, $P_{y,t}$ is the price index of finished goods, and $N_t(i)$ represents hours worked; γ, b, μ and η are positive structural parameters, and γ and η represent the constant elasticity of substitution between consumption and real balances, and the inverse of the elasticity of labor supply, respectively. The representative household's total time available is normalized to one in each period.

The preference shock, κ_t, has the time-series representation:

$$\log(\kappa_t) = \rho_\kappa \log(\kappa_{t-1}) + \varepsilon_{\kappa,t}, \tag{2}$$

where $\varepsilon_{\kappa,t}$ is a serially uncorrelated independent and identically distributed process with a mean-zero and a standard error σ_κ.

The household $i's$ budget constraint is

$$\begin{aligned} & C_t(i) + I_t(i) + CAC_t(i) + \frac{M_t(i)}{P_{y,t}} + \frac{B_{t+1}(i)}{P_{y,t}} \\ = \; & \frac{W_t(i)}{P_{y,t}} N_t(i) + \frac{Q_t}{P_{y,t}} K_t(i) + \frac{M_{t-1}(i)}{P_{y,t}} + R_{t-1} \frac{B_t(i)}{P_{y,t}} + \frac{D_{y,t}(i)}{P_{y,t}} + \frac{D_{z,t}(i)}{P_{y,t}} + \frac{T_t(i)}{P_{y,t}}, \end{aligned} \tag{3}$$

where $I_t(i)$ is real investment, $CAC_t(i)$ is the real cost of adjusting the stock of physical capital $K_t(i)$, $B_{t+1}(i)$ represents bonds carried in period $t+1$, $W_t(i)$ is the nominal wage, Q_t is the nominal rental rate of capital, R_{t-1} is the gross nominal interest rate between period $t-1$ and period t, $D_{y,t}(i)$ and $D_{z,t}(i)$ are the nominal dividends paid to the household by the finished-good firms and by the intermediate-good firms, respectively, and $T_t(i)$ is a lump-sum nominal transfer from the monetary authority.

The cost-function for the adjusting the stock of physical capital is

$$CAC_t(i) = \frac{\varphi_k}{2} \left(\frac{K_{t+1}(i)}{K_t(i)} - 1 \right)^2 K_t(i), \tag{4}$$

where $\varphi_k > 0$ is the cost parameter.

The investment technology is

$$I_t(i) = K_{t+1}(i) - (1-\delta) K_t(i), \tag{5}$$

where $\delta \in (0,1)$ is the rate of depreciation of physical capital.

Aggregate labor input, N_t, is a composite of all labor skills,

$$N_t = \left(\int_0^1 N_t(i)^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}, \tag{6}$$

where σ represents the elasticity of substitution between skills. Labor demand for skill i is,

$$N_t(i) = \left(\frac{W_t(i)}{W_t}\right)^{-\sigma} N_t, \tag{7}$$

where W_t is the wage rate of the composite skill, given by

$$W_t = \left(\int_0^1 W_t(i)^{1-\sigma} di\right)^{\frac{1}{1-\sigma}}. \tag{8}$$

Household i chooses $C_t(i)$, $M_t(i)$, $B_{t+1}(i)$, $K_{t+1}(i)$ and $W_t(i)$ (whenever nominal wages can be adjusted), that maximize the expected discounted sum of utility flows, subject to the budget constraint and the firms' labor demand for skill i. The first-order conditions for this problem are:

$$\frac{\kappa_t C_t(i)^{\frac{-1}{\gamma}}}{C_t(i)^{\frac{\gamma-1}{\gamma}} + b_t^{\frac{1}{\gamma}} \left(\frac{M_t(i)}{P_{y,t}}\right)^{\frac{\gamma-1}{\gamma}}} = \lambda_t(i), \tag{9}$$

$$\frac{\kappa_t b_t^{\frac{1}{\gamma}} \left(\frac{M_t(i)}{P_{y,t}}\right)^{\frac{-1}{\gamma}}}{C_t(i)^{\frac{\gamma-1}{\gamma}} + b_t^{\frac{1}{\gamma}} \left(\frac{M_t(i)}{P_{y,t}}\right)^{\frac{\gamma-1}{\gamma}}} = \lambda_t(i)\left(1 - \frac{1}{R_t}\right), \tag{10}$$

$$\begin{aligned} &\beta E_t \frac{\lambda_{t+1}(i)}{\lambda_t(i)} \left[q_{t+1} + 1 - \delta + \varphi_k \left(\frac{K_{t+2}(i)}{K_{t+1}(i)} - 1\right)\frac{K_{t+2}(i)}{K_{t+1}(i)} - \frac{\varphi_k}{2}\left(\frac{K_{t+2}(i)}{K_{t+1}(i)} - 1\right)^2\right] \\ = \;& 1 + \varphi_k E_t \left(\frac{K_{t+1}(i)}{K_t(i)} - 1\right), \end{aligned} \tag{11}$$

and

$$\lambda_t(i) = \beta R_t E_t \left(\frac{\lambda_{t+1}(i)}{\pi_{y,t+1}}\right), \tag{12}$$

where $\lambda_t(i)$ is the nonnegative Lagrange multiplier associated with the budget constraint, $q_t = Q_t/P_{y,t}$, and $\pi_{y,t+1}$ is the rate of inflation of finished goods.

2.1.1 The Wage Decision

In each period, nominal wages are adjusted with probability $1 - d_w$. The first-order condition with respect to $W_t(i)$ is

$$\widetilde{W}_t(i) = \frac{\sigma}{\sigma - 1} \frac{E_t \sum_{q=0}^{\infty} (\beta d_w)^q N_{t+q}(i)^{\eta+1}}{E_t \sum_{q=0}^{\infty} (\beta d_w)^q N_{t+q}(i) \lambda_{t+q}(i) \frac{1}{P_{y,t+q}}}. \tag{13}$$

At the symmetric equilibrium, the aggregate nominal wage is given by the following recursive equation:

$$W_t = \left[d_w W_{t-1}^{1-\sigma} + (1-d_w) \widetilde{W}_t^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \tag{14}$$

where $\widetilde{W}_t$ is the optimal or average nominal wage of workers who can revise their wages in period t.

2.2 Firms and Processing Stages

Firms producing at different stages are related by an input-output linkage. Producers also charge different prices at each stage. In each period, the nominal prices of finished goods have a probability d_y of staying constant, while the probability that the prices of intermediate goods stay put is d_z.

2.2.1 Final Stage

Final output, Y_t, is a composite of all finished goods $Y_t(j)$, $j \in (0,1)$ denoting a particular type of finished good,

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{\theta_y - 1}{\theta_y}} dj \right)^{\frac{\theta_y}{\theta_y - 1}},$$

where θ_y is the elasticity of substitution between finished goods.

Finished-good firm j maximizes profits, solving the following problem

$$\max_{Y_t(j)} P_{y,t} \left(\int_0^1 Y_t(j)^{\frac{\theta_y - 1}{\theta_y}} dj \right)^{\frac{\theta_y}{\theta_y - 1}} - \int_0^1 P_{y,t}(j) Y_t(j) dj.$$

where $P_{y,t}(j)$ is the nominal price of finished good j. Profit maximization yields the following first-order condition for the demand of finished good j,

$$Y_t(j) = \left(\frac{P_{y,t}(j)}{P_{y,t}} \right)^{-\theta_y} Y_t. \tag{15}$$

The price index $P_{y,t}$ is given by

$$P_{y,t} = \left(\int_0^1 P_{y,t}(j)^{1-\theta_y} dj \right)^{\frac{1}{1-\theta_y}}.$$

2.2.2 Intermediate Stage

Output at the intermediate stage of processing, Z_t, is a composite of all intermediate goods $Z_t(l)$, $l \in (0,1)$ denoting a type of intermediate good,

$$Z_t = \left(\int_0^1 Z_t(l)^{\frac{\theta_z - 1}{\theta_z}} dl \right)^{\frac{\theta_z}{\theta_z - 1}},$$

where θ_z is the elasticity of substitution between intermediate goods.

Profit maximization yields the following first-order condition for the demand of intermediate good l,

$$Z_t(l) = \left(\frac{P_{z,t}(l)}{P_{z,t}} \right)^{-\theta_z} Z_t, \tag{16}$$

where $P_{z,t}(l)$ is the price of intermediate good l, and $P_{z,t}$ represents the price index of intermediate goods. Price index $P_{z,t}$ is given by

$$P_{z,t} = \left(\int_0^1 P_{z,t}(l)^{1-\theta_z} dl \right)^{\frac{1}{1-\theta_z}}.$$

2.2.3 Finished–Good Firms

The production of finished good j requires the use of labor $N_{y,t}(j)$, capital $K_{y,t}(j)$, and intermediate inputs, $Z_t(j)$. Finished-good firms utilize the following constant returns to scale (CRS) technology

$$Y_t(j) = Z_t(j)^{\phi} \left[A_{y,t} K_{y,t}(j)^{\alpha_y} N_{y,t}(j)^{1-\alpha_y} \right]^{1-\phi}, \tag{17}$$

where the parameter $\phi \in (0,1)$ measures the elasticity of output with respect to intermediate inputs, and $\alpha_y \in (0,1)$.

