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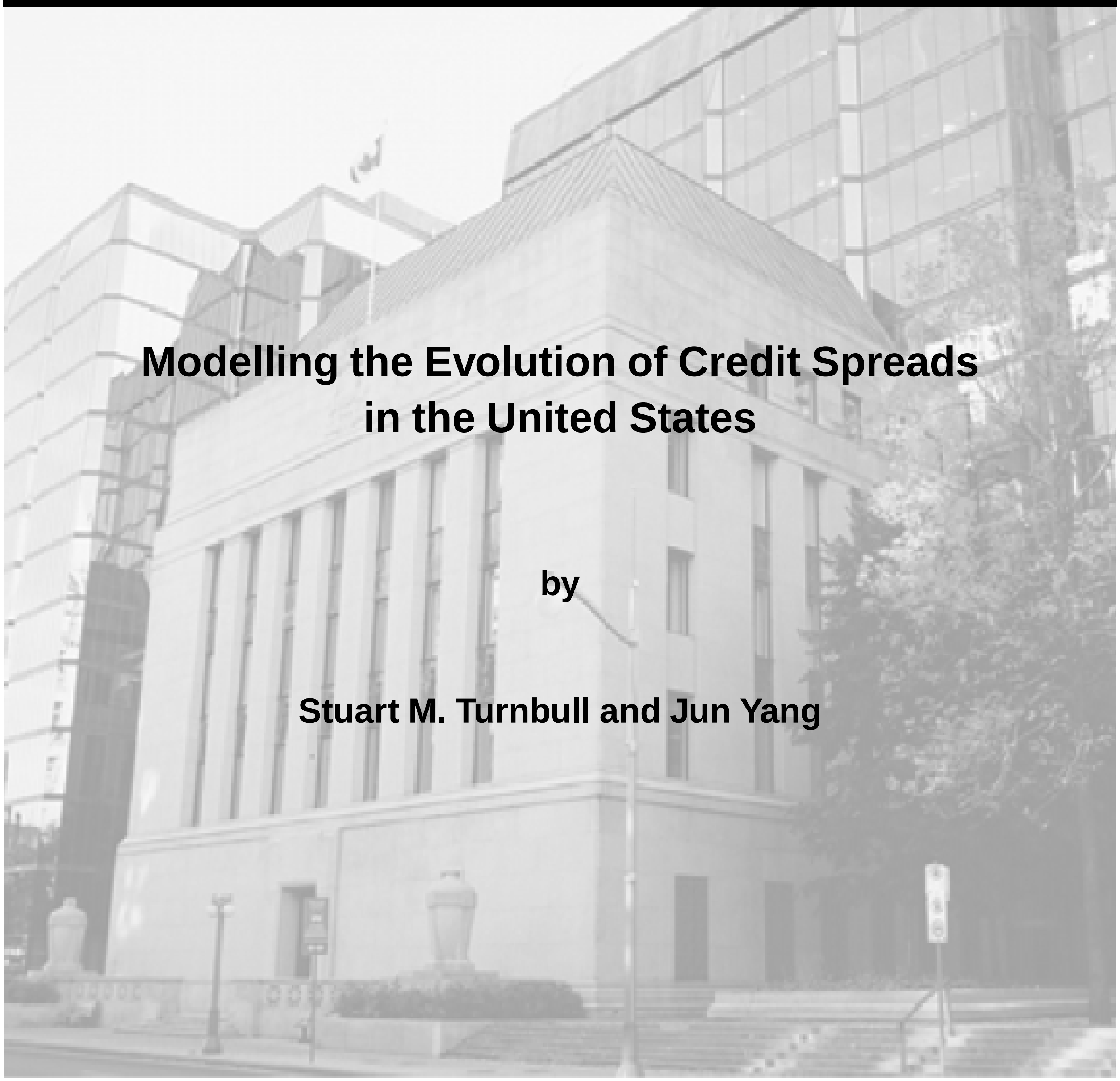

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Contents

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Abstract

The authors use Jarrow and Turnbull's (1995) reduced-form methodology to model the evolution of the term structure of interest rates in the United States for different credit classes and different industries. The authors also estimate a liquidity function for each credit class and industry. Using data from individual firms, the authors estimate the probability of default under the natural measure and compare it with the estimated default frequencies produced by KMV.

JEL classification: G12, G13
Bank classification: Financial markets; Market structure and pricing

Résumé

Partant du modèle de forme réduite de Jarrow et Turnbull (1995), les auteurs représentent l'évolution de la structure par terme des taux d'intérêt aux États-Unis selon la catégorie de notation et le secteur d'activité. Ils estiment aussi une fonction de liquidité pour chaque catégorie de notation et secteur concerné. Au moyen d'une mesure naturelle tirée des données d'entreprises sélectionnées, ils calculent par ailleurs la probabilité de défaillance de chaque entreprise, puis la comparent à celle estimée à l'aide du modèle KMV.

Classification JEL : G12; G13
Classification de la Banque : Marchés financiers; Structure de marché et fixation des prix

1. Introduction

There has been extensive development in the credit-risk literature since Black and Scholes (1973) and Merton (1974) published their pioneering works. Two basic approaches have been proposed to model corporate default risk. The first approach, known as the structural approach, defines default as occurring either at maturity (Merton 1974) or when the firm's asset value falls below a pre-specified threshold level (Kim, Ramaswamy, and Sundaresan 1992, Leland 1994, and Longstaff and Schwartz 1995). This approach has been applied in Merton (1974), Cooper and Mello (1991), and many other studies. An attractive feature of these models is that they explain the default time of a company in terms of firm-specific variables. One critical assumption of these models, however, is that the evolution of firm value follows a diffusion process. Since a diffusion process does not allow a sudden drop in firm value, the probability of the firm defaulting in the near term is negligible (Duffie and Lando 2001). Therefore, these models generate near-zero credit spread for short-term debt, which is strongly rejected by empirical evidence (Jones, Mason, and Rosenfeld 1984). Alternatively, Zhou (1997) obtains positive short-term credit spreads by modelling the asset value as a jump-diffusion process. This comes at the cost of tractability, since multiple jumps must be allowed to determine the asset value.

The second approach, the reduced-form approach first introduced by Jarrow and Turnbull (1992, 1995), proposes an exogenous model for the default process and allows for the possibility of default in the immediate future. This framework has been expanded by Madan and Unal (1998, 2000), Duffee (1999), Duffie and Singleton (1999), and Hughston and Turnbull (2000). A major advantage of this approach is that it generates realistic short-term credit spreads. In addition, the reduced-form models have flexibility in specifying the source of default. Jarrow and Turnbull (2000) model the default process as a Cox process (Lando 1998) by incorporating two state variables — the spot rate and an equity index — into the intensity function, and allowing the market risk (unexpected changes in interest rates and firm values) to affect the default probability.

Since the corporate bond market is not as liquid as the Treasury bond market, Jarrow and Turnbull (2000) include a convenience yield to account for the liquidity premium. Duffie and Lando (2001) provide a bridge between the structural and reduced-form approaches by assuming informational asymmetry.

Jarrow and Turnbull (2000) construct a reduced-form model that incorporates both default risk and liquidity risk. Numerous studies (e.g., Duffee 1998, and Vassalou and Xing 2004) have found that default risk is influenced by systematic factors. Jarrow and Turnbull (2000) assume that the default intensity of a firm depends on two state variables: the instantaneous interest rate and instantaneous unexpected excess return of an equity index. In addition, default risk may not be the sole determinant of the credit spread. Jarrow and Turnbull incorporate a convenient yield as one of the determinants of the credit spread. We describe an empirical implementation of an extended version of Jarrow and Turnbull's (2000) model. Jarrow and Turnbull assume that the instantaneous interest rate follows a one-factor Vasicek model. Empirical studies (Chen and Scott 1993, Pearson and Sun 1994, and Dai and Singleton 2000), however, have found that at least two factors are needed to explain the movement of the yield curve of government bonds. In this paper, the instantaneous interest rate is assumed to follow a two-factor Vasicek model. In addition, the default intensity of a firm depends on the unexpected one-year excess return of an equity index (the Standard and Poor's (S&P) 500 Index), because its instantaneous unexpected excess return is very volatile. Chordia, Sarkar, and Subrahmanyam (2003) show that common factors drive liquidity in both equity and bond markets. In this paper, we assume that the liquidity premium depends on a liquidity measure of the bond market, the yield spread between "on-the-run" and "off-the-run" U.S. 30-year Treasury bonds, and a common macro-factor, the one-month average volatility of the S&P 500 Index.

The data used for this study are from the Bridge Fixed Income Database that consists of daily prices and yields to maturity for various fixed-income securities, including the U.S. government and corporate bonds. We use bond data that is pooled and from individual firms to estimate the intensity function and the liquidity function of corporate bonds. The pooled data set groups corporate bonds with a given credit rating and a particular industry. The data set from individual firms uses corporate bonds with a given firm. The time period covered in the study is January 1995 to May 2001.

Using pooled data, we find that default risk is related to the two systematic factors. In addition, the two liquidity proxies seem to capture the existence of liquidity premiums in corporate bond prices. Furthermore, the relationship between the default risk of a specific firm and the two systematic factors is found to be significant. However, the effect of the two liquidity proxies on the bond prices of a particular firm is not significant.

This paper is organized as follows. Section 2 briefly summarizes the extended version of Jarrow and Turnbull's (2000) credit-risk model. Section 3 describes the data-construction process. The econometric methodology is discussed in section 4. Section 5 provides the estimation results for the evolution of the term structure curves for each credit class and industry. It also provides estimation results using data from individual firms. Section 6 offers some conclusions.

2. The Structure of the Credit-Risk Model

Consider an economy with the time horizon $[0,\bar{T}]$. The economy is assumed to be frictionless, with no arbitrage opportunity, but with illiquidities present. Default-free zero-coupon bonds and risky zero-coupon bonds of all maturities are traded. The default-free bond pays a dollar with certainty at maturity T, for $0 \le T \le \bar{T}$, with a time t price $p(t,T)$. A firm issues the risky bond with a promise that it will pay a dollar at maturity, T. The bond is risky because, if the firm goes bankrupt prior to time T, the promised one dollar may not be paid. Let Γ represent the first time the firm defaults. The default time is a random variable. Let

$$N(t) = 1_{\{\Gamma \le t\}} = \begin{cases} 1 & if \quad \Gamma \le t \\ 0 & otherwise \end{cases}.$$

The random variable, $N(t)$, is a point process that indicates whether default occurred prior to time t. We let $h(t)$ represent its intensity process. The time t intensity process, $h(t)\Delta$, gives the approximate probability of default for this firm over the interval $[t, t+\Delta]$.

If default occurs, the bondholder will receive a fractional recovery $(L(\Gamma))$ of the market value of the bond just prior to default. In other words, the bond is worth only a fraction of its pre-default value when default occurs.

Under the assumption of no arbitrage, standard arbitrage pricing theory (Duffie and Singleton 1999) implies that there exists an equivalent probability measure (risk-neutral measure), Q, such that the values of default-free and risky zero-coupon bonds are martingale, which in turn implies that

$$p(t,T) = E_t^Q\left[\exp\left(-\int_t^T r(u)du\right)\right],$$

$$v(t,T) = E_t^Q\left[\exp\left(-\int_t^T \left(r(u) + h(u)L(u)\right)du\right)\right],$$

where $r(t)$ is the instantaneous interest rate at t, and $r(u) + h(u)L(u)$ is the so-called "default-adjusted discount rate."

The U.S. government and corporate bonds used in the study are coupon-bearing bonds. A coupon bond pays coupons of c_i dollars at time T_i, for $i = 1,2,\dots,n$, where $T_n = T$. Standard no-arbitrage arguments give the prices of default-free and defaultable coupon bonds as

$$P(t,T) = \sum_{i=1}^{n} c_i\, p(t, T_i) \tag{1}$$

and $$V(t,T) = \sum_{i=1}^{n} c_i\, v(t, T_i) \ , \tag{2}$$

respectively.