The final-stage productivity shock, $A_{y,t}$, follows a log-difference stationary process

$$\log(A_{y,t}) = (1 - \rho_{A,y}) \log(A_y) + \rho_{A,y} \log(A_{y,t-1}) + \varepsilon_{y,t}, \tag{18}$$

where $\varepsilon_{y,t}$ is a mean–zero, iid normal process that is independent, with a standard error σ_y. Firms can adjust the labor input only by paying a cost. Labor adjustment costs are measured as a proportional loss of final output:

$$LAC_{y,t}(j) = \frac{\varphi_y}{2} \left(\frac{N_{y,t}(j)}{N_{y,t-1}(j)} - 1 \right)^2 Y_t, \qquad \varphi_y \geqslant 0, \tag{19}$$

where $\varphi_y > 0$ is the cost parameter.

Firms are price-takers in the markets for inputs and monopolistic competitors in the markets for products. Nominal prices at each stage of processing are chosen optimally in a randomly staggered fashion.

The finished-good firm j solves the following problem :

$$\max_{\{K_{y,t}(j),N_{y,t}(j),Z_t(j),P_{y,t}(j)\}} E_t \sum_{q=0}^{\infty} (\beta d_y)^q \frac{\lambda_{t+q}}{\lambda_t} \frac{D_{y,t+q}(j)}{P_{t+q}},$$

subject to:

$$D_{y,t}(j) = P_{y,t}(j)Y_t(j) - Q_t K_{y,t}(j) - W_t N_{y,t}(j) - P_{z,t} Z_t(j) - P_{y,t} LAC_{y,t}(j),$$

and equations (15) and (17).

The corresponding first–order conditions are:

$$\begin{aligned} w_t &= (1-\alpha_y)(1-\phi)\zeta_{y,t}(j)\frac{Y_t(j)}{N_{y,t}(j)} - \varphi_y \frac{Y_t}{N_{y,t-1}(j)}\left(\frac{N_{y,t}(j)}{N_{y,t-1}(j)} - 1\right) + \\ & \quad \beta\varphi_y E_t \frac{\lambda_{t+1}}{\lambda_t}\frac{Y_{t+1}}{N_{y,t}(j)}\frac{N_{y,t+1}(j)}{N_{y,t}(j)}\left(\frac{N_{y,t+1}(j)}{N_{y,t}(j)} - 1\right), \end{aligned} \tag{20}$$

$$q_t = \alpha_y(1-\phi)\zeta_{y,t}(j)\frac{Y_t(j)}{K_{y,t}(j)}, \tag{21}$$

and

$$p_{z,t} = \phi\zeta_{y,t}(j)\frac{Y_t(j)}{Z_t(j)}, \tag{22}$$

where $w_t = \frac{W_t}{P_{y,t}}$ is the real wage, $p_{z,t} = \frac{P_{z,t}}{P_{y,t}}$ is the relative price of intermediate goods to final goods, and $\zeta_{y,t}(j)$ is firm $j's$ real marginal cost .

2.2.4 Finished-Good Pricing

The first-order condition for $P_{y,t}(j)$ is:

$$\tilde{P}_{y,t}(j) = \frac{\theta_y}{\theta_y - 1}\frac{E_t\sum_{q=0}^{\infty}(\beta d_y)^q\frac{\lambda_{t+q}}{\lambda_t}\zeta_{y,t}(j)Y_{t+q}(j)}{E_t\sum_{q=0}^{\infty}(\beta d_y)^q\frac{\lambda_{t+q}}{\lambda_t}Y_{t+q}(j)\frac{1}{P_{y,t+q}}}. \tag{23}$$

At the symmetric equilibrium, the average price of finished goods is

$$P_{y,t} = \left[d_y P_{y,t-1}^{1-\theta_y} + (1-d_y)\tilde{P}_{y,t}^{1-\theta_y}\right]^{\frac{1}{1-\theta_y}}, \tag{24}$$

where $\tilde{P}_{y,t}$ is the optimal or average price of finished-good firms that are allowed to revise their prices in period t.

2.2.5 Intermediate–Good Firms

Intermediate-good firm l rents capital $K_{z,t}(l)$ and hires workers $N_{z,t}(l)$, using a CRS technology to produce intermediate good $Z_t(l)$,

$$Z_t(l) = A_{z,t} K_{z,t}(l)^{\alpha_z} N_{z,t}(l)^{1-\alpha_z}, \tag{25}$$

where $\alpha_z \in (0,1)$.

The intermediate-stage productivity shock, $A_{z,t}$, follows a log-difference stationary process

$$\log(A_{z,t}) = (1-\rho_{A,z})\log(A_z) + \rho_{A,z}\log(A_{z,t-1}) + \varepsilon_{z,t}, \tag{26}$$

where $\varepsilon_{z,t}$ is a mean–zero, iid normally distributed process with a standard error σ_z.

Intermediate-good firms also have to pay a cost to adjust hours:

$$LAC_{z,t}(l) = \frac{\varphi_z}{2}\left(\frac{N_{z,t}(l)}{N_{z,t-1}(l)} - 1\right)^2 Z_t, \qquad \varphi_z \geqslant 0, \tag{27}$$

where φ_z is the cost parameter.

The intermediate-good firm l solves the following problem:

$$\max_{\{K_{z,t}(l), N_{z,t}(l), P_{z,t}(l)\}} E_t \sum_{q=0}^{\infty} (\beta d_z)^q \frac{\lambda_{t+q}}{\lambda_t} \frac{D_{z,t+q}(l)}{P_{y,t+q}},$$

subject to:

$$D_{z,t}(l) = P_{z,t}(l) Z_t(l) - Q_t K_{z,t}(l) - W_t N_{z,t}(l) - P_{z,t} LAC_{z,t}(l),$$

and equations (16) and (25).

The first–order conditions corresponding to this problem are:

$$\begin{aligned} w_t \quad = \quad & (1-\alpha_z)\zeta_{z,t}(l)\frac{Z_t(l)}{N_{z,t}(l)} - \varphi_z \frac{p_{z,t} Z_t}{N_{z,t-1}(j)}\left(\frac{N_{z,t}(l)}{N_{z,t-1}(l)} - 1\right) + \\ & \beta\varphi_z E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{p_{z,t+1} Z_{t+1}}{N_{z,t}(j)} \frac{N_{z,t+1}(l)}{N_{z,t}(l)}\left(\frac{N_{z,t+1}(l)}{N_{z,t}(l)} - 1\right), \end{aligned} \tag{28}$$

and,

$$q_t = \alpha_z \zeta_{z,t}(l) \frac{Z_t(l)}{K_{z,t}(l)}. \tag{29}$$

where $\zeta_{z,t}(l)$ is firm $l's$ real marginal cost.

2.2.6 Intermediate-Good Pricing

The first-order condition with respect to $P_{z,t}(l)$ is

$$\tilde{P}_{z,t}(l) = \frac{\theta_z}{\theta_z - 1} \frac{E_t \sum_{q=0}^{\infty} (\beta d_z)^q \frac{\lambda_{t+q}}{\lambda_t} \zeta_{z,t}(l) Z_{t+q}(l)}{E_t \sum_{q=0}^{\infty} (\beta d_z)^q \frac{\lambda_{t+q}}{\lambda_t} Z_{t+q}(l) \frac{1}{P_{z,t+l}}}. \tag{30}$$

At the symmetric equilibrium, the average price of intermediate goods is

$$P_{z,t} = \left[d_z P_{z,t-1}^{1-\theta_z} + (1 - d_z) \tilde{P}_{z,t}^{1-\theta_z} \right]^{\frac{1}{1-\theta_z}}, \tag{31}$$

where $\tilde{P}_{z,t}$ is the optimal or average price of intermediate-good firms that are allowed to revise their prices in period t.

2.3 The Policy Rule

The monetary authority sets the short-term nominal interest rate, R_t, in response to finished-good inflation and final output, both measured in deviations from their respective steady-state values π^* and y^*. The policy rule also includes an interest-rate smoothing term and a serially correlated policy shock. The rule is

$$\log\left(\frac{R_t}{R^*}\right) = \rho_R \left(\frac{R_{t-1}}{R^*}\right) + (1 - \rho_R) \left[\rho_\pi \log\left(\frac{\pi_{y,t}}{\pi_y^*}\right) + \rho_y \log\left(\frac{y_t}{y^*}\right) \right] + v_t, \tag{32}$$

where

$$v_t = \rho_v v_{t-1} + \varepsilon_{v,t}. \tag{33}$$

R^* is the steady-state gross nominal interest rate, and $\varepsilon_{v,t}$ is a mean–zero, iid normally distributed process with a standard error σ_v.

2.4 Closing the Model

At the symmetric equilibrium, the market-clearing conditions are:

$$K_t = K_{y,t} + K_{z,t}, \tag{34}$$

where $K_{y,t} = \int K_{y,t}(j)dj$ and $K_{z,t} = \int K_{z,t}(l)dl$,

$$N_t = N_{y,t} + N_{z,t}, \tag{35}$$

where $N_{y,t} = \int N_{y,t}(j)dj$ and $N_{z,t} = \int N_{z,t}(l)dl$,

$$Y_t = C_t + I_t + CAC_t + LAC_{y,t} + LAC_{z,t}, \tag{36}$$

and

$$M_t - M_{t-1} = T_t. \tag{37}$$

2.5 Equilibrium

An equilibrium consists in a set of allocations $\{C_t, N_t, B_t, M_t/P_{y,t}, K_{t+1}, Y_t, I_t, Z_t, K_{k,t}, N_{k,t}, \pi_{k,t}, \pi_{w,t}, p_{z,t}, w_t, \zeta_{k,t}, q_t, R_t\}_{t=0}^{\infty}$, for $k = y, z$, that satisfies the following conditions: (i) the household's allocations solve its utility maximization problem; (ii) each finished-good producer's allocations and price solve its profit maximization problem taking the wage and all prices but its own as given; (iii) each intermediate-good producer's allocations and price solve its profit maximization problem; and (iv) all markets clear.