The prices in expressions (1) and (2) are for coupon bonds traded in a perfectly liquid market. This may not be a good approximation for U.S. corporate bonds, however, due to problems of liquidity. Following Jarrow and Turnbull (2000), we introduce a liquidity function, $l(t,T)$, to accommodate the effect of liquidity risk on risky zero-coupon bonds. The price of an illiquid risky zero-coupon bond, $v^l(t,T)$, is given by

$$v^l(t,T) = e^{-l(t,T)} v(t,T) .$$

Consequently, the price of an illiquid risky coupon bond, $V^l(t,T)$, is given by

$$V^l(t,T) = \sum_{i=1}^{n} c_i v^l(t,T_i) = \sum_{i=1}^{n} c_i e^{l(t,T_i)} v(t,T_i) . \tag{3}$$

In this study, we assume that the probability of default for a company depends on two state variables: the instantaneous interest rate and the unexpected one-year excess return of an equity index. Next, we describe the stochastic evolution of the default-free spot rate, the specification of the intensity function, and the specification of the liquidity function.

2.1 Spot rate process

The instantaneous spot rate, $r(t)$, is assumed to be an affine function of two unobserved latent factors, $y_1(t)$ and $y_2(t)$,

$$r(t) = w_0 + w_1 y_1(t) + w_2 y_2(t) , \tag{4}$$

where w_0 controls the long-term mean of the spot rate, and w_i controls the volatility of the latent variable y_i, $i = 1,2$. The latent factors $y_i(t)$ are assumed to follow Gaussian diffusions,

$$dy_i(t) = -\kappa_i y_i(t) dt + dW_i(t) , \; i = 1,2 , \tag{5}$$

where $dW_1(t)$ and $dW_2(t)$ are standard Brownian motions under the natural measure, with the instantaneous correlation coefficient φ. Let λ_i denote the market price of risk for the latent variable, $y_i(t)$, $i=1,2$. Under the equivalent martingale measure, Q, the latent variable $y_i(t)$ follows

$$dy_i(t) = (-\lambda_i - \kappa_i y_i(t))dt + d\tilde{W}_i(t),\ i=1,2, \tag{6}$$

where $d\tilde{W}_1(t)$ and $d\tilde{W}_2(t)$ are standard Brownian motions under the equivalent martingale measure, Q, with the instantaneous correlation coefficient, φ.

2.2 Equity index process

Let $I(t)$ denote a market index. Under the equivalent martingale measure, Q, it is assumed that changes in the index are described by

$$\frac{dI(t)}{I(t)} = r(t)dt + \sigma_I d\tilde{W}_I(t), \tag{7}$$

where $r(t)$ is the default-free spot rate, σ_I is the volatility of the rate of return of the index, and $d\tilde{W}_I(t)$ is a standard Brownian motion under the equivalent martingale measure, Q. The Brownian motions, $d\tilde{W}_I(t)$ and $d\tilde{W}_i(t)$, have instantaneous correlation coefficients ϕ_i, $i=1,2$. Let $x(t) = \ln(I(t))$, so that

$$dx(t) = \left(r(t) - \sigma_I^2/2\right)dt + \sigma_I d\tilde{W}_I(t). \tag{8}$$

Let λ_I denote the market price of risk of the equity index. Under the natural measure, $x(t)$ follows

$$dx(t) = (r(t) + \lambda_I \sigma_I - \sigma_I^2/2)dt + \sigma_I dW_I(t), \tag{9}$$

where $dW_I(t)$ is a standard Brownian motion under the natural measure.

2.3 Intensity function

The intensity function in this study is assumed to be of the form

$$h(t) = a_0 + a_1 r(t) + \beta M(t), \tag{10}$$

where $M(t) \equiv \frac{1}{A}\int_{t-A}^{t} d\widetilde{W}_I(t)$ is the average unexpected accumulative return of the equity index over the period $[t-A, t]$. If the past average unanticipated return has been negative, it is hypothesized that the probability of default over the next interval will increase, which implies that we expect the coefficient β to be negative. In the empirical estimation, we take A to be one year. The choice of one year is arbitrary.

The fractional recovery rate, $L(t)$, is assumed to be constant; that is, $L(t) = L$.

2.4 Liquidity function

Chordia, Sarkar, and Subrahmanyam (2003) find that common factors drive liquidity in both stock and bond markets. It is assumed in this study that the liquidity function is of the form

$$l(t,T) = [\ \delta_1 \sigma_I^M + \delta_2 S(t)\ ](T-t), \tag{11}$$

where σ_I^M is the one-month average instantaneous volatility of the equity index, and $S(t)$ is the current yield spread between the off-the-run and on-the-run 30-year U.S. Treasury bonds. This is a measure of the lack of liquidity in the Treasury market.

Let $\tau = T - t$ and $B_i(\tau) = \frac{1}{\kappa_i}(1 - e^{-\kappa_i \tau})$, $i = 1,2$. Given these specifications, it can be shown (Duffie and Singleton 1999, and Jarrow and Turnbull 2000) that the time t price of the default-free zero-coupon bond in a perfectly liquid market is

$$p(t,T) = \exp\Big\{ -w_0\tau - w_1 B_1(\tau) y_1(t) - w_2 B_2(\tau) y_2(t) + \frac{\lambda_1 w_1}{\kappa_1}\big(\tau - B_1(\tau)\big) + \frac{\lambda_2 w_2}{\kappa_2}\big(\tau - B_2(\tau)\big)$$

$$+ (1/2)\sum_{i=1}^{2}\sum_{j=1}^{2}\frac{\varphi_{ij} w_i w_j}{\kappa_i + \kappa_j}\left[\frac{\tau - B_i(\tau)}{\kappa_i} + \frac{\tau - B_j(\tau)}{\kappa_j} - B_i(\tau)B_j(\tau)\right]\Bigg\}, \qquad (12)$$

where $\varphi_{ii} = 1$ for $i = 1,2$, and $\varphi_{12} = \varphi_{21} = \varphi$.

If no default has occurred at or prior to time t, the price of the risky zero-coupon bond in a perfectly liquid market is

$$v(t,T) = \exp\{\left(-a_0 L - w_0(1 + a_1 L)\right)\tau - (1 + a_1 L)w_1 B_1(\tau)y_1(t) - (1 + a_1 L)w_2 B_2(\tau)y_2(t)$$

$$-\beta\frac{1}{A}\int_{-A}^{t}(s + A)d\widetilde{W}_I(t) + \frac{1}{2}\beta^2\tau - \frac{1}{3}\beta^2 A$$

$$+\frac{\lambda_1(1 + a_1 L)w_1}{\kappa_1}\left(\tau - B_1(\tau)\right) + \frac{\lambda_2(1 + a_1 L)w_2}{\kappa_2}\left(\tau - B_2(\tau)\right)$$

$$+\sum_{i=1}^{2}\phi_i(1 + a_i L)w_i\beta\left[\tau - \frac{A}{2} - B_i(\tau) + \frac{A - B_i(A)}{A\kappa_i}\right]/\kappa_i$$

$$+ (1/2)\sum_{i=1}^{2}\sum_{j=1}^{2}\frac{\varphi_{ij}(1 + a_1 L)^2 w_i w_j}{\kappa_i + \kappa_j}\left[\frac{\tau - B_i(\tau)}{\kappa_i} + \frac{\tau - B_j(\tau)}{\kappa_j} - B_i(\tau)B_j(\tau)\right]\Bigg\}, \qquad (13a)$$

when $\tau \ge A$, and

$$v(t,T) = \exp\{\left(-a_0 L - w_0(1 + a_1 L)\right)\tau - (1 + a_1 L)w_1 B_1(\tau)y_1(t) - (1 + a_1 L)w_2 B_2(\tau)y_2(t)$$

$$-\beta\frac{1}{A}\int_{-A}^{-A+\tau}(s + A)d\widetilde{W}_I(t) - \beta\tau\frac{1}{A}\int_{-A+\tau}^{t}d\widetilde{W}_I(t) + \frac{1}{6}\beta^2\tau^3 / A^2$$

$$+\frac{\lambda_1(1 + a_1 L)w_1}{\kappa_1}\left(\tau - B_1(\tau)\right) + \frac{\lambda_2(1 + a_1 L)w_2}{\kappa_2}\left(\tau - B_2(\tau)\right)$$

$$+\sum_{i=1}^{2}\phi_i(1 + a_i L)w_i\beta\frac{1}{A}\left[\tau\left(\frac{\tau}{2} - B_i(\tau)\right) + \frac{\tau - B_i(\tau)}{\kappa_i}\right]/\kappa_i$$

$$+ (1/2)\sum_{i=1}^{2}\sum_{j=1}^{2}\frac{\varphi_{ij}(1 + a_1 L)^2 w_i w_j}{\kappa_i + \kappa_j}\left[\frac{\tau - B_i(\tau)}{\kappa_i} + \frac{\tau - B_j(\tau)}{\kappa_j} - B_i(\tau)B_j(\tau)\right]\Bigg\}, \qquad (13b)$$

when $\tau \le A$.

2.5 Expected probability of default

Given the estimated intensity function, $h(t)$, we can infer the probability of default over a specified horizon, T, under the equivalent martingale measure, Q, as

$$\Pr^{Q}(\Gamma \le T) = 1 - E_t^{Q}\left[\exp\left(-\int_t^{t+T} h(u)du\right)\right].$$

If we can estimate the market prices of risk of the underlying state variables, we can also compute the probability of default over a specified horizon, T, under the natural measure, P, as

$$\Pr^{P}(\Gamma \le T) = 1 - E_t^{P}\left[\exp\left(-\int_t^{t+T} h(u)du\right)\right].$$

With the specification of the intensity function in this study, the probability of default over a time horizon, T, under the natural measure when $T = A$, is

$$\begin{aligned}
\Pr(\Gamma \le T) = 1 - \exp\Bigg\{ -\left(a_0 + a_1 w_0 + \frac{1}{2}\beta\lambda_I\right)T \\
- a_1 w_1 B_1(T) y_1(t) - a_1 w_2 B_2(T) y_2(t) - \beta \frac{1}{T}\int_{t-T}^{t} (u+T) d\tilde{W}_I(u) \\
+ \sum_{i=1}^{2}\sum_{j=1}^{2} \frac{\varphi_{ij} a_1^2 w_i w_j}{\kappa_i + \kappa_j}\left[\frac{T - B_i(T)}{\kappa_i} + \frac{T - B_j(T)}{\kappa_j} - B_i(T)B_j(T)\right] \\
+ \sum_{i=1}^{2} \phi_i a_i w_i \beta \frac{1}{T}\left[T(T/2 - B_i(T)) + \frac{T - B_i(T)}{\kappa_i}\right] / \kappa_i,
\end{aligned} \qquad (14)$$

where $\varphi_{ii} = 1$ for $i = 1,2$, and $\varphi_{12} = \varphi_{21} = \varphi$.

3. Data Description

The Treasury and corporate data used in this study are from the Bridge Fixed Income Database that consists of daily prices and yields to maturity of various fixed-income securities, including U.S. government and corporate bonds. Debt issues are classified as callable, putable, convertible, sinkable, and straight. Each debt contract is assigned an industry and a credit class. In this study, we use the Standard and Poor's credit rating. The time period covered in this study is January 1995 to May 2001.

Daily prices on seven on-the-run U.S. Treasury bills and bonds that have maturities of 3 months, 6 months, 1 year, 2 years, 5 years, 10 years, and 30 years, respectively, are used to estimate the parameters in the spot rate process. The data-construction process for the U.S. government bonds is referred to in Turnbull, Turetsky, and Yang (2001). In this paper, we describe the data-construction processes for the U.S. corporate bonds.

We construct two data sets. The first groups corporate bonds with a given credit rating and in a particular industry. The second uses data from individual firms. We use several exclusionary filters to construct the two data sets. First, we exclude all debt issues that contain embedded options. This filter leaves only straight coupon-bearing bonds. Second, we exclude bonds that have a very short maturity (less than 6 months) and a very long maturity (longer than 30 years), since the market for them is extremely illiquid. We also exclude long-term discount bonds (having a maturity longer than one year), and bonds that have monthly or quarterly coupons, because of the irregularity exhibited in their prices. These filters leave only semi-annual coupon bonds with a maturity of between 6 months and 30 years. Third, we employe a median-yield filter of 2.5 per cent to remove debt issues whose yields to maturity are larger or smaller than the median yield by this percentage, because of probable data-collecting errors.

For the credit class data set, the median yield is calculated every day using bonds issued by companies in the same industry and credit class. Applying the median-yield filter to those bonds, we are able to construct several subsets that contain daily bond prices for different industries and credit classes. However, there are still too many bonds left in each subset every day. To reduce computing time in our estimation, we randomly choose as many as 30 bonds across different maturities per day from each subset to construct the pooled data subsets used in the study. The seven industries chosen in this study are banks, consumer goods, energy, manufacturing, services, telephone, and transportation. The five credit classes chosen are shown in Table 1. Table 2 shows the average number of bonds per day in each subset.