3 Econometric Procedure

3.1 Estimation

The model is solved through log-linearization of its equilibrium conditions around a symmetric steady state in which all variables are constant. The steady-state rate of inflation of finished goods is set equal to one. The linearized system yields the following state space representation:

$$\mathcal{X}_t = \mathbf{A}\mathcal{X}_{t-1} + \mathbf{B}\epsilon_t, \tag{38}$$

$$\mathcal{Y}_t = \mathbf{C}\mathcal{X}_t, \tag{39}$$

where $\mathcal{X}_t$ is a vector that keeps track of the model's predetermined and exogenous variables, and $\mathcal{Y}_t$ is a vector that includes the remaining endogenous variables. The Kalman filter is used to evaluate the likelihood function $\mathcal{L}(Y^T|\Theta)$, associated with the state-space solution. Prior to the estimation, we define the following vector of observables:

$$\mathcal{Z}_t = \begin{bmatrix} \hat{c}_t & \widehat{y}_t & \widehat{R}_t & \hat{\pi}_{y,t} & \hat{y}_t - \hat{n}_t & \hat{w}_t \end{bmatrix}',$$

which is composed of real consumption, final output, the nominal interest rate, the rate of inflation of finished goods, the average productivity of labor, and real wages, each variable being measured in percentage-deviations from its steady-state value.

Since the model has four structural shocks, the number of variables to use in the estimation to avoid stochastic singularity should in principle be limited to four. However, the number of variables can be increased through the addition of measurement errors [see also Altug (1989), Sargent (1989), McGrattan (1994), Hall (1996), and Ireland (2004b)]. Hence, we augment the model with a vector of two measurement errors, e_t. The system of equations for the selected variables is

$$\mathcal{Z}_t = \mathbf{K} \begin{pmatrix} \mathcal{X}_t \\ \mathcal{Y}_t \end{pmatrix} + \mathbf{L} \begin{pmatrix} \epsilon_t \\ e_t \end{pmatrix}, \tag{40}$$

where the matrices $\mathbf{K}$ and $\mathbf{L}$ are obtained after selecting the appropriate variables in $\mathcal{X}_t$, $\mathcal{Y}_t$, and the vector of errors.[5] The measurement errors, which are assumed to be independent from the structural shocks, follow the autoregressive process:

$$e_{t+1} = \mathbf{M} e_t + \upsilon_t, \tag{41}$$

$$E\left(\upsilon_t \upsilon_t'\right) = \Sigma_\upsilon, \tag{42}$$

where $\mathbf{M}$ and Σ_υ are diagonal matrices.

3.2 Data

The model is estimated with U.S. quarterly data for the period 1960:I to 2004:IV. The nominal interest rate is measured by the 3-month Treasury Bill Rate. The rate of inflation of finished goods is measured by the quarterly rate of change of the consumer price index. Consumption is the sum of real personal consumption expenditures on nondurable goods and services. Output is measured by the sum of total personal consumption expenditures and private fixed investment. The real wage is the ratio of the nonfarm business sector compensation to the consumer price index. All series, except the nominal interest rate, are seasonally adjusted. Consumption, output and hours worked are converted into per capita terms after each variable has been divided by the civilian population.

[5]The two measurement errors are associated to inflation rate of finished goods and labor productivity. Once the parameters of the model are estimated, we compute the variance decomposition to make sure that the non-structural shocks explain only a small fraction of the variability of finished-good inflation and labor productivity.

All series, except the interest rate and the rate of inflation, are logged and detrended using the HP filter.

3.3 Calibration

It can be difficult, when estimating relatively large structural models by maximum likelihood, to obtain sensible estimates of all the structural parameters either because some parameters are not easy to identify or because the optimization algorithm fails to locate the maximum due the complexity of the objective function, so that the algorithm breaks down. To deal with this issue, we calibrate some parameters prior to the estimation. First, the subjective discount rate β is set to 0.995, implying a steady-state annual real interest rate of 2%. The value assigned to μ, the weight on leisure in the utility function, implies that the representative household spends approximately one third of its time working in the steady state. The rate of depreciation of physical capital is 0.025. The parameters θ_y and θ_z determining the steady-state markups of prices over marginal costs are each assigned a value of 8, implying a steady-state markup of 14% at each stage [see also Basu 1995 and Huang, Liu and Phaneuf (2004)].[6] The parameter σ, denoting the elasticity of substitution between labor skills, is taken to be 6.0, which is in line with the microeconomic evidence of Griffin (1992) and the macroeconomic estimates obtained by Ambler, Guay and Phaneuf (2006).

4 Empirical Results

4.1 The Benchmark Model

We call *benchmark model* the one which features all the theoretical ingredients that were described in Section 2. This model is driven by four structural shocks: a preference shock, a shock to final-stage technology, a shock to intermediate-stage technology and a monetary policy shock. We seek to estimate the following group of structural parameters $\{\rho_{A,z}, \rho_{A,y}, \rho_\kappa, \rho_\upsilon, \sigma_{A,z}, \sigma_{A,y}, \sigma_\kappa, \sigma_\upsilon, b, \gamma, \eta, \alpha_z, \phi, \alpha_y, d_z, d_y, d_w, \varphi_k, \varphi_z, \varphi_y, \rho_R, \rho_\pi, \rho_y\}$.

Table 1 displays the parameter estimates of the benchmark model. The structural parameters of the model are estimated quite precisely. The point estimate of γ is 0.0701, implying an interest elasticity of money demand of -0.0754, consistent with the evidence in Ireland (2003) and Kim (2000). The parameter b, which determines the relative importance of consumption with respect to

[6] Basu and Fernald (2002) find that the steady-state markup is about 5% when factor utilization rates are controlled for, while it is about 12% without correction for factor utilization. The value proposed by Rotemberg and Woodford (1997) is 20% without correction for factor utilization.

real balances, is 0.0744. The point estimate $\eta = 0.8831$ implies that the elasticity of labor supply is about 1.13, which is consistent with the evidence reported in Mulligan (1998).

The point estimates of $\rho_{A,y}$, $\rho_{A,z}$, $\sigma_{A,y}$ and $\sigma_{A,z}$ suggest that the shock to intermediate-stage technology is somewhat more persistent than the shock to final-stage technology and has a slightly larger innovation. The point estimate of α_z, associated with the stock of physical capital in the production function of intermediate-good firms, is 0.3407. The parameter ϕ, associated with the intermediate inputs in the production function of finished-good firms, is 0.2416. The point estimate of α_y is 0.13. The point estimates of ϕ and α_y imply a share of hours worked in the production of finished goods of about 0.66.

The probability that the prices of finished goods stay put in each period, d_y, is 0.6561, implying an average duration of price contracts at the final stage of 2.9 quarters. The corresponding probability for nominal prices at the intermediate stage of processing, d_z, is 0.6992, implying that the average duration of contracts is 3.3 quarters. These estimates suggest a moderate amount of nominal price stickiness at both stages of processing. Wage contracts last longer than price contracts, with an estimated probability d_w of 0.8461, corresponding to an average duration of 6.5 quarters.

The length of price contracts implied by our model estimates is broadly consistent with the evidence reported in Christiano, Eichenbaum and Evans (2005) who find that nominal prices are adjusted once every 2.5 quarters on average.[7] However, they are shorter than in the sticky-price models of Galí and Gertler (1999) and Eichenbaum and Fisher (2004) who find that contracts last six quarters on average, or in the model of Smets and Wouters (2003) in which prices are adjusted only once every nine quarters on average. Relying on microeconomic evidence, Bils and Klenow (2004) argue that nominal prices adjust more frequently than our point estimates suggest.[8]

The estimated parameter for the adjustment cost of physical capital, $\varphi_k = 9.5827$, is statistically significant and allows a reasonable match of the volatility of investment by the model. The point estimates of the labor adjustment-cost parameters are $\varphi_y = 5.7406$ and $\varphi_z = 3.3746$, respectively, meaning that labor seems more costly to adjust at the final stage than at the intermediate stage of processing.

The point estimate $\rho_\pi = 1.4702$ in the monetary policy rule is close to the value of 1.5 proposed by Taylor (1993). The parameter ρ_y is close to zero and statistically insignificant. We find no

[7] Christiano *et al.* (2005) estimate a one-stage model featuring sticky nominal prices, sticky nominal wages, and several other real frictions. In their model, aggregate fluctuations are driven solely by monetary shocks.

[8] It is difficult, however, to establish a direct comparison between our evidence and theirs. For example, Bils and Klenow (2004) examine the frequency of price changes for 350 categories of goods and services covering about 70% of consumer spending between the years 1995 and 1997.

strong evidence of interest-rate smoothing, our point estimate of ρ_R being 0.0918 and statistically insignificant, or of persistence in the unsystematic intervention of the monetary authority, with a point estimate of ρ_v of only 0.1571.