For the data set from individual firms, the median yield is calculated from the bonds issued by the same company every day, and the median-yield filter is applied to these bonds. Because of the data limitation, only two companies are used in the study: General Motors and Merrill Lynch. Table 2 also shows the average number of bonds per day each year for both companies.

For the equity market index, we use daily observations on the S&P 500 index, obtained from Bloomberg. Since we assume that the intensity function depends on the unexpected one-year excess return of the equity index, the sample period for the S&P 500 index is January 1994 to May 2001.

4. Econometric Methodology

4.1 Spot interest rate process

The parameters of the interest rate are common to all firms. We use only seven on-the-run U.S. Treasury securities to estimate the interest rate parameters. The detailed procedures in estimating one-, two-, and three-factor Vasicek models are described in Turnbull, Turetsky, and Yang (2001). In this section, we briefly summarize the estimating procedure for the two-factor model.

Chen and Scott (1993) and Pearson and Sun (1994) have developed a maximum-likelihood estimator for the parameters that drive the processes of the interest rate. They derive the likelihood function for the observed bond prices as functions of the unobservable latent variables. This technique enables them to estimate all the parameters, including the market prices of risk, in their Cox, Ingersoll, and Ross (CIR) models. We implement the same methodology to estimate the two-factor Vasicek model. Specifically, we assume that the two-factor model exactly prices two portfolios constructed from the seven on-the-run U.S. treasuries. The first portfolio consists of on-the-run Treasury bills with maturities of 3 months, 6 months, and 1 year. The second portfolio consists of on-the-run Treasury bonds with maturities of 2 years, 5 years, 10 years, and 30 years.

Let $\overline{P}_i(t)$ and $e_i(t)$ denote the observed market price and measurement error for treasury i at time t, $i = 1,2,\ldots,7$. Our assumption implies the following econometric model,

$$\begin{aligned}
\overline{P}_1(t) &= P(t,T_1) + e_1(t) \\
\overline{P}_2(t) &= P(t,T_2) + e_2(t) \\
\overline{P}_3(t) &= P(t,T_3) - e_1(t) - e_2(t) \\
\overline{P}_4(t) &= P(t,T_4) + e_4(t) \\
\overline{P}_5(t) &= P(t,T_5) + e_5(t) \\
\overline{P}_6(t) &= P(t,T_6) + e_6(t) \\
\overline{P}_7(t) &= P(t,T_7) - e_4(t) - e_5(t) - e_6(t),
\end{aligned} \tag{15}$$

where $P(t,T)$ is the default-free coupon-bond formula defined in (1), and T_i is the time-to-maturity of treasury i, $i = 1,2,\ldots,7$. The first portfolio is

$$\overline{P}_1(t) + \overline{P}_2(t) + \overline{P}_3(t) = P(t,T_1) + P(t,T_2) + P(t,T_3), \tag{16}$$

and the second portfolio

$$\overline{P}_4(t) + \overline{P}_5(t) + \overline{P}_6(t) + \overline{P}_7(t) = P(t,T_4) + P(t,T_5) + P(t,T_6) + P(t,T_7). \tag{17}$$

The two latent variables, $y_1(t)$ and $y_2(t)$, are recovered by simultaneously solving a system of two non-linear equations, as given by expressions (16) and (17).

The unrestricted measurement errors in the two-factor models — see expression (15) — are assumed to follow $AR(1)$ processes:

$$e_k(t) = \rho_k e_k(t-1) + \varepsilon_k(t), \tag{18}$$

where the innovations $\varepsilon_k(t)$ are assumed to be independently and normally distributed with mean μ_k and variance σ_k^2. The measurement errors are also assumed to be independent of the latent variables. Let $E(t)$ denote the vector of the unrestricted measurement errors $(e_1(t), e_2(t), e_4(t), e_5(t), e_6(t))$. The log-likelihood function for a sample of observations on $E(t)$ for $t = t_1, t_2, \ldots, t_N$ is

$$\ln L(E(t_1), E(t_2), \ldots, E(t_N)) = \ln f_0(E(t_1)) + \sum_{i=2}^{N} \ln f(E(t_i) \mid E(t_{i-1})),$$

where $f_0(E(t_1))$ is the joint unconditional density of the unrestricted measurement errors and takes the form $f_0(E(t_1)) = \prod_{k=1}^{I} \frac{1}{\sqrt{2\pi \cdot \sigma_k^2 / (1-\rho_k^2)}} e^{-\frac{1}{2} \cdot \frac{\left(e_k(t_1) - \frac{\mu_k}{1-\rho_k}\right)^2}{\sigma_k^2 / (1-\rho_k^2)}}$, and $f(E(t_i) \mid E(t_{i-1}))$ is the joint conditional density of the unrestricted measurement errors and takes the form

$$f(E(t_i) \mid E(t_{i-1})) = \prod_{k=1}^{I} \frac{1}{\sqrt{2\pi \cdot \sigma_k^2}} e^{-\frac{1}{2} \cdot \frac{(e_k(t_i) - \rho_k e_k(t_{i-1}) - \mu_k)^2}{\sigma_k^2}}.$$

Assuming that the latent variables follow stationary processes (i.e., $\kappa_i > 0$), we can derive the conditional density function of the state variables $(y_1(s), y_2(s))$ given $(y_1(t), y_2(t))$, $s > t$, in the two-factor Vasicek model as a joint normal distribution:

$$f\left[\begin{pmatrix} y_1(s) \\ y_2(s) \end{pmatrix} \middle| \begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix}\right] \sim N\left[\begin{pmatrix} e^{-\kappa_1(s-t)} y_1(t) \\ e^{-\kappa_2(s-t)} y_2(t) \end{pmatrix}, \begin{pmatrix} \frac{1-e^{-2\kappa_1(s-t)}}{2\kappa_1} & \frac{\varphi(1-e^{-(\kappa_1+\kappa_2)(s-t)})}{\kappa_1+\kappa_2} \\ \frac{\varphi(1-e^{-(\kappa_1+\kappa_2)(s-t)})}{\kappa_1+\kappa_2} & \frac{1-e^{-2\kappa_2(s-t)}}{2\kappa_2} \end{pmatrix}\right],$$

where $\kappa_i > 0$, $i = 1,2$.

Letting s in the above expression approach infinite, we can also derive the unconditional density function of the state variables as a joint normal distribution:

$$g\begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix} \sim N\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \frac{1}{2\kappa_1} & \frac{\varphi}{\kappa_1 + \kappa_2} \\ \frac{\varphi}{\kappa_1 + \kappa_2} & \frac{1}{2\kappa_2} \end{pmatrix}\right].$$

Let $Y(t)$ denote the vector of the two latent variables $(y_1(t), y_2(t))$. The joint distribution for a sample of observations on the state variables for $t = t_1, t_2, \ldots, t_N$ is

$$f\left(Y(t_1), Y(t_2), \ldots, Y(t_N)\right) = g\left(Y(t_1)\right) \cdot \prod_{i=2}^{N} f\left(Y(t_i) \mid Y(t_{i-1})\right),$$

and the log-likelihood function is

$$\ln L\left(Y(t_1), Y(t_2), \ldots, Y(t_N)\right) = \ln g\left(Y(t_1)\right) + \sum_{i=2}^{N} \ln f\left(Y(t_i) \mid Y(t_{i-1})\right).$$

Given the assumptions of the latent variables and the measurement errors, we can derive the log-likelihood function for a sample of N observations on the prices of the seven on-the-run treasuries at time $t_1, t_2, \ldots, t_N$ as

$$\log L = \ln L\left(\hat{Y}(t_1), \hat{Y}(t_2), \ldots, \hat{Y}(t_N)\right) - \sum_{i=1}^{N} \ln\left(abs \mid J_i \mid\right)$$
$$+ \ln L\left(E(t_1), E(t_2), \ldots, E(t_N)\right), \qquad (19)$$

where $\hat{Y}(t)$ is the vector of recovered state variables, and J_i is the Jacobian of the transformation from the state variables and the unrestricted measurement errors to the observed bond prices.

We apply the maximum-likelihood estimation technique to estimate the parameters in the interest rate process by maximizing expression (19). After obtaining the estimated parameters, we also recover the daily latent variables, $y_1(t)$ and $y_2(t)$, and compute the daily instantaneous interest rate, $r(t)$, from equation (4).

4.2 Equity index process

Using the daily S&P 500 index and the recovered spot rate from the two-factor Vasicek model, we apply the maximum-likelihood technique to estimate the parameters of the equity index process as given in expression (7). Under the equivalent martingale measure, Q, the conditional density function of the logarithm of the equity index $x(t+\Delta)$, given $x(t)$, $\Delta > 0$, can be approximated as a normal distribution:

$$f^Q(x(t+\Delta) \mid x(t)) = N\left(x(t) + \left(r(t) - \sigma_I^2 / 2\right)\Delta, \quad \sigma_I^2 \Delta\right).$$

The log-likelihood function for a sample of observations on the equity index for $t = t_1, t_2, \ldots, t_N$ is

$$\ln L\left(x(t_2), x(t_3), \ldots, x(t_N)\right) = \sum_{i=2}^{N} \ln f^Q\left(x(t_i) \mid x(t_{i-1})\right).$$

Given the parameter estimate for the market volatility, σ_I, and daily spot rates, the daily $d\widetilde{W}_I(t)$ process is computed using the following formula:

$$d\widetilde{W}_I(t) = \left[x(t) - x(t-\Delta) - \left(r(t) - \sigma_I^2 / 2\right)\Delta\right] / \sigma_I .$$

To estimate the market price of risk of the equity index, we apply the maximum-likelihood technique to estimate the equity index process as given in expression (8). Under the natural measure, the conditional density function of the index $x(t+\Delta)$, given $x(t)$, $\Delta > 0$, can be approximated as a normal distribution:

$$f(x(t+\Delta) \mid x(t)) = N\left(x(t) + \left(r(t) - \sigma_I^2 / 2 + \lambda_I \sigma_I\right)\Delta, \quad \sigma_I^2 \Delta\right).$$

The log-likelihood function for a sample of observations on the equity index for $t = t_1, t_2, \ldots, t_N$ is

$$\ln L\big(x(t_2), x(t_3), \ldots, x(t_N)\big) = \sum_{i=2}^{N} \ln\ f\big(x(t_i) \mid x(t_{i-1})\big).$$

After obtaining the estimated parameters and recovered state variables $\big(y_1(t), y_2(t)\big)$ and $d\tilde{W}_I(t)$, we compute the simple correlation coefficients between $d\tilde{W}_i(t)$ and $d\tilde{W}_I(t)$ as the estimates for ϕ_i, $i = 1, 2$, in the risky zero-coupon bond formula.

4.3 Intensity function and liquidity function

Given the estimated parameters and recovered state variables in the spot rate process and the equity index process, the remaining task is to estimate the intensity function and liquidity function of corporate bonds. The parameters in the intensity function are $a_0 L$, $a_1 L$, and β. The parameters in the liquidity function are δ_1 and δ_2. First, however, we need to compute the two liquidity measures, the one-month average volatility of the equity index, σ_I^M, and the yield spread between on-the-run and off-the-run 30-year Treasury bonds, $S(t)$. At time t, we estimate σ_I^M by applying the maximum-likelihood technique described in section 4.2 on past one-month observations of the equity index. To compute $S(t)$ at time t, we first need to choose an off-the-run 30-year Treasury bond as the candidate, since there are many off-the-run 30-year bonds every day. Among all available off-the-run 30-year Treasury bonds that have a maturity of at least 28 years, we choose the one that has a coupon rate closest to that of the on-the-run 30-year Treasury bond. Then we use the difference between the yield-to-maturity of the chosen off-the-run 30-year bond and that of the on-the-run 30-year bond as the approximation for $S(t)$.

For the estimation of the intensity function and liquidity function of corporate bonds, a non-linear regression procedure is implemented with the parameters in the state variable processes and the recovered state variables fixed. To estimate these parameters, we minimize the summation of the mean-squared percentage pricing error:

$$\min_{\{a_o L, a_1 L, \delta_1, \delta_2\}} \sum_{t=1}^{T} \sum_{k=1}^{K_t} (\varepsilon_{k,t})^2 , \qquad t = 1, \ldots, T \text{ , and } k = 1, \ldots K_t ,$$

where K_t is the number of bonds on day t, and $\varepsilon_{k,t} = \dfrac{\overline{P}(t, T_k) - V^l(t, T_k)}{\overline{P}(t, T_k)}$. $\overline{P}(t, T_k)$ is the market price of the risky coupon bond, and $V^l(t, T_k)$ is the theoretical price in our extended Jarrow and Turnbull (2000) credit-risk model.

5. Parameter Estimation

5.1 Estimation with pooled corporate data

Four different models for the default intensity and liquidity discount are estimated using the pooled corporate bond data. The models differ regarding the number of state variables and liquidity measures in the intensity function and liquidity function, respectively. Model 1 has $\beta = \delta_1 = \delta_2 = 0$. This is the case with one state variable (the spot rate) in the intensity function and no liquidity discount. Model 2 has two state variables (the spot rate and equity index) and no liquidity discount with $\delta_1 = \delta_2 = 0$. Model 3 has $\delta_2 = 0$, and Model 4 includes all parameters. The different models are summarized in Table 3. For each pooled corporate subset, we estimate all four models. The estimation procedure is as follows.

First, we use daily prices on seven on-the-run U.S. Treasury bills and bonds over the sample period January 1995 to May 2001, to estimate the parameters in the spot rate process. The estimation results are reported in Table 4. All of the coefficients are statistically significant, with the exception of the market price of risk for the second state variable. The recovered daily latent variables and spot rates are shown in Figures 1 and 2, respectively.

Second, we use the daily S&P 500 index to estimate the parameters in the equity index process. These parameters are assumed to be constant over the sample period January 1994 to

May 2001. The estimated results are reported in Table 4, and the recovered unexpected one-year excess returns of the S&P 500 index are plotted in Figure 3. The estimated coefficients are statistically significant. The correlation coefficients between the Brownian motions in the spot rate process and the one in the equity index process are also computed and reported in Table 4.

Third, we compute the two liquidity measures: the one-month average instantaneous volatility of the equity index, σ_I^M, and the yield spread between off-the-run and on-the-run 30-year U.S. Treasury bonds, $S(t)$. The results are reported in Figures 4 and 5, respectively.

Fourth, given the estimated parameters in the state variable processes, recovered state variables, and constructed liquidity measures, we apply non-linear regressions to estimate the parameters in the intensity and liquidity functions using the pooled corporate bond data subsets constructed from bonds issued by companies in the same industry and credit class. The estimation results are reported in Table 5.

As Longstaff and Schwartz (1995) point out, the static effect of a higher spot rate is to increase the risk-neutral drift of the firm value process. A higher drift reduces the default probability. In addition, an increase in the short-term interest rate usually indicates a decreased risk of an economic recession in the medium term. Therefore, the sign of a_1 is expected to be negative. A higher unexpected return of an equity index, on average, increases the value of a firm and reduces the default probability. Thus, the sign of β is expected to be negative. The estimation results support this prediction. The estimates for a_1L and β are negative and statistically significant across industries and credit classes. In addition, the estimate for a_1L in general increases in absolute magnitude while the credit rating falls, which suggests that the low-quality bonds are more sensitive to the spot rate than the high-quality bonds. For Model 1, the estimated coefficient $a_1 L$ is negative across industries. For Model 2, the estimated coefficients

$a_1 L$ and β are also negative across industries. For Models 3 and 4, the coefficients are generally statistically significant and have the expected sign.

There is no clear trend in the estimates for β for different credit classes. The results also show that the estimates for $a_1 L$ and β differ across industries, which suggests that the spot rate and the return of the S&P 500 index have a larger impact on the default probability for some industries than for others.

The parameter estimates for the two liquidity measures are positive and statistically significant for most industries and credit classes. The results show that the two liquidity measures seem to capture the presence of illiquidity in the U.S. corporate bond market.

5.2 Derived credit spreads

After obtaining the parameter estimates in the intensity and liquidity functions, we estimate the 1-year, 5-year, and 10-year yield spreads between corporate discount bonds and treasury discount bonds for the manufacturing industry in all four models over the sample period January 1995 to May 2001. The two liquidity measures in our model account for only a few basis points in the predicted corporate spreads. Therefore, the results in Models 3 and 4 are very similar to those in Model 2. Only the time series of the estimated credit yield spreads in Models 1 and 2 are reported in Figures 6 and 7, respectively.

Model 1 produces positive short-, medium-, and long-term credit spreads for all credit classes over the sample period. The credit spread increases with declining credit quality. Model 2 also produces positive medium- and long-term credit spreads for all credit classes over the sample period. Sometimes, however, it produces negative short-term credit spreads for high-quality bonds, because the intensity function in our model specifies that the default probability depends on the unexpected one-year excess return of an equity index. From the derived risky bond price formula, we can see that the unexpected one-year excess return has a larger impact on the short-term yield than the long-term yield, and its effect declines very quickly when the time-to-maturity of a bond increases. Most corporate bonds used in this study are long-term bonds. Therefore, the estimate for β mainly reflects the effect of the unexpected one-year return of the equity index on the long-term coupons, and results in negative credit spreads for short-term discount bonds. Alternative specifications are an interesting topic for future research.

5.3 Estimation with data from individual firms

The results described section 5.2 show that the intensity function with two state variables — the spot rate and the unexpected one-year return of the S&P 500 index — produces negative short-term yield spreads for the corporate discount bonds over the treasury discount bonds because of the high volatility in the second state variable. In section 5.4, we will want to compute the estimated probability of default (EPD) of each company over a one-year horizon and compare it with that reported by Moody's KMV. Therefore, we exclude the second state variable, the unexpected one-year return of the S&P 500 index, from the intensity function when we use the data from individual firms to estimate the intensity and liquidity functions. Three different models for the default intensity and liquidity discount are estimated using the data from individual firms. Model 1 has $\delta_1 = \delta_2 = 0$. This is the case with one state variable (the spot rate) in the intensity function and no liquidity discount. Model 2 has one liquidity measure with $\delta_2 = 0$. Model 3 includes all parameters. The different models are summarized in Table 6.

To estimate the intensity function and liquidity function using the data from individual firms, we implement a rolling forward estimation procedure to accommodate possible structural changes in our models.

First, we apply a rolling forward procedure to estimate the parameters in the spot rate process. At the first trading day of month t, we use the past year of daily treasury data (month $t-12$ to month t) to obtain the maximum-likelihood estimates of the parameters in the spot rate process. Then, moving forward one month to the first day of month $t+1$, we estimate these parameters again using the past year of data (month $t-11$ to month $t+1$). Applying this procedure, we obtain parameter estimates each month from January 1996 to May 2001, for a total of 65 months. The average values of the estimated parameters are reported in Table 7. The recovered latent variables and spot rates from the procedure are recorded.

Second, we apply the one-month rolling forward procedure to estimate the intensity function and liquidity function using one year of daily data from individual firms. The non-linear regression described in section 5.2 is implemented to obtain the parameter estimates with data on two companies: General Motors and Merrill Lynch. Table 8 reports the average values of the estimated parameters in the three models. The estimates for $a_1 L$ are negative and statistically significant for both companies, which indicates that a higher spot rate reduces the default probability for both companies. The estimate of $a_1 L$ is larger in absolute magnitude for General Motors than for Merrill Lynch, as is the estimate of the coefficient, $a_0 L$. There are differences in the magnitude of the liquidity coefficients for the two firms.

5.4 Comparison with Moody's KMV

Finally, with the estimated market price of the underlying state variable, we compute the estimated probability of default of each company over a one-year horizon. In the econometric estimation, we estimate the product of the hazard function and the loss function. To estimate the probability of default, we must make some assumptions about the magnitude of the loss function. For a given a value of the loss function, the estimated probabilities of default in Models 2 and 3 are very similar to those in Model 1. Consequently, we will show the results for only Model 1. The similarity of the results suggests that the two market liquidity measures constructed in our model cannot capture the liquidity discount for the two companies. In other words, the liquidity discount seems to be firm-specific.

Figure 8 shows the sensitivity of the estimated probability of default to different assumptions about the magnitude of the loss function. The results are quite sensitive to the value of the loss function. For each firm, we also plot Moody’s KMV estimates of the expected probability of default, and Moody's credit ratings for the two firms.

The estimated probabilities of default for the two companies follow a very similar pattern, because in the hazard function only one common state variable, the spot rate, affects the probability of default. Over the period, the estimated probability of default for General Motors is, in general, larger than that for Merrill Lynch. This is consistent with the fact that General Motors has a lower credit rating than Merrill Lynch over the sample period. However, using the KMV estimates, post-May 2000, the expected probability of default for General Motors is larger than that for Merrill Lynch, though the reverse holds for almost three years prior to that date.

There is a large difference in the orders of magnitude between the estimated probability of default produced by Model 1 and that produced by KMV. Janosi, Jarrow, and Yildirim (2002) find a similar difference.

We also compute the simple correlation coefficients between the monthly estimated probability of default in our model and those reported by KMV over the sample period. For General Motors, the correlation coefficient is 0.738; for Merrill Lynch, it is 0.335. KMV’s estimate is derived using the equity price of the firm, whereas our model uses the firm-specific credit spread.

6. Conclusion

This study has used bond data that is pooled and from individual firms to estimate an extended version of Jarrow and Turnbull's (2000) reduced-form credit-risk model that includes both default risk and liquidity risk. The results have shown that the default probability of a firm is related to the changes in the spot rate and the return on an equity index. Our model captures the integration of market risk and credit risk. The two market liquidity measures constructed in our model seem to capture the presence of illiquidity in the U.S. corporate bond market when pooled data are used. In addition, the estimation method enables us to estimate the market prices of risk for the underlying state variables. We are able to infer the expected probability of default under the natural measure. This has an important practical implication, since it is necessary for a risk manager to predict the default probability under the natural measure.

Some aspects of the model need to be improved to reduce the pricing errors for corporate bonds. First, we could include an industry-specific index in the hazard function; second, we could specify a hazard function that does not permit negative values; and third, we could specify a firm-specific liquidity function.

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Table 1. Credit Ratings

This table reports Moody's and Standard and Poor's credit ratings for corporate bonds. It also shows the credit ratings used in this study.

Moody's ratings	**S&P ratings**	**Ratings used**
Investment grade		
Aaa	AAA	
Aa1	AA+	
Aa2	AA	Aa2
Aa3	AA-	
A1	A+	
A2	A	A2
A3	A-	
Baa1	BBB+	
Baa2	BBB	Baa2
Baa3	BBB-	
Non-investment grade		
Ba1	BB+	Ba1
Ba2	BB	
Ba3	BB-	
B1	B+	B1
B2	B	
B3	B-	

Table 2. Statistics Based on Data from Pooled and Individual Firms

This table reports the average numbers of bonds per day for pooled corporate subsets and individual firm subsets over the sample period January 1995 to May 2001. The pooled corporate data subsets are constructed from bonds issued by the companies in the same industry and credit class. The table has seven industries (banks, consumer goods, energy, manufacturing, services, telephone, and transportation) and five credit classes (Aa2, A2, Baa2, Ba1, and B1). The data subsets from individual firms are constructed from bonds issued by the same company. There are two subsets for individual firms in this table. GM represents General Motors, and MER represents Merrill Lynch.

	Banks	**Consumer goods**	**Energy**	**Manufacturing**	**Services**	**Telephone**	**Transportation**
Aa2	21	10	20	12	27	23	8
A2	30	30	30	30	30	30	30
Baa2	22	15	30	30	29	19	30
Ba1	9	7	15	21	28	10	6
B1	4	8	17	25	27	9	5
	1995	1996	1997	1998	1999	2000	2001
GM	22	23	23	21	23	22	20
MER	26	56	80	97	108	89	90

Table 3. Model Description with Pooled Corporate Data

This table reports the parameters to be estimated in the intensity function and liquidity function for the four models with pooled corporate data. The intensity function is $h(t) = a_0 + a_1 r(t) + \beta \frac{1}{A}\int_{t-A}^{t} d\widetilde{W}_I(t)$, where $r(t)$ is the spot rate and $\frac{1}{A}\int_{t-A}^{t} d\widetilde{W}_I(t)$ is the unexpected one-year excess return of the S&P 500 index. The liquidity function is $l(t,T) = (\delta_1 \sigma_I^M + \delta_2 S_{OnOff})(T-t)$, where σ_I^M is the one-month average instantaneous volatility of the S&P 500 index, and S_{OnOff} is the yield spread between the off-the-run and the on-the-run 30-year U.S. Treasury bonds.