4.2 Sources of Business Cycles

Which shock contributes most to aggregate fluctuations? To answer this question, we first look at the variance decomposition of several variables over the infinite horizon. These results are presented in Table 2. The intermediate-stage technology shock ε_z explains 72.3% of the variance of final output, 44.9% of the variance of total hours, 67% of the variance of consumption and 80.7% of the variance of investment. It also explains 76.2% of the variance of intermediate-stage hours and 84.2% of the variance of intermediate-stage output. Notice that, while ε_z contributes 37.4% of the variance of final-stage hours, it explains a much higher percentage of the cyclical variability of final output. This suggests that the use of intermediate inputs by finished-good firms substantially magnifies the effect of ε_z on the variance of final output.

The policy shock ε_v explains only 14.7% of the variability of final output, and a larger proportion of the volatility of total hours at 21.9%. This shock, however, is an important determinant of inflation variability, contributing to 71.6% of the total volatility of finished-good inflation and 89.5% of the volatility of intermediate-good inflation. Still, the two technology shocks ε_z and ε_y explain a non negligible fraction of the variability of finished-good inflation when their effects are combined. Preference shocks explain a very small percentage of the variance of all variables, except for final-stage hours with 20.2% and total hours with 13.6%.

Table 3 focuses on the forecast error variance decompositions of Y_t, Z_t, $\pi_{y,t}$ and $\pi_{z,t}$ at a shorter horizon of one to forty quarters. Intermediate-stage technology shocks clearly are the driving force at business cycle frequencies (of say, one to twelve quarters), explaining 31%, 52.1%, 62.3% and 65.7% of the one, four-, eight-, and twelve-quarter ahead forecast error variance, respectively. Once their effects are combined, technology shocks account for 47%, 59%, 68% and 72% of the variability of final output at the same horizons. The policy shock has a significant impact at an horizon of one quarter, contributing to 50.7% of the variability of final output, but this percentage rapidly declines to 35.8%, 24.7% and 20.2% at the four-, eight-, and twelve-quarter horizon, respectively. However, the policy shock is the dominant source of inflation variability at all horizons. Finally, preference shocks explain 8% or less of the variance of final output and less than 4 percent of the variability of finished-good inflation at all horizons.

4.3 The Effects of Stage-Specific Technology Shocks

Technology shocks have very different effects on aggregate variables depending on the source of technological change. Figure 1 displays the impulse-response functions of several variables to a positive one percent intermediate-stage technology shock. A positive ε_z has a strong impact on final output, total investment and total hours worked. More precisely, final output rises persistently, with an initial response of 0.57% and a peak increase of 1.12% in the sixth quarter. Following the same shock, investment rises, with an initial response of 2.18% and a peak increase of 3.49% in the fourth quarter. Total hours rise by 0.39% initially, reach a maximum increase of 1.01% in the fifth quarter, and then gradually return to their preshock level. The rise in consumption is somewhat smaller, with an initial response of 0.38% and a maximum increase of 0.84% in the sixth quarter. Hence, ε_z produces persistent, hump-shaped responses of final output, consumption, investment and total hours. Thus, the benchmark model generates rich business-cycle dynamics in response to an intermediate-stage technology shock, and is therefore able to meet the criterion of evaluating a model's performance proposed by King, Plosser and Rebelo (1988) and Cogley and Nason (1995), among others.

The impact of ε_z on intermediate-stage hours N_z and intermediate-stage output Z is of course stronger. That is, following a positive ε_z, intermediate-stage output rises persistently, with an initial response of 1.54% and a peak response of about 2.52% in the seventh quarter. The increase in output remains above 1% even after forty quarters. Intermediate-stage hours also rise significantly, with an initial response of 0.68% and a peak response of about 2.03% in the seventh quarter. The strong impact of ε_z on both N_z and Z results mostly from two factors. First, intermediate goods become cheaper following an intermediate-stage technology improvement, inducing a sharp, persistent, hump-shaped decline in the relative price of intermediate goods to final goods, p_z. Second, real wages fall initially, which further stimulates N_z.

This shock also affects the rates of inflation of finished goods and intermediate goods quite differently. It has a direct impact on the real marginal costs of intermediate-good firms, inducing a persistent decline in the rate of inflation of intermediate goods, π_z. In contrast, the rate of inflation of finished goods, π_y, initially increases by 0.22%. The increase in π_y is attributed mostly to the rise in the real rental rate of capital r^k and to the increase in labor adjustment costs which is caused by the boom in final-stage hours. The nominal interest, which is linked to the rate of inflation of finished goods through the Taylor rule, rises on impact and then begins to fall after three periods.

Figure 2 displays the impulse responses to a positive one-percent final-stage technology shock ε_y. A positive one-percent shock ε_y is followed for a few periods by a relatively small increase in final output, consumption and investment. This shock does not have the strong expansionary effects of its intermediate-stage counterpart. A positive shock ε_y even generates a persistent, hump-shaped *decline* in final-stage and total hours. It leads to a relatively small increase in relative price of intermediate goods to final goods, with an initial increase of p_z of 0.32% and a peak-response of 0.60% in the fifth quarter. Also, unlike ε_z which makes real wages fall initially, a positive ε_y produces a persistent, hump-shaped rise in real wages. The relatively small increase in p_z and the rise in real wages lead to a fall in both Z and N_z.

Thus, a key feature of the benchmark model is that technology shocks may have either an expansionary or contractionary effect on hours worked depending on the source of technological change. In contrast, standard RBC models predict that hours rise, while sticky-price models predict that they will likely fall if nominal price rigidity is combined to a weakly accommodative monetary policy [e.g., Galí (1999)].[9]

What are the key theoretical ingredients behind our main findings? We try first to answer this question by shutting down some channels in the benchmark model while keeping the other estimated parameters unchanged. Later, we estimate a variant of model that features only one stage of production (the final stage) and nominal rigidities, and a second variant that incorporates the two stages of processing while assuming perfectly flexible wages and prices.

We first look at the role of the input-output linkage by comparing the impulse responses implied by the benchmark model and those obtained after imposing an arbitrarily small value of ϕ (i.e. $\phi = 0.01$). Figure 3 reports the impulse responses for some selected variables. With a small ϕ, finished-good firms are almost completely insulated from the intermediate stage. Both Z and N_z still rise significantly following a positive intermediate-stage technology shock. But, the boom in intermediate-stage output Z is weakly transmitted to the final stage since ϕ is small. Thus, final output is almost unaffected. These results suggest that the input-output linkage between firms producing at different stages is a strong channel that propagates the effects of intermediate-stage technology shocks. Meanwhile, the effects of final-stage technology shocks on final output, total hours, consumption and investment are almost insensitive to a change in ϕ.

Next, we look at the role of nominal rigidities. First, we focus on nominal wage rigidity by imposing $d_y = d_z = 0$. The results are presented in Figure 4. Our main findings are little affected by assuming that nominal prices are perfectly flexible. In particular, a final-stage technology improvement still is followed by a persistent, hump-shaped decline in hours worked. Hence, in our

[9]However, as our sensitivity analysis below clearly shows, sticky prices is not the main reason why in our two-stage model a final-stage technology shock yields a fall in hours.

two-stage framework, sticky prices are not a key factor leading to a fall in hours following a positive shock ε_y.

Alternatively, we impose $d_w = 0$, while assuming sticky nominal prices. The results are presented in Figure 5. Nominal wage rigidity clearly is a more important factor for our main findings. With perfectly flexible nominal wages, a positive shock ε_z now induces an initial increase in real wages. With real wages rising, the increase in N_z is smaller than under sticky wages and the rise in N_y obtained with nominal wage rigidity disappears. Therefore, the increase in total hours is smaller, and so is the rise in final output. Notice that following a positive shock ε_y, there is no persistent decline in final-stage hours and total hours.

Finally, we look at the effect of costly labor adjustment on our results by imposing $\varphi_y = \varphi_z = 0$. Figure 6 shows that labor adjustment costs help obtaining hump-shaped impulse responses of final output, final-stage hours, total hours, consumption and investment following an intermediate-stage technology shock ε_z. However, labor adjustment costs have no strong impact on the effects of ε_y.

4.4 Business Cycle Statistics

In the literature on stochastic DGE models, an important criterion of evaluating a model's performance is to look at a fairly comprehensive set of business-cycle statistics within a single model. Table 4 reports some business-cycle facts for the postwar period and compares them with those predicted by the benchmark model. All series are detrended using the HP filter. The model does well in matching the size of fluctuations in investment, hours worked, real wages and productivity relative to fluctuations in output. It also correctly reproduces the relative variability of CPI-inflation to PPI-inflation. Overall, the benchmark model provides an accurate description of the size of economic fluctuations during the postwar era.

Turning to comovments, the benchmark model correctly predicts that consumption, investment and hours worked are all highly correlated with output. It also implies that the average labor productivity is mildly procyclical and that finished-good inflation and intermediate-good inflation are both highly correlated, as found in the data.

The model also provides a successful account of some critical comovements. In particular, it predicts that real wages are weakly procyclical–consistent with the Dunlop-Tarshis observation–despite the fact that technology shocks are the main source of fluctuations in our model. Specifically, the correlation between output and the real wages is 0.25 according to the benchmark model, while it is 0.37 in the data. The benchmark model is also consistent with the modern reincarnation of the Dunlop-Tarshis observation–the near-zero correlation between hours and productivity [e.g., Christiano and Eichenbaum (1992)]. The model predicts a correlation between hours and productivity of -0.116, while the actual correlation is -0.054.