Pooled corporate data				
	Intensity function		Liquidity function	
	$a_1 L$	β	δ_1	δ_2
Model 1	Yes			
Model 2	Yes	Yes		
Model 3	Yes	Yes	Yes	
Model 4	Yes	Yes	Yes	Yes

Table 4. Estimation of the State Variable Processes

This table reports the maximum-likelihood estimates for the parameters in the spot rate and equity index processes. The spot rate process is a two-factor Vasicek model, $r(t) = w_0 + w_1 y_1(t) + w_2 y_2(t)$. $y_1(t)$ and $y_2(t)$ are two latent variables, and are assumed to follow a mean-reverting Gaussian diffusion process, $dy_i(t) = (-\lambda_i - \kappa_i y_i(t))dt + d\tilde{W}_i(t)$, $i = 1,2$. $d\tilde{W}_1(t)$ and $d\tilde{W}_2(t)$ are standard Brownian motions under the equivalent martingale measure, Q, with the instantaneous correlation coefficient, φ. λ_i denotes the market price of risk for the latent variable, $y_i(t)$, $i = 1,2$. Time-series data of prices on seven on-the-run U.S. Treasury bills and bonds with maturities of 3 months, 6 months, 1 year, 2 years, 5 years, 10 years, and 30 years are used to estimate the spot rate process. The equity index model is $\frac{dI(t)}{I(t)} = r(t)dt + \sigma_I d\tilde{W}_I(t)$, where $r(t)$ is the spot interest rate, and $d\tilde{W}_I(t)$ is a standard Brownian motion under the equivalent martingale measure, Q. The market price of risk for the equity index is denoted by λ_I, and the instantaneous correlation coefficients between $d\tilde{W}_i(t)$ and $d\tilde{W}_I(t)$ are denoted by ϕ_i, $i = 1,2$. Daily observations on the S&P 500 index are used to estimate the equity index process. The asymptotic standard errors are reported in parentheses below the estimates. The sample period for the treasury data is January 1995 to May 2001, and the sample period for the S&P 500 index is January 1994 to May 2001.

			Spot rate model				
w_0	w_1	w_2	κ_1	κ_2	λ_1	λ_2	φ
0.0476	0.0168	0.0153	0.9262	0.0571	-0.7145	-0.0731	-0.8469
(0.0390)	(0.0004)	(0.0005)	(0.0496)	(0.0029)	(0.2618)	(0.1552)	(0.0106)
			Equity index model				
			σ_I	λ_I			
			0.1711	0.5011			
			(0.0001)	(0.0023)			
			Correlation between state variables				
			ϕ_1	ϕ_2			
			0.1385	-0.1063			
			(0.0783)	(0.0632)			

Table 5. Estimation of Intensity Function and Liquidity Function with Pooled Corporate Data

This table reports the non-linear regression estimates for the parameters in the intensity function and liquidity function with pooled corporate data. There are 43 pooled corporate data sets constructed from bonds issued by companies in the same industry and credit class. The intensity function is $h(t) = a_0 + a_1 r(t) + \beta \frac{1}{A}\int_{t-A}^{t} d\widetilde{W}_I(t)$, where $r(t)$ is the spot rate and $\frac{1}{A}\int_{t-A}^{t} d\widetilde{W}_I(t)$ is the unexpected one-year excess return of the S&P 500 index. The liquidity function is $l(t,T) = (\delta_1 \sigma_I^M + \delta_2 S_{OnOff})(T-t)$, where σ_I^M is the one-month average instantaneous volatility of the S&P 500 index, and S_{OnOff} is the yield spread between the off-the-run and the on-the-run 30-year U.S. Treasury bonds. Model 1 has one state variable — the spot rate — in the intensity function, and no liquidity discount. Model 2 has two state variables — the spot rate and unexpected one-year excess of the S&P 500 index — in the intensity function, and no liquidity discount. Model 3 has two state variables in the intensity function and one liquidity measure in the liquidity function. Model 4 has two state variables in the intensity function and two liquidity measures in the liquidity function. The asymptotic standard errors are reported in parentheses below the estimates. The average pricing errors in the models are reported in the last column. The sample period is January 1995 to May 2001.

(continued)

Table 5 (continued)

Banks							
Credit class	Model	a_0L	a_1L	β	δ_1	δ_2	Pricing error (%)
Aa2	**Model 1**	0.0260 (0.0002)	-0.3080 (0.0032)				1.64
	Model 2	0.0257 (0.0002)	-0.2898 (0.0031)	-0.0104 (0.0002)			1.56
	Model 3	0.0224 (0.0002)	-0.2553 (0.0036)	-0.0095 (0.0002)	0.0067 (0.0003)		1.47
	Model 4	0.0183 (0.0003)	-0.2145 (0.0035)	-0.0081 (0.0002)	0.0047 (0.0003)	2.3044 (0.0445)	1.45
A2	**Model 1**	0.0325 (0.0002)	-0.3991 (0.0034)				2.03
	Model 2	0.0313 (0.0002)	-0.3463 (0.0031)	-0.0216 (0.0002)			1.96
	Model 3	0.0221 (0.0002)	-0.2549 (0.0034)	-0.0170 (0.0002)	0.0187 (0.0003)		1.90
	Model 4	0.0193 (0.0002)	-0.2390 (0.0033)	-0.0149 (0.0002)	0.0146 (0.0003)	2.6757 (0.0395)	1.87
Baa2	**Model 1**	0.0326 (0.0002)	-0.3736 (0.0037)				2.10
	Model 2	0.0335 (0.0002)	-0.3603 (0.0035)	-0.0188 (0.0002)			1.90
	Model 3	0.0285 (0.0003)	-0.3088 (0.0041)	-0.0169 (0.0003)	0.0106 (0.0004)		1.85
	Model 4	0.0253 (0.0003)	-0.2877 (0.0038)	-0.0147 (0.0002)	0.0045 (0.0004)	3.5700 (0.0492)	1.81
Ba1	**Model 1**	0.0623 (0.0010)	-0.5788 (0.0174)				4.90
	Model 2	0.0583 (0.0010)	-0.4577 (0.0171)	-0.0447 (0.0010)			4.03
	Model 3	0.0508 (0.0011)	-0.4196 (0.0172)	-0.0370 (0.0011)	0.0232 (0.0014)		3.89
	Model 4	0.0484 (0.0011)	-0.4465 (0.0165)	-0.0318 (0.0011)	0.0155 (0.0014)	5.1558 (0.1665)	3.80
B1	**Model 1**	0.0999 (0.0019)	-1.0272 (0.0319)				5.95
	Model 2	0.0918 (0.0019)	-0.8339 (0.0326)	-0.0414 (0.0024)			4.66
	Model 3	0.0805 (0.0021)	-0.7952 (0.0321)	-0.0275 (0.0026)	0.0375 (0.0032)		4.50
	Model 4	0.0733 (0.0019)	-0.7977 (0.0290)	-0.0129 (0.0024)	0.0249 (0.0029)	9.1066 (0.3072)	4.38

(continued)

Table 5 (continued)

Consumer goods							
Credit class	Model	a_0L	a_1L	β	δ_1	δ_2	Pricing error (%)
Aa2	**Model 1**	0.0311 (0.0003)	-0.3679 (0.0052)				3.11
	Model 2	0.0289 (0.0003)	-0.2990 (0.0049)	-0.0343 (0.0006)			2.75
	Model 3	0.0242 (0.0003)	-0.2532 (0.0051)	-0.0294 (0.0006)	0.0096 (0.0004)		2.64
	Model 4	0.0223 (0.0003)	-0.2448 (0.0048)	-0.0267 (0.0006)	0.0062 (0.0004)	1.9457 (0.0492)	2.61
A2	**Model 1**	0.0329 (0.0002)	-0.3803 (0.0040)				2.52
	Model 2	0.0306 (0.0002)	-0.3096 (0.0036)	-0.0282 (0.0002)			2.18
	Model 3	0.0233 (0.0003)	-0.2361 (0.0039)	-0.0241 (0.0003)	0.0149 (0.0003)		2.07
	Model 4	0.0216 (0.0003)	-0.2288 (0.0038)	-0.0229 (0.0002)	0.0124 (0.0003)	1.8266 (0.0438)	2.03
Baa2	**Model 1**	0.0442 (0.0004)	-0.5339 (0.0074)				3.73
	Model 2	0.0366 (0.0004)	-0.3732 (0.0075)	-0.0311 (0.0005)			3.35
	Model 3	0.0308 (0.0005)	-0.3245 (0.0076)	-0.0267 (0.0005)	0.0136 (0.0006)		3.20
	Model 4	0.0290 (0.0005)	-0.3206 (0.0075)	-0.0244 (0.0005)	0.0109 (0.0006)	2.0932 (0.0784)	3.15
Ba1	**Model 1**	0.0744 (0.0009)	-0.6878 (0.0151)				6.73
	Model 2	0.0760 (0.0009)	-0.6194 (0.0145)	-0.0756 (0.0017)			5.66
	Model 3	0.0634 (0.0012)	-0.5001 (0.0163)	-0.0671 (0.0018)	0.0243 (0.0015)		5.52
	Model 4	0.0581 (0.0011)	-0.4756 (0.0152)	-0.0594 (0.0017)	0.0106 (0.0015)	5.8502 (0.1650)	5.43
B1	**Model 1**	0.0911 (0.0012)	-0.7892 (0.0191)				7.45
	Model 2	0.0908 (0.0011)	-0.7360 (0.0193)	-0.0346 (0.0017)			6.83
	Model 3	0.0658 (0.0016)	-0.4935 (0.0221)	-0.0174 (0.0018)	0.0501 (0.0023)		6.70
	Model 4	0.0612 (0.0015)	-0.4873 (0.0211)	-0.0107 (0.0017)	0.0338 (0.0022)	7.0420 (0.2388)	6.62

(continued)

Table 5 (continued)

Energy							
Credit class	Model	a_0L	a_1L	β	δ_1	δ_2	Pricing error (%)
Aa2	**Model 1**	0.0255 (0.0003)	-0.2837 (0.0053)				2.05
	Model 2	0.0270 (0.0003)	-0.2806 (0.0051)	-0.0170 (0.0003)			1.95
	Model 3	0.0235 (0.0004)	-0.2395 (0.0063)	-0.0161 (0.0003)	0.0065 (0.0006)		1.90
	Model 4	0.0209 (0.0004)	-0.2242 (0.0062)	-0.0144 (0.0003)	0.0022 (0.0006)	2.4046 (0.0610)	1.87
A2	**Model 1**	0.0372 (0.0002)	-0.4387 (0.0031)				3.22
	Model 2	0.0350 (0.0002)	-0.3499 (0.0027)	-0.0476 (0.0003)			2.98
	Model 3	0.0318 (0.0002)	-0.3186 (0.0029)	-0.0447 (0.0003)	0.0061 (0.0003)		2.84
	Model 4	0.0291 (0.0002)	-0.3039 (0.0028)	-0.0416 (0.0003)	0.0029 (0.0003)	2.3297 (0.0324)	2.78
Baa2	**Model 1**	0.0407 (0.0002)	-0.4740 (0.0041)				3.13
	Model 2	0.0399 (0.0002)	-0.4142 (0.0037)	-0.0377 (0.0003)			2.85
	Model 3	0.0304 (0.0003)	-0.3232 (0.0040)	-0.0314 (0.0003)	0.0193 (0.0004)		2.77
	Model 4	0.0274 (0.0003)	-0.3076 (0.0038)	-0.0283 (0.0003)	0.0143 (0.0004)	3.0839 (0.0421)	2.65
Ba1	**Model 1**	0.0848 (0.0006)	-1.0009 (0.0090)				5.15
	Model 2	0.0797 (0.0005)	-0.8721 (0.0088)	-0.0512 (0.0007)			4.66
	Model 3	0.0631 (0.0006)	-0.7577 (0.0087)	-0.0343 (0.0007)	0.0417 (0.0007)		4.56
	Model 4	0.0586 (0.0006)	-0.7462 (0.0081)	-0.0306 (0.0007)	0.0339 (0.0007)	5.1384 (0.0912)	4.50
B1	**Model 1**	0.0940 (0.0006)	-0.9851 (0.0095)				5.45
	Model 2	0.0923 (0.0006)	-0.9480 (0.0099)	-0.0091 (0.0007)			4.53
	Model 3	0.0841 (0.0007)	-0.8753 (0.0105)	-0.0032 (0.0008)	0.0181 (0.0010)		4.42
	Model 4	0.0830 (0.0007)	-0.8898 (0.0103)	-0.0011 (0.0008)	0.0123 (0.0010)	3.0485 (0.1068)	4.37

(continued)

Table 5 (continued)

Manufacturing							
Credit class	Model	a_0L	a_1L	β	δ_1	δ_2	Pricing error (%)
Aa2	**Model 1**	0.0305 (0.0003)	-0.3749 (0.0052)				3.51
	Model 2	0.0287 (0.0003)	-0.3105 (0.0049)	-0.0339 (0.0006)			3.25
	Model 3	0.0229 (0.0004)	-0.2520 (0.0054)	-0.0286 (0.0006)	0.0115 (0.0005)		3.15
	Model 4	0.0209 (0.0003)	-0.2435 (0.0052)	-0.0251 (0.0006)	0.0079 (0.0004)	2.2700 (0.0585)	3.08
A2	**Model 1**	0.0359 (0.0002)	-0.4256 (0.0039)				3.20
	Model 2	0.0345 (0.0002)	-0.3565 (0.0035)	-0.0397 (0.0003)			2.48
	Model 3	0.0295 (0.0003)	-0.3061 (0.0039)	-0.0363 (0.0003)	0.0100 (0.0003)		2.34
	Model 4	0.0273 (0.0003)	-0.2943 (0.0038)	-0.0341 (0.0003)	0.0065 (0.0003)	2.2038 (0.0418)	2.23
Baa2	**Model 1**	0.0435 (0.0003)	-0.4862 (0.0051)				4.51
	Model 2	0.0431 (0.0003)	-0.4132 (0.0046)	-0.0527 (0.0004)			4.15
	Model 3	0.0372 (0.0004)	-0.3538 (0.0051)	-0.0489 (0.0004)	0.0117 (0.0005)		4.02
	Model 4	0.0339 (0.0003)	-0.3331 (0.0050)	-0.0454 (0.0004)	0.0064 (0.0004)	3.0672 (0.0569)	3.87
Ba1	**Model 1**	0.0649 (0.0004)	-0.6743 (0.0067)				5.31
	Model 2	0.0620 (0.0004)	-0.5885 (0.0066)	-0.0355 (0.0005)			4.86
	Model 3	0.0507 (0.0005)	-0.4792 (0.0073)	-0.0286 (0.0006)	0.0234 (0.0007)		4.70
	Model 4	0.0482 (0.0005)	-0.4736 (0.0070)	-0.0263 (0.0005)	0.0167 (0.0007)	3.4015 (0.0759)	4.59
B1	**Model 1**	0.0827 (0.0005)	-0.7198 (0.0080)				5.45
	Model 2	0.0785 (0.0005)	-0.5930 (0.0079)	-0.0411 (0.0006)			4.53
	Model 3	0.0750 (0.0006)	-0.5604 (0.0086)	-0.0388 (0.0006)	0.0074 (0.0008)		4.37
	Model 4	0.0730 (0.0006)	-0.5589 (0.0085)	-0.0360 (0.0006)	0.0020 (0.0008)	2.8720 (0.0923)	4.31

(continued)

Table 5 (continued)

Services							
Credit class	Model	a_0L	a_1L	β	δ_1	δ_2	Pricing error (%)
Aa2	**Model 1**	0.0274 (0.0002)	-0.3239 (0.0039)				3.05
	Model 2	0.0254 (0.0002)	-0.2694 (0.0039)	-0.0209 (0.0003)			2.78
	Model 3	0.0220 (0.0003)	-0.2395 (0.0041)	-0.0180 (0.0003)	0.0078 (0.0004)		2.64
	Model 4	0.0212 (0.0003)	-0.2370 (0.0041)	-0.0167 (0.0003)	0.0064 (0.0004)	0.8908 (0.0421)	2.57
A2	**Model 1**	0.0352 (0.0002)	-0.4097 (0.0040)				3.22
	Model 2	0.0341 (0.0002)	-0.3549 (0.0038)	-0.0332 (0.0003)			2.98
	Model 3	0.0288 (0.0003)	-0.3028 (0.0042)	-0.0295 (0.0003)	0.0111 (0.0004)		2.87
	Model 4	0.0269 (0.0003)	-0.2933 (0.0041)	-0.0275 (0.0003)	0.0076 (0.0004)	1.9585 (0.0429)	2.80
Baa2	**Model 1**	0.0429 (0.0003)	-0.4785 (0.0058)				4.77
	Model 2	0.0410 (0.0003)	-0.3799 (0.0054)	-0.0519 (0.0005)			4.15
	Model 3	0.0315 (0.0004)	-0.2882 (0.0059)	-0.0451 (0.0005)	0.0190 (0.0005)		3.99
	Model 4	0.0291 (0.0004)	-0.2814 (0.0057)	-0.0416 (0.0005)	0.0133 (0.0005)	2.9209 (0.0623)	3.87
Ba1	**Model 1**	0.0431 (0.0003)	-0.3657 (0.0050)				4.15
	Model 2	0.0413 (0.0003)	-0.3104 (0.0050)	-0.0222 (0.0004)			3.88
	Model 3	0.0324 (0.0004)	-0.2120 (0.0056)	-0.0177 (0.0005)	0.0158 (0.0005)		3.70
	Model 4	0.0301 (0.0004)	-0.1981 (0.0055)	-0.0154 (0.0004)	0.0119 (0.0005)	2.3138 (0.0637)	3.59
B1	**Model 1**	0.0749 (0.0004)	-0.6767 (0.0068)				5.45
	Model 2	0.0712 (0.0004)	-0.5656 (0.0067)	-0.0360 (0.0005)			4.53
	Model 3	0.0682 (0.0005)	-0.5353 (0.0076)	-0.0343 (0.0006)	0.0057 (0.0007)		4.47
	Model 4	0.0680 (0.0005)	-0.5350 (0.0076)	-0.0342 (0.0006)	0.0055 (0.0007)	0.1311 (0.0847)	4.47

(continued)

Table 5 (continued)

Telephone							
Credit class	Model	a_0L	a_1L	β	δ_1	δ_2	Pricing error (%)
Aa2	**Model 1**	0.0251 (0.0002)	-0.3017 (0.0032)				2.53
	Model 2	0.0301 (0.0002)	-0.3230 (0.0028)	-0.0406 (0.0004)			2.28
	Model 3	0.0304 (0.0003)	-0.3255 (0.0035)	-0.0408 (0.0004)	-0.0005 (0.0003)		2.19
	Model 4	0.0287 (0.0003)	-0.3154 (0.0034)	-0.0377 (0.0004)	-0.0032 (0.0003)	1.4793 (0.0370)	2.11
A2	**Model 1**	0.0354 (0.0002)	-0.4506 (0.0037)				2.82
	Model 2	0.0334 (0.0002)	-0.3638 (0.0032)	-0.0399 (0.0003)			2.18
	Model 3	0.0271 (0.0002)	-0.3036 (0.0035)	-0.0355 (0.0003)	0.0137 (0.0003)		1.97
	Model 4	0.0254 (0.0002)	-0.2960 (0.0034)	-0.0339 (0.0003)	0.0112 (0.0003)	1.7968 (0.0453)	1.89
Baa2	**Model 1**	0.0571 (0.0003)	-0.6345 (0.0049)				3.94
	Model 2	0.0555 (0.0003)	-0.5806 (0.0049)	-0.0285 (0.0005)			3.65
	Model 3	0.0450 (0.0004)	-0.4800 (0.0051)	-0.0194 (0.0005)	0.0222 (0.0005)		3.51
	Model 4	0.0444 (0.0004)	-0.4784 (0.0051)	-0.0186 (0.0005)	0.0206 (0.0005)	0.8104 (0.0499)	3.42
Ba1	**Model 1**	0.0518 (0.0012)	-0.3872 (0.0210)				7.15
	Model 2	0.0378 (0.0011)	-0.0849 (0.0200)	-0.0670 (0.0012)			5.68
	Model 3	0.0413 (0.0011)	-0.0808 (0.0201)	-0.0728 (0.0013)	-0.0145 (0.0013)		5.53
	Model 4	0.0411 (0.0011)	-0.0972 (0.0206)	-0.0720 (0.0013)	-0.0147 (0.0013)	1.0138 (0.1772)	5.43
B1	**Model 1**	0.1146 (0.0014)	-1.2552 (0.0234)				7.15
	Model 2	0.0791 (0.0015)	-0.6177 (0.0277)	-0.0584 (0.0015)			6.83
	Model 3	0.0778 (0.0016)	-0.6321 (0.0283)	-0.0550 (0.0017)	0.0085 (0.0017)		6.74
	Model 4	0.0763 (0.0016)	-0.5655 (0.0290)	-0.0572 (0.0017)	0.0091 (0.0017)	-2.0848 (0.2254)	6.65

(continued)

Table 5 (continued)

Transportation							
Credit class	Model	$a_0 L$	$a_1 L$	β	δ_1	δ_2	Pricing error (%)
Aa2	**Model 1**	0.0482 (0.0005)	-0.6288 (0.0079)				3.05
	Model 2	0.0458 (0.0004)	-0.5270 (0.0062)	-0.0554 (0.0006)			2.78
	Model 3	0.0326 (0.0005)	-0.3980 (0.0067)	-0.0419 (0.0006)	0.0251 (0.0007)		2.64
	Model 4	0.0284 (0.0005)	-0.3642 (0.0064)	-0.0399 (0.0006)	0.0207 (0.0006)	2.9265 (0.0725)	2.57
A2	**Model 1**	0.0272 (0.0002)	-0.3034 (0.0038)				3.22
	Model 2	0.0259 (0.0002)	-0.2447 (0.0035)	-0.0276 (0.0002)			2.98
	Model 3	0.0200 (0.0003)	-0.1893 (0.0038)	-0.0241 (0.0003)	0.0128 (0.0004)		2.87
	Model 4	0.0174 (0.0003)	-0.1769 (0.0036)	-0.0216 (0.0002)	0.0074 (0.0003)	2.9916 (0.0413)	2.80
Baa2	**Model 1**	0.0341 (0.0003)	-0.2917 (0.0050)				4.77
	Model 2	0.0345 (0.0002)	-0.2161 (0.0042)	-0.0564 (0.0004)			4.15
	Model 3	0.0324 (0.0003)	-0.1945 (0.0047)	-0.0553 (0.0004)	0.0041 (0.0004)		3.99
	Model 4	0.0306 (0.0003)	-0.1873 (0.0046)	-0.0535 (0.0004)	-0.0009 (0.0004)	2.3007 (0.0529)	3.87
Ba1	**Model 1**	0.0681 (0.0009)	-0.7695 (0.0147)				4.15
	Model 2	0.0591 (0.0009)	-0.5519 (0.0150)	-0.0601 (0.0012)			3.88
	Model 3	0.0445 (0.0011)	-0.4083 (0.0162)	-0.0491 (0.0013)	0.0292 (0.0014)		3.70
	Model 4	0.0359 (0.0011)	-0.3345 (0.0156)	-0.0463 (0.0012)	0.0200 (0.0013)	6.2037 (0.1803)	3.59
B1	**Model 1**	0.0877 (0.0014)	-0.6934 (0.0235)				5.45
	Model 2	0.0786 (0.0013)	-0.4330 (0.0230)	-0.0692 (0.0017)			4.53
	Model 3	0.0722 (0.0017)	-0.3725 (0.0250)	-0.0652 (0.0019)	0.0127 (0.0024)		4.47
	Model 4	0.0721 (0.0017)	-0.3753 (0.0250)	-0.0648 (0.0019)	0.0118 (0.0025)	0.4239 (0.2571)	4.47

Table 6. Model Description with Data from Individual Firms

This table reports the parameters to be estimated in the intensity function and liquidity function for the three models with data from individual firms. The intensity function is $h(t) = a_0 + a_1 r(t) + \beta \frac{1}{A}\int_{t-A}^{t} d\tilde{W}_I(t)$, where $r(t)$ is the spot rate and $\frac{1}{A}\int_{t-A}^{t} d\tilde{W}_I(t)$ is the unexpected one-year excess return of the S&P 500 index. The liquidity function is $l(t,T) = (\delta_1 \sigma_I^M + \delta_2 S_{OnOff})(T-t)$, where σ_I^M is the one-month average instantaneous volatility of the S&P 500 index, and S_{OnOff} is the yield spread between the off-the-run and the on-the-run 30-year U.S. Treasury bonds.