Why is the benchmark model able to explain these critical comovements? Let us consider the correlation between hours and productivity. The benchmark model predicts that the two technology shocks affect hours worked very differently, ε_z producing a rise in hours while ε_y leads to a fall. Table 5 reports the correlations between hours and productivity conditional on the type of shock. The intermediate-stage technology shock generates a correlation between hours and productivity which is mildly positive at 0.507. However, the same correlation is -0.835 conditional on the final-stage technology shock. Hence, the correlation between hours and productivity conditional both on ε_z and ε_y is -0.025 according to the model, which is close to the actual correlation. These findings contrast sharply with those of standard RBC models that predict highly procyclical real wages and a correlation between hours and productivity which is both positive and high.[10]

4.5 Alternative Models

Our model's main driving mechanism can also be assessed by comparing the results obtained with the benchmark model with those of two model variants. *Model I* features only final-stage production, sticky nominal prices and sticky nominal wages. It is estimated under the following parameter restrictions: $\rho_{A,z} = \sigma_{A,z} = \alpha_z = \phi = d_z = \varphi_z = 0$. This model is driven by three structural shocks since the intermediate-stage technology shock is excluded from the model. It is also estimated with a vector of three measurement errors. This variant resembles new keynesian models.

Model II combines the two stages of processing, perfectly flexible prices at both stages of processing and perfectly flexible wages. Model II is estimated under the following parameter restrictions: $d_z = d_y = d_w = 0$. This model is closer in the spirit to RBC models. The parameter estimates of Model I and Model II are presented in Table 1.

Reported under the label "*Model I*", the point estimate of d_w is now 0.9250, implying excessively long wage contracts of 13.3 quarters on average. With a point estimate of d_y of 0.7325, nominal price contracts last 3.74 quarters on average. The other significant change in parameter estimates

[10]Christiano and Eichenbaum (1992) lower this correlation by incorporating a government spending shock which shifts the labor supply curve. They find that it is at best reduced to 0.58. Braun (1994) and McGrattan (1994) add shocks to the tax rates on capital and labor. This helps reduce the correlation between hours and productivity, but at the cost of significantly lowering the contribution of technology shocks to the cyclical variance of hours and output. Finally, Cho, Cooley and Phaneuf (1997) argue that combining technology and monetary shocks in a DGE model with sticky nominal wages could improve the correlation between hours and productivity.

concerns ρ_π in the monetary policy rule, which is now much higher than in the benchmark model (2.13 *vs* 1.47). The business-cycle statistics generated by Model I are reported in Table 4. Hours worked are too volatile relative to output, whereas the relative volatility of investment is much too low. Also, real wages are strongly countercylical, while the correlation between hours and productivity is both negative and strong. But more importantly, based on the likelihood ratio test (see the bottom of Table 1), the benchmark model is strongly preferred by the data to Model I.

Model II does not perform well either, facing many of the anomalies of standard RBC models. The relative volatility of hours is much too low. The relative volatility of real wages is much too high. Real wages and productivity are both strongly procyclical. The correlation between hours and productivity is both positive and high. Once more, based on the likelihood ratio test, the benchmark model is strongly preferred to Model II.

5 Conclusion

Real-business-cycle theory claims that technology shocks have accounted for the bulk of output variability at business cycle frequencies in the postwar U.S. economy. However, a recent of literature has questioned the empirical relevance of technology shocks as a source of aggregate fluctuations [e.g., Galí (1999), Basu, Fernald and Kimball (2004), and Christiano, Eichenbaum and Vigfusson (2004)]. Moreover, business cycle models in which technological change is the main driving force have been confronted to several problems, predicting for example highly procyclical real wages, and a correlation between hours and labor productivity which is strongly positive. Finally, this class of models has failed to generate interesting business-cycle dynamics.

We have proposed a framework in which firms realistically operate at two stages of processing: a stage of intermediate goods and a stage of finished goods. Firms at the two stages are related through an input-output linkage. A distinctive characteristic of our framework is that it allows changes in the pace of technology to take place at different stages of processing. An estimated version of model that features stage-specific nominal price rigidities and nominal wage rigidity suggests that shocks to intermediate-stage technology, the stage previously missing from DGE models, are the main source of business cycles.

Our analysis has helped identify two important factors propagating the effects of intermediate-stage technology shocks. One is the input-output linkage between firms at different stages, finished-good firms using intermediate inputs. The other is nominal wage rigidity. We have shown that technology shocks impact very differently on hours worked depending on the source of technological change. An intermediate-stage technology improvement drives hours up, whereas at the final stage, it drives hours down. Therefore, the two-stage framework accounts for the weak cyclical pattern in real wages and the near-zero correlation between hours and productivity.

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Table 1: Parameter Estimation Results

	Benchmak Model		Model I		Model II	
Parameter	Estimate	Std Error	Estimate	Std Error	Estimate	Std Error
$\rho_{A,y}$	0.8716	0.0177	0.8573	0.0090	0.8524	0.0145
$\rho_{A,z}$	0.9600	0.0711	$---$	$---$	0.9600	0.0014
ρ_v	0.1571	0.0335	0.6232	0.0366	0.3177	0.0302
ρ_κ	0.9512	0.0171	0.9108	0.0137	0.7188	0.0445
$\sigma_{A,y}$	0.0181	0.0007	0.0187	0.0010	0.0123	0.0008
$\sigma_{A,z}$	0.0197	0.0060	$---$	$---$	0.0086	0.0003
σ_v	0.0232	0.0020	0.0059	0.0003	0.0079	0.0002
σ_κ	0.0133	0.0006	0.0101	0.0010	0.0052	0.0002
ρ_R	0.0918	0.0767	0.0000	$---$	0.0363	0.0597
ρ_π	1.4702	0.0793	2.1285	0.0574	0.9984	0.0013
ρ_y	-0.0050	0.0060	-0.0122	0.0039	-0.0153	0.0020
d_w	0.8461	0.0079	0.9250	0.0313	$---$	$---$
d_y	0.6561	0.0256	0.7325	0.0063	$---$	$---$
d_z	0.6992	0.0539	$---$	$---$	$---$	$---$
φ_k	9.5827	0.6927	11.1243	0.2791	7.4139	0.5207
φ_y	5.7406	1.8588	2.4015	0.2979	5.9127	0.8304
φ_z	3.3746	1.1554	$---$	$---$	1.7069	0.5428
ϕ	0.2416	0.1312	$---$	$---$	0.4954	0.0245
b	0.0744	0.0389	0.2521	0.0595	0.1792	0.0198
γ	0.0701	0.1537	0.2974	0.0450	0.1131	0.0215
α_y	0.1300	0.0128	0.2564	0.0229	0.1333	0.0520
α_z	0.3407	0.0461	$---$	$---$	0.6110	0.0298
η	0.8831	0.4621	0.7120	0.3003	1.3040	0.0659
	$\mathcal{L} = 3567.40$		$\mathcal{L}_I = 3506.73$		$\mathcal{L}_{II} = 3387.33$	

Benchmark Model: Two-stage model with nominal rigidities; Model I: One-stage model with nominal rigidities; Model II: Two-stage model with flexible wages and prices

$\mathcal{L}$ denotes the maximized value of the log likelihood function. Then, the likelihood ratio statistic for the null hypothesis that the benchmark model is preferred to model I is equal to $2(\mathcal{L} - \mathcal{L}_I)$ that has a $\chi^2(4)$ distribution which gives a $p - value = 0.9999$.

Table 2: Benchmark Model: Variance Decomposition (Infinite Horizon)

Variable	$\varepsilon_{y,t}$	$\varepsilon_{z,t}$	$\varepsilon_{v,t}$	$\varepsilon_{\tau,t}$
Y_t	5.12	72.38	14.76	7.74
Z_t	3.86	84.22	11.46	0.46
C_t	4.83	67.03	14.54	13.60
I_t	5.46	80.69	13.35	0.50
N_t	19.64	44.91	21.87	13.57
$N_{y,t}$	19.48	37.36	22.94	20.23
$N_{z,t}$	10.39	76.19	11.54	1.88
w_t	12.93	70.06	14.41	2.60
$\frac{Y_t}{N_t}$	47.89	48.48	1.98	1.64
$\pi_{y,t}$	14.67	10.25	71.58	3.50
$\pi_{z,t}$	0.70	7.80	89.51	1.99

Table 3: Benchmark Model: Variance Decomposition (Different Horizons)

Final-goods sector output (Y_t)

Quarters ahead	$\varepsilon_{y,t}$	$\varepsilon_{z,t}$	$\varepsilon_{v,t}$	$\varepsilon_{\tau,t}$
1	15.64	30.95	50.63	2.78
4	6.52	52.07	35.81	5.60
8	5.80	62.26	24.71	7.22
12	6.29	65.69	20.20	7.82
20	6.00	68.48	17.38	8.13
40	5.36	71.21	15.45	7.98

Intermediary-goods sector output (Z_t)

Quarters ahead	$\varepsilon_{y,t}$	$\varepsilon_{z,t}$	$\varepsilon_{v,t}$	$\varepsilon_{\tau,t}$
1	0.41	20.41	78.43	0.75
4	3.61	46.07	49.18	1.14
8	6.21	61.42	31.27	1.09
12	6.57	68.64	23.86	0.93
20	5.66	75.98	17.64	0.71
40	4.35	82.13	12.99	0.53

Final-goods sector inflation ($\pi_{y,t}$)