Data from individual firms			
	Hazard function	**Liquidity function**	
	$a_1 L$	δ_1	δ_2
Model 1	Yes		
Model 2	Yes	Yes	
Model 3	Yes	Yes	Yes

Table 7. Rolling Forward Estimation Results of the Spot Rate Process

This table reports the rolling forward maximum-likelihood estimates for the parameters in the spot rate process. The spot rate process is a two-factor Vasicek model, $r(t) = w_0 + w_1 y_1(t) + w_2 y_2(t)$. $y_1(t)$ and $y_2(t)$ are two latent variables, and are assumed to follow the mean-reverting Gaussian diffusion process, $dy_i(t) = (-\lambda_i - \kappa_i y_i(t))dt + d\tilde{W}_i(t)$, $i = 1,2$. $d\tilde{W}_1(t)$ and $d\tilde{W}_2(t)$ are standard Brownian motions under the equivalent martingale measure, Q, with the instantaneous correlation coefficient, φ. λ_i denotes the market price of risk for the latent variable $y_i(t)$, $i = 1,2$. A one-month rolling forward estimation procedure is implemented to estimate the spot rate process. The average values of the parameter estimates are reported in this table. The standard errors, computed with 3 Newey-West (1987) lags, are reported in parentheses below the estimates. The sample period is January 1995 to May 2001.

Spot rate process							
w_0	w_1	w_2	κ_1	κ_2	λ_1	λ_2	φ
0.0492	0.0165	0.0148	1.0938	0.0522	-0.5487	-0.0863	-0.8750
(0.0373)	(0.0007)	(0.0008)	(0.0231)	(0.0036)	(0.2030)	(0.1554)	(0.0090)

Table 8. Rolling Forward Estimation Results of Intensity Function and Liquidity Function with Data from Individual Firms

This table reports the non-linear regression estimates for the parameters in the intensity function and liquidity function with data from individual firms. Two data sets from individual firms are constructed, from bonds issued by General Motors and Merrill Lynch, respectively. The intensity function is $h(t) = a_0 + a_1 r(t)$, where $r(t)$ is the spot. The liquidity function is $l(t,T) = (\delta_1 \sigma_I^M + \delta_2 S(T))(T-t)$, where σ_I^M is the one-month average instantaneous volatility of the S&P 500 index, and S_{OnOff} is the yield spread between the off-the-run and the on-the-run 30-year U.S. Treasury bonds. Model 1 has one state variable, the spot rate, in the intensity function, and no liquidity discount. Model 2 has one state variable in the intensity function and one liquidity measure in the liquidity function. Model 3 has one state variable in the intensity function and two liquidity measures in the liquidity function. The standard errors, computed with 3 Newey-West (1987) lags, are reported in parentheses below the estimates. The average pricing errors in the models are reported in the last column. The sample period is January 1995 to May 2001.

General Motors					
	$a_0 L$	$a_1 L$	δ_1	δ_2	Pricing error (%)
Model 1	0.0285 (0.0007)	-0.2898 (0.0117)			1.12
Model 2	0.0273 (0.0007)	-0.2805 (0.0122)	0.0035 (0.0009)		1.08
Model 3	0.0274 (0.0008)	-0.2868 (0.0134)	0.0024 (0.0009)	0.8883 (0.1525)	1.05
Merrill Lynch					
	$a_0 L$	$a_1 L$	δ_1	δ_2	Pricing error (%)
Model 1	0.0265 (0.0005)	-0.2739 (0.0080)			0.97
Model 2	0.0253 (0.0005)	-0.2674 (0.0083)	0.0041 (0.0007)		0.93
Model 3	0.0256 (0.0006)	-0.2740 (0.0090)	0.0026 (0.0007)	0.6714 (0.1134)	0.92

Figure 1. Recovered Two Latent Variables in the Spot Rate Model

This figure plots the recovered daily latent variables, $y_1(t)$ and $y_2(t)$, in the two-factor Vasicek model. Given the parameter estimates in the Vasicek model, the two latent variables are recovered by solving a system of two non-linear equations: $\overline{P}_1(t)+\overline{P}_2(t)+\overline{P}_3(t)=P(t,T_1)+P(t,T_2)+P(t,T_3)$, and $\overline{P}_4(t)+\overline{P}_5(t)+\overline{P}_6(t)+\overline{P}_7(t)=P(t,T_4)+P(t,T_5)+P(t,T_6)+P(t,T_7)$, simultaneously. $\overline{P}_1(t)$, $\overline{P}_2(t)$, and $\overline{P}_3(t)$ are the observed time t prices of U.S. Treasury bills with maturities of 3 months, 6 months, and 1 year, respectively. $\overline{P}_1(t)$, $\overline{P}_2(t)$, $\overline{P}_3(t)$, and $\overline{P}_4(t)$ are the observed time t prices of U.S. Treasury bonds with maturities of 2 years, 5 years, 10 years, and 30 years, respectively. $P(\cdot,\cdot)$ is the default-free coupon bond formula. The sample period is January 1995 to May 2001.

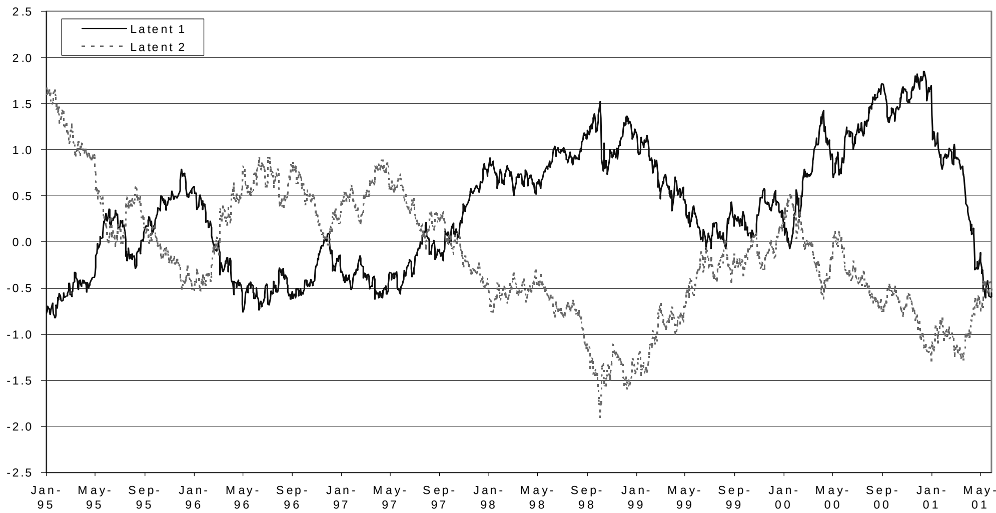


Figure 2. Recovered Daily Instantaneous Interest Rates in the Spot Rate Model

This figure plots the recovered daily instantaneous interest rates in the two-factor Vasicek model. Given the estimated parameters and recovered two latent variables, $y_1(t)$ and $y_2(t)$, the spot rate is computed by $r(t)=w_0+w_1y_1(t)+w_2y_2(t)$. The sample period is January 1995 to May 2001.

Figure 3. Unexpected One-Year Excess Returns of the S&P 500 Index (Percentage)

This figure plots the unexpected one-year excess returns of the S&P 500 index. Given the parameter estimate for the market volatility, σ_I, and daily spot rates, $r(t)$, the daily $d\tilde{W}_I(t)$ process is computed using the following formula: $d\tilde{W}_I(t) = \left[x(t) - x(t-\Delta) - \left(r(t) - \sigma_I^2/2\right)\Delta\right] / \sigma_I$. The unexpected one-year excess return of the S&P 500 index at time t is approximated by the summation of $d\tilde{W}_I(t)$ in the past 250 trading days, $\sum_{j=0}^{250} d\tilde{W}_I(t-j)$. The sample period is January 1994 to May 2001. The reported time series of the unexpected one-year excess returns of the S&P 500 index starts in January 1995, since the past one-year data are used to calculate the starting point.

Figure 4. One-Month Average Instantaneous Volatility of the S&P 500 Index

This figure plots the time series of the maximum-likelihood estimate for the one-month average instantaneous volatility of the S&P 500 index. At time t , the maximum-likelihood estimation technique is implemented using past one-month data on the S&P 500 index. The sample period is January 1995 to May 2001.

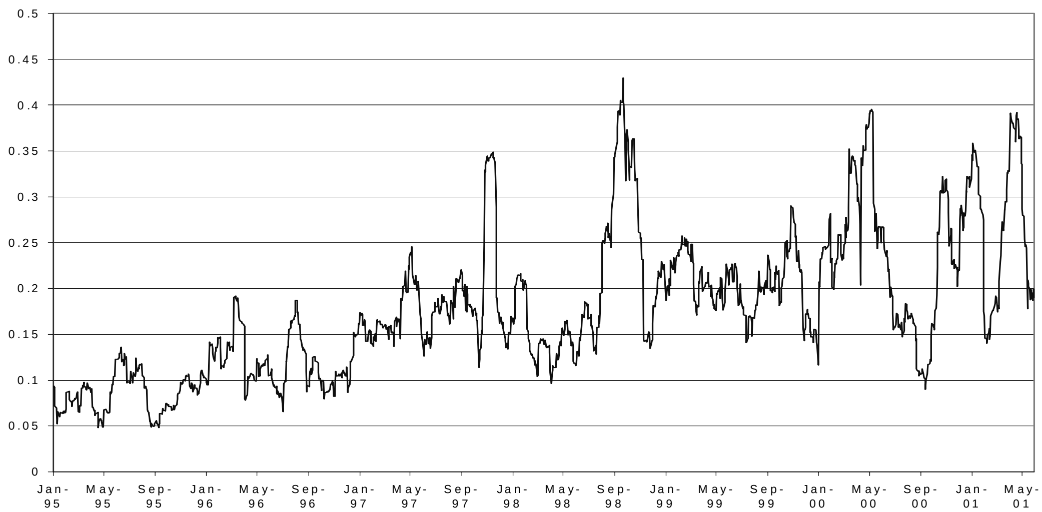


Figure 5. Yield Spread (Percentage) Between Off-the-Run and On-the-Run U.S. 30-Year Treasury Bonds

This figure plots the difference between the yields to maturity of the off-the-run and on-the-run U.S. 30-year Treasury bonds. The sample period is January 1995 to May 2001.