Quarters ahead	$\varepsilon_{y,t}$	$\varepsilon_{z,t}$	$\varepsilon_{v,t}$	$\varepsilon_{\tau,t}$
1	12.09	6.73	77.98	3.19
4	14.99	6.49	74.84	3.67
8	14.39	8.10	73.97	3.52
12	14.51	9.45	72.59	3.44
20	14.73	9.92	71.89	3.44
40	14.69	10.08	71.70	3.50

Intermediary-goods sector inflation ($\pi_{z,t}$)

Quarters ahead	$\varepsilon_{y,t}$	$\varepsilon_{z,t}$	$\varepsilon_{v,t}$	$\varepsilon_{\tau,t}$
1	0.11	2.25	95.71	1.94
4	0.62	4.49	92.83	2.06
8	0.67	6.67	90.65	2.01
12	0.67	7.43	89.90	1.99
20	0.68	7.54	89.79	1.99
40	0.70	7.68	89.63	1.99

Table 4: Second-Order Unconditional Moments in the Benchmark and Alternative Models

Moments	US data	Benchmark Model	Model I	Model II
$\frac{std(C)}{std(Y)}$	0.5062 (0.0204)	0.8334	0.9104	0.7642
$\frac{std(I)}{std(Y)}$	2.8681 (0.0836)	2.6380	2.2057	2.1711
$\frac{std(N)}{std(Y)}$	0.8543 (0.0611)	0.9277	1.3100	0.2184
$\frac{std(w)}{std(Y)}$	0.6372 (0.0712)	0.7965	0.8293	1.0218
$\frac{std(Y/N)}{std(Y)}$	0.5152 (0.0815)	0.4965	0.6780	0.8115
$\frac{std(\pi_y)}{std(\pi_z)}$	0.5677 (0.0405)	0.5540	$---$	0.9687
$Corr(Y, C)$	0.9105 (0.2345)	0.9909	0.9875	0.9615
$Corr(Y, I)$	0.9630 (0.2645)	0.9341	0.8378	0.9287
$Corr(Y, N)$	0.8192 (0.1860)	0.8700	0.8612	0.8909
$Corr(Y, Y/N)$	0.5188 (0.1856)	0.3886	−0.1891	0.9925
$Corr(N, Y/N)$	−0.0535 (0.1033)	−0.1163	−0.6619	0.8287
$Corr(Y, w)$	0.3721 (0.1804)	0.2472	−0.6873	0.9710
$Corr(\pi_y, \pi_z)$	0.7503 (0.2694)	0.8055	$---$	0.8848

Table 5: Second-Order Conditional Moments in the Benchmark Model

Moments	US data	Benchmark Model				
		All shocks	ε_y	ε_z	Supply shocks	Demand shocks
$\frac{std(C)}{std(Y)}$	0.5062 (0.0204)	0.8334	0.8090	0.8020	0.8025	0.9320
$\frac{std(I)}{std(Y)}$	2.8681 (0.0836)	2.6380	2.7228	2.7854	2.7813	2.0698
$\frac{std(N)}{std(Y)}$	0.8543 (0.0611)	0.9277	1.8165	0.7308	0.8467	1.1644
$\frac{std(w)}{std(Y)}$	0.6372 (0.0712)	0.7965	1.2652	0.7836	0.8242	0.6927
$\frac{std(Y/N)}{std(Y)}$	0.5677 (0.0405)	0.4965	1.5179	0.4064	0.5536	0.1993
$\frac{std(\pi_y)}{std(\pi_z)}$	0.5677 (0.0405)	0.5540	2.5391	0.6351	0.9486	0.5019
$Corr(Y,C)$	0.9105 (0.2345)	0.9909	0.9951	0.9948	0.9948	0.9877
$Corr(Y,I)$	0.9630 (0.2645)	0.9341	0.9691	0.9691	0.9691	0.8057
$Corr(Y,N)$	0.8192 (0.1860)	0.8700	0.5493	0.9366	0.8329	0.9946
$Corr(Y,Y/N)$	0.5188 (0.1856)	0.3886	0.0015	0.7765	0.5325	−0.7931
$Corr(N,Y/N)$	−0.0535 (0.1033)	−0.1163	−0.8348	0.5065	−0.0250	−0.8523
$Corr(Y,w)$	0.3721 (0.1804)	0.2472	−0.2865	0.6248	0.5257	−0.8916
$Corr(\pi_y,\pi_z)$	0.7503 (0.2694)	0.8055	0.4612	−0.0250	0.0861	0.9568

Figure 1: Impulse Responses to an Intermediate-Stage Technology Shock

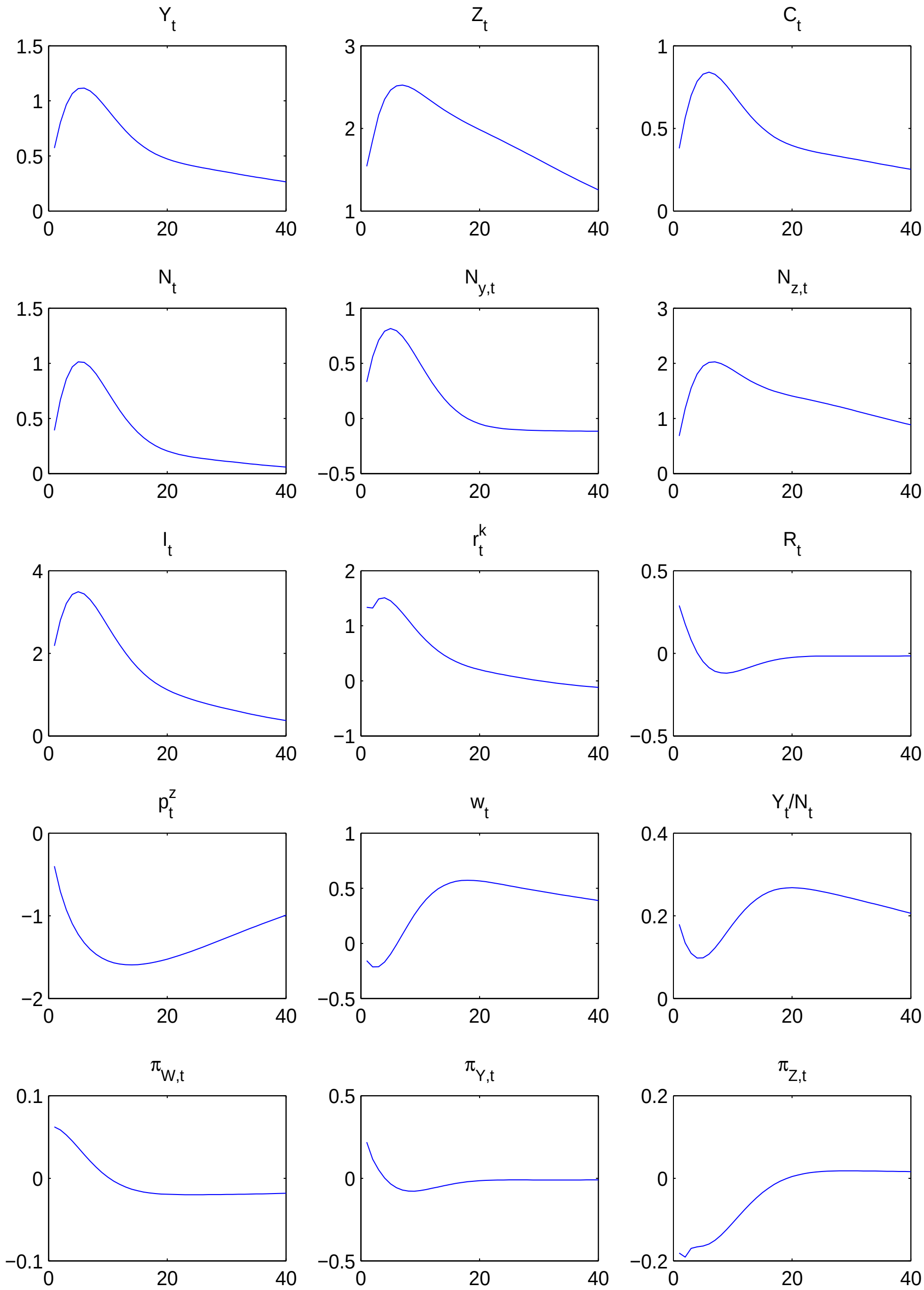


Figure 2: Impulse Responses to a Final-Stage Technology Shock

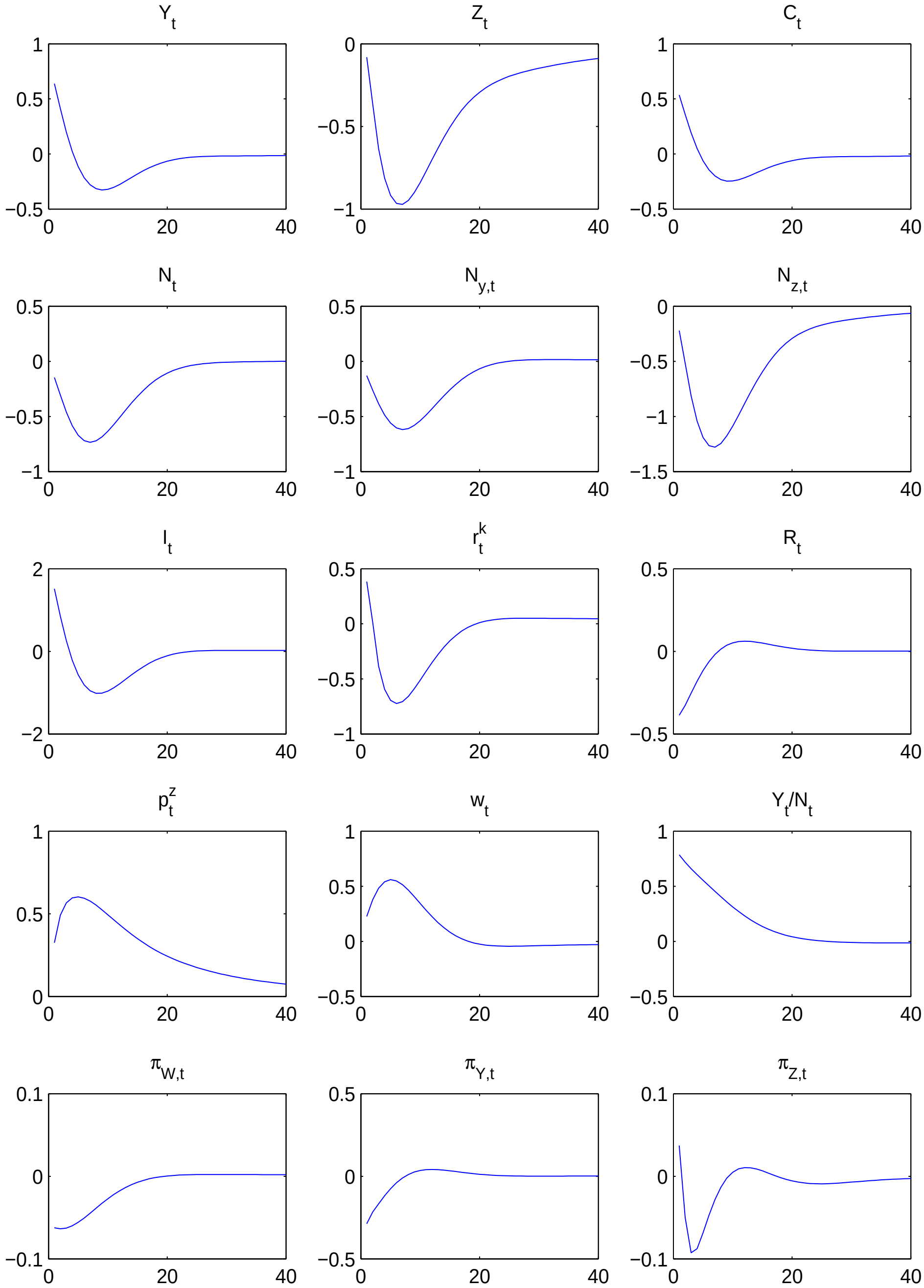
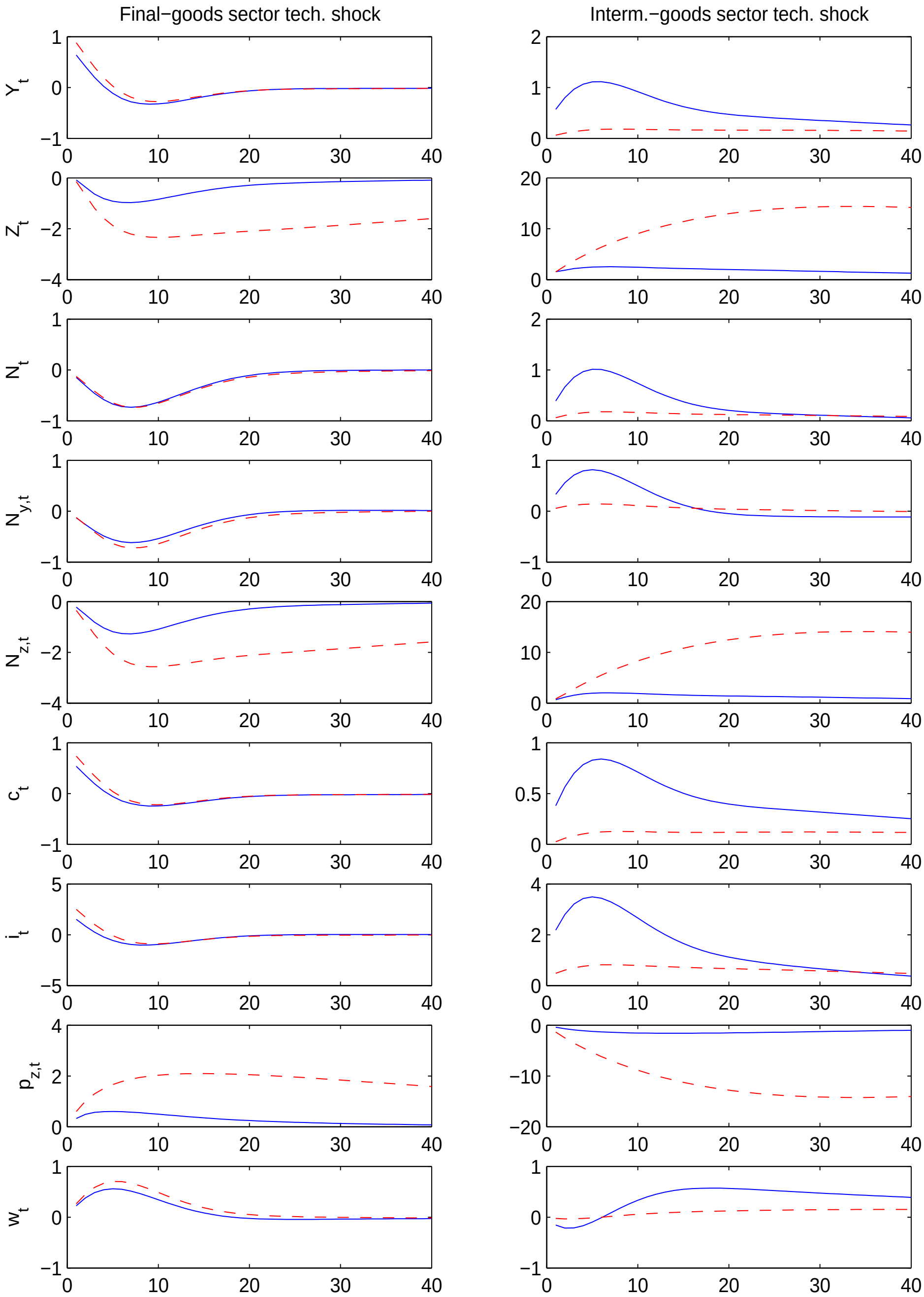
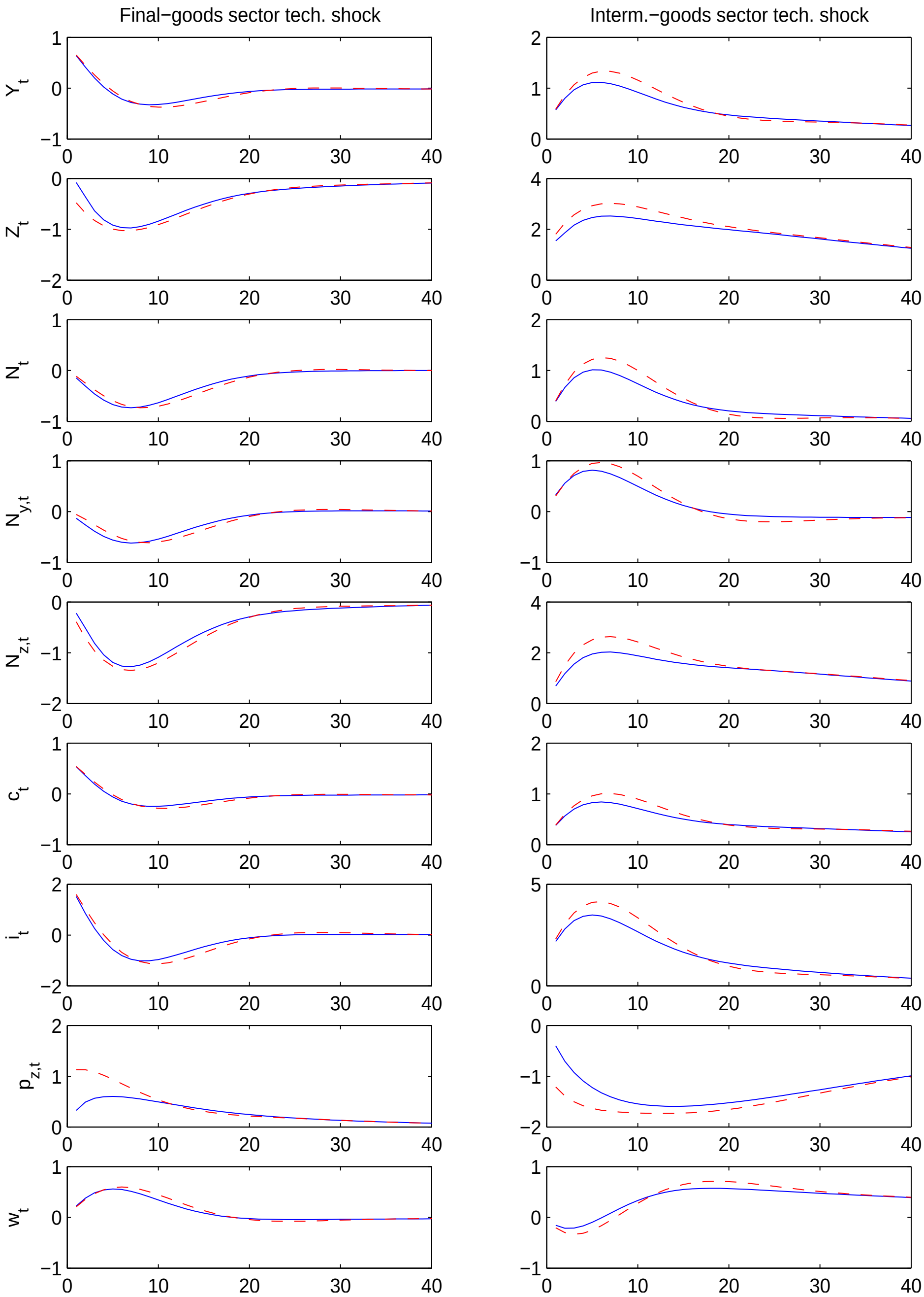


Figure 3: The Role of the Share of Intermediate Goods, ϕ



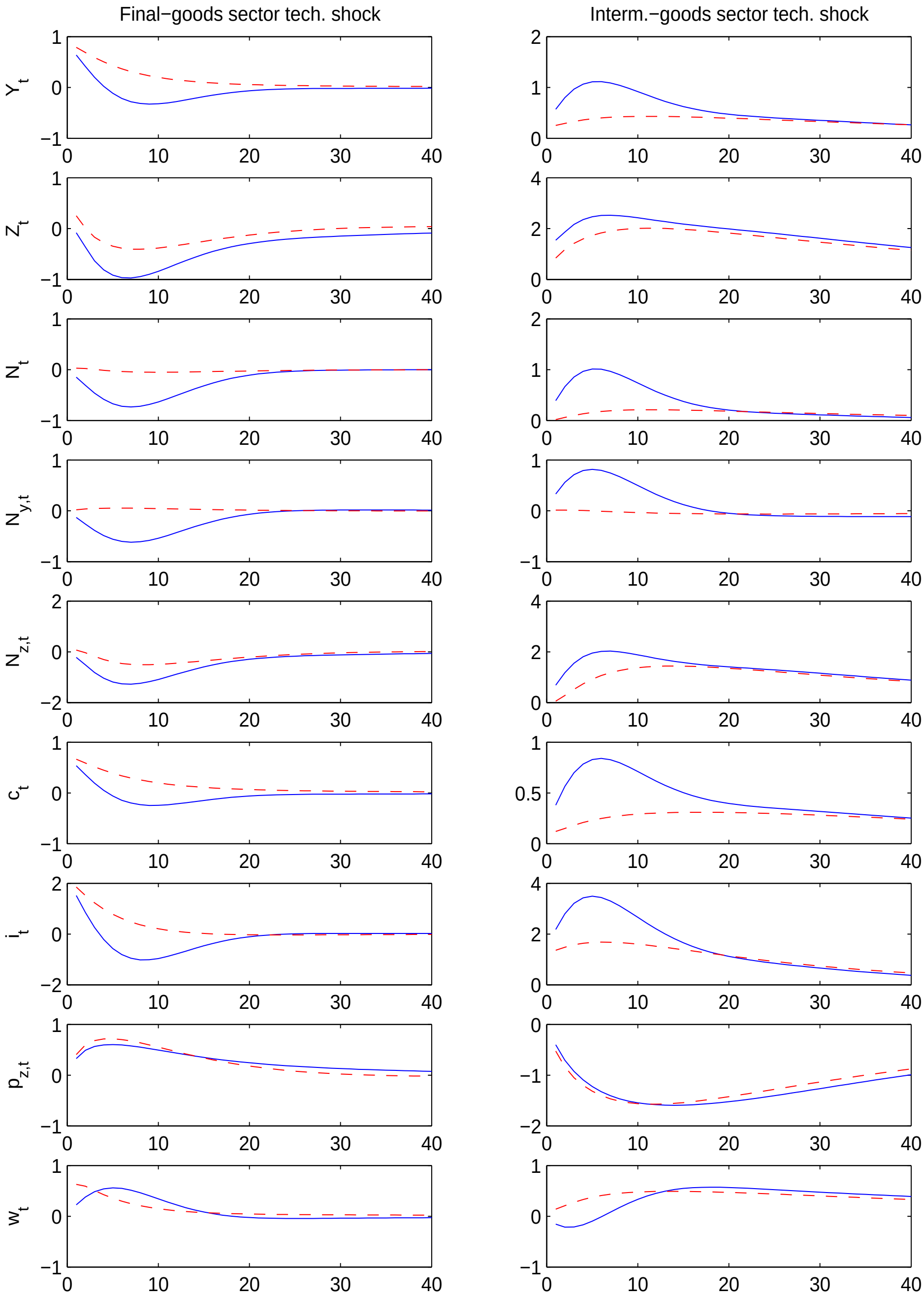
solid line: $\phi = 0.2416$; dashed line: $\phi = 0.0100$

Figure 4: The Role of Nominal Price Rigidities, d_y and d_z



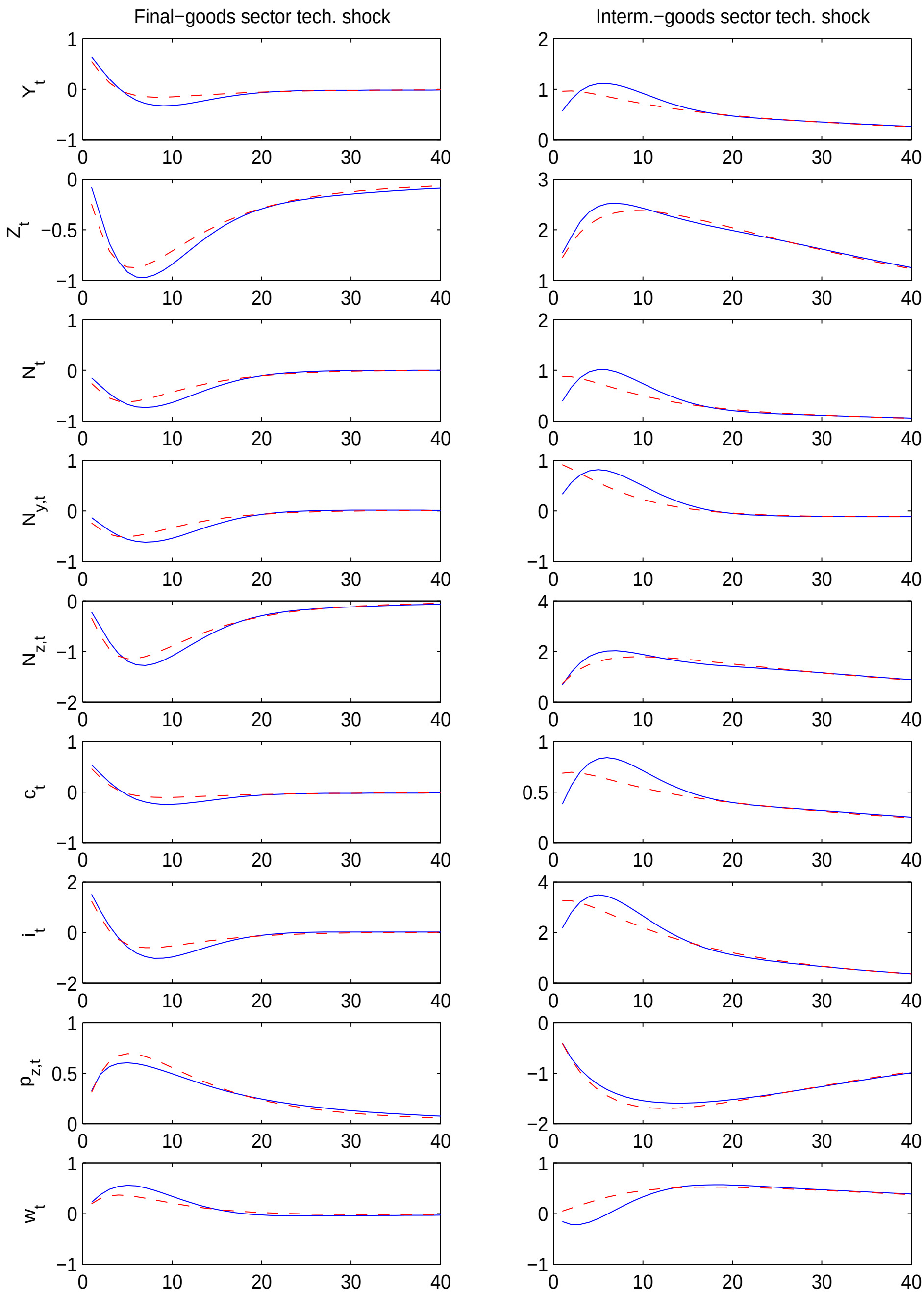
solid line: $d_y = 0.6561$ and $d_z = 0.6992$; dashed line: $d_y = d_z = 0.0000$

Figure 5: The Role of Nominal Wage Rigidity, d_w



solid line: $d_w = 0.8461$; dashed line: $d_w = 0.0000$

Figure 6: The Role of Labor Adjustment Costs, φ_y and φ_z



solid line: $\psi_y = 5.7406$ and $\psi_z = 3.3746$; dashed line: $\psi_y = \psi_z = 0.0000$