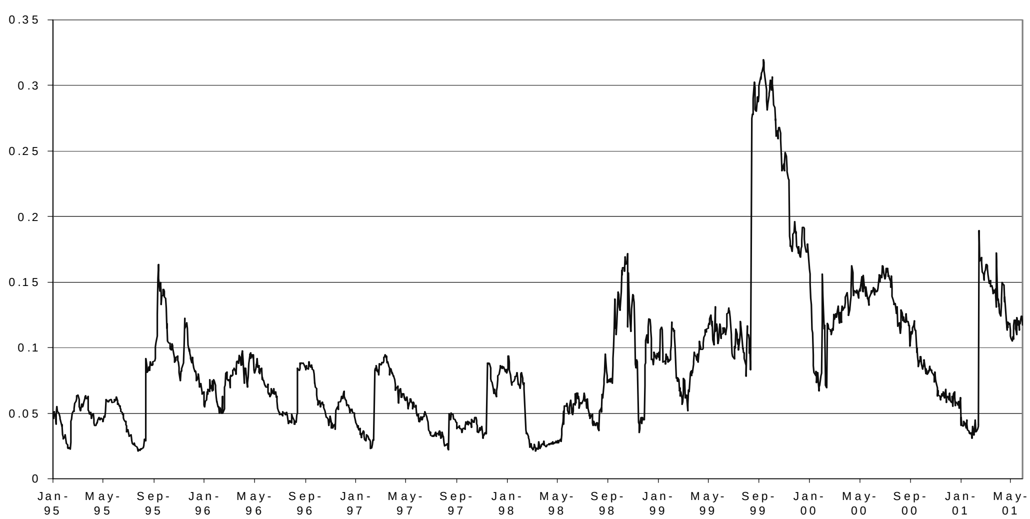
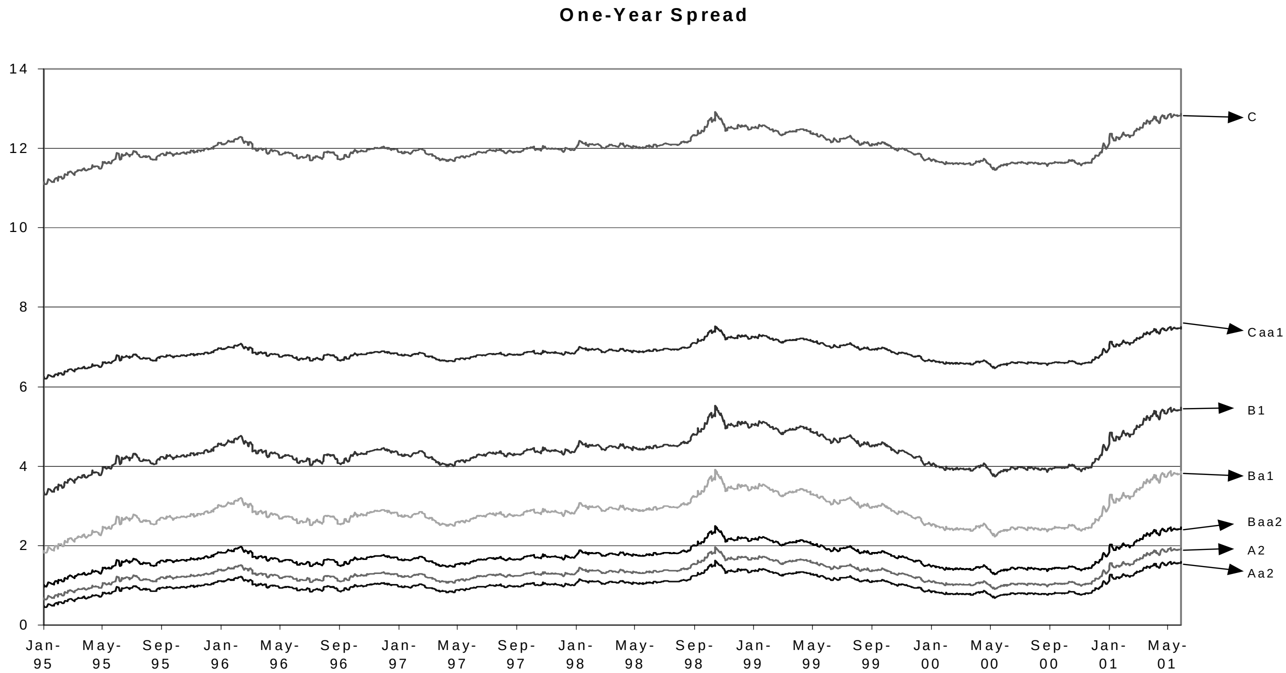


Figure 6. Estimated Credit Yield Spreads (Percentage) for Manufacturing Industry in Model 1

Given the estimated intensity function in Model 1, the predicted one-year, five-year, and ten-year credit yield spreads over the sample period for different credit classes in the manufacturing industry are plotted in the three graphs, respectively. The sample period is January 1995 to May 2001.



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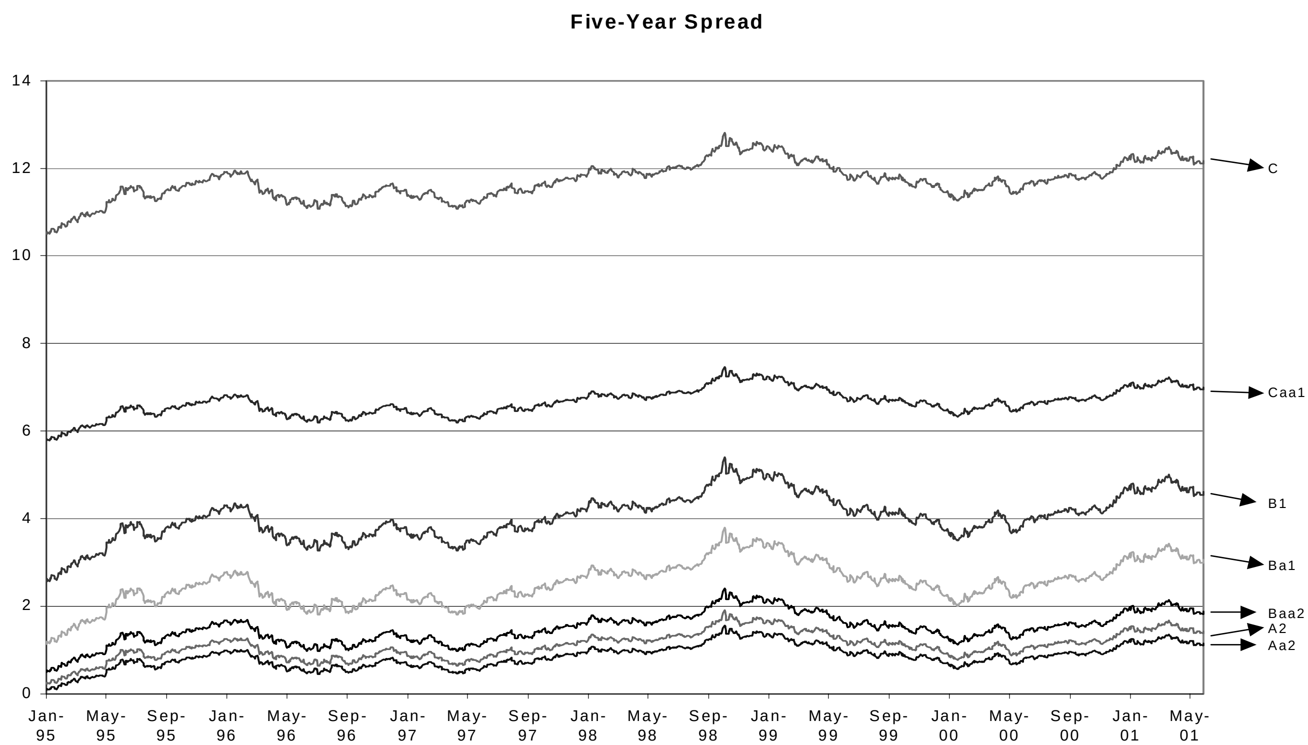

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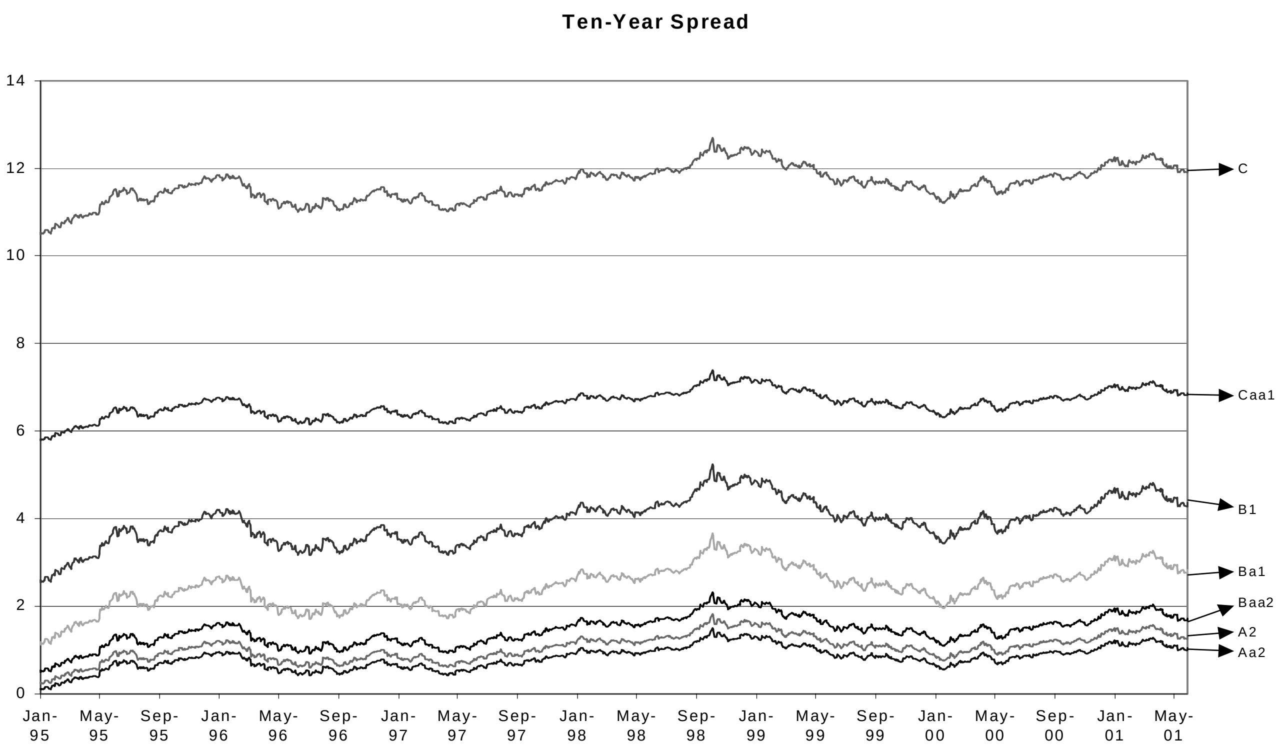
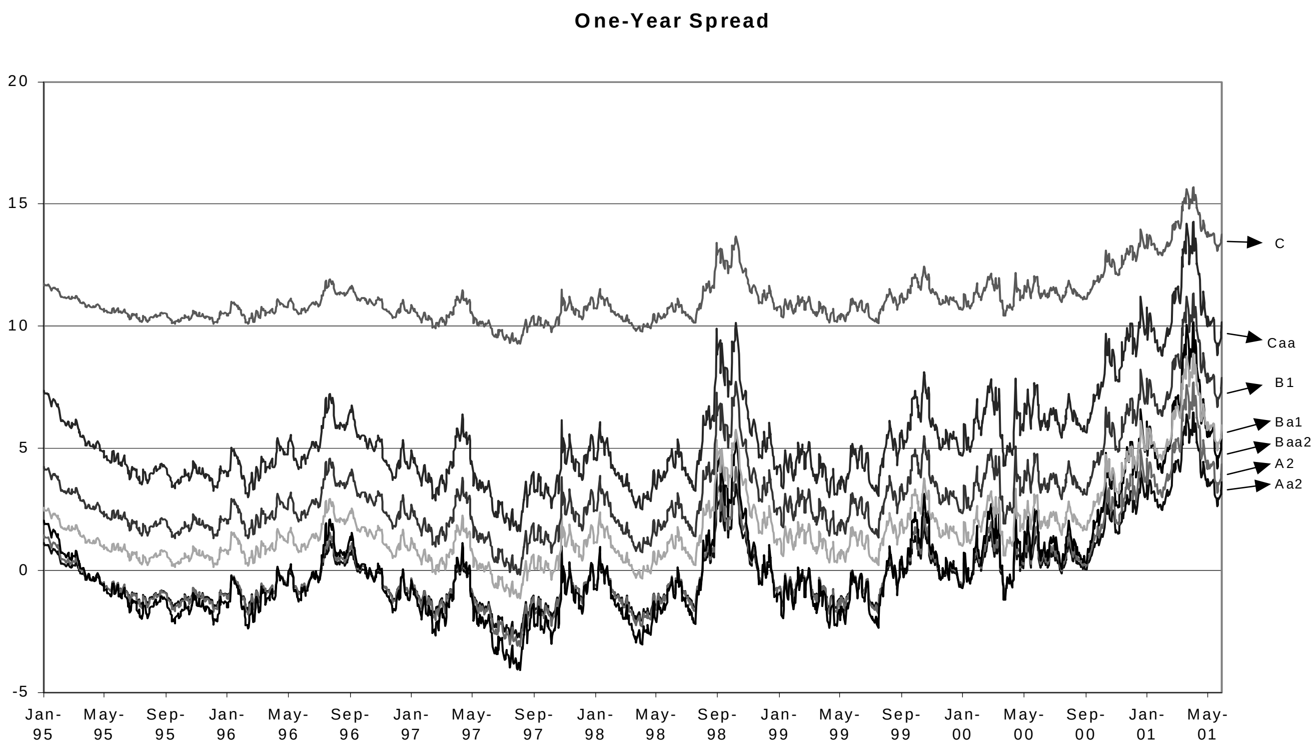


Figure 7. Estimated Credit Yield Spreads (Percentage) for Manufacturing Industry in Model 2

Given the estimated intensity function in Model 2, the predicted one-year, five-year, and ten-year credit yield spreads over the sample period for different credit classes in the manufacturing industry are plotted in the three graphs, respectively. The sample period is January 1995 to May 2001.



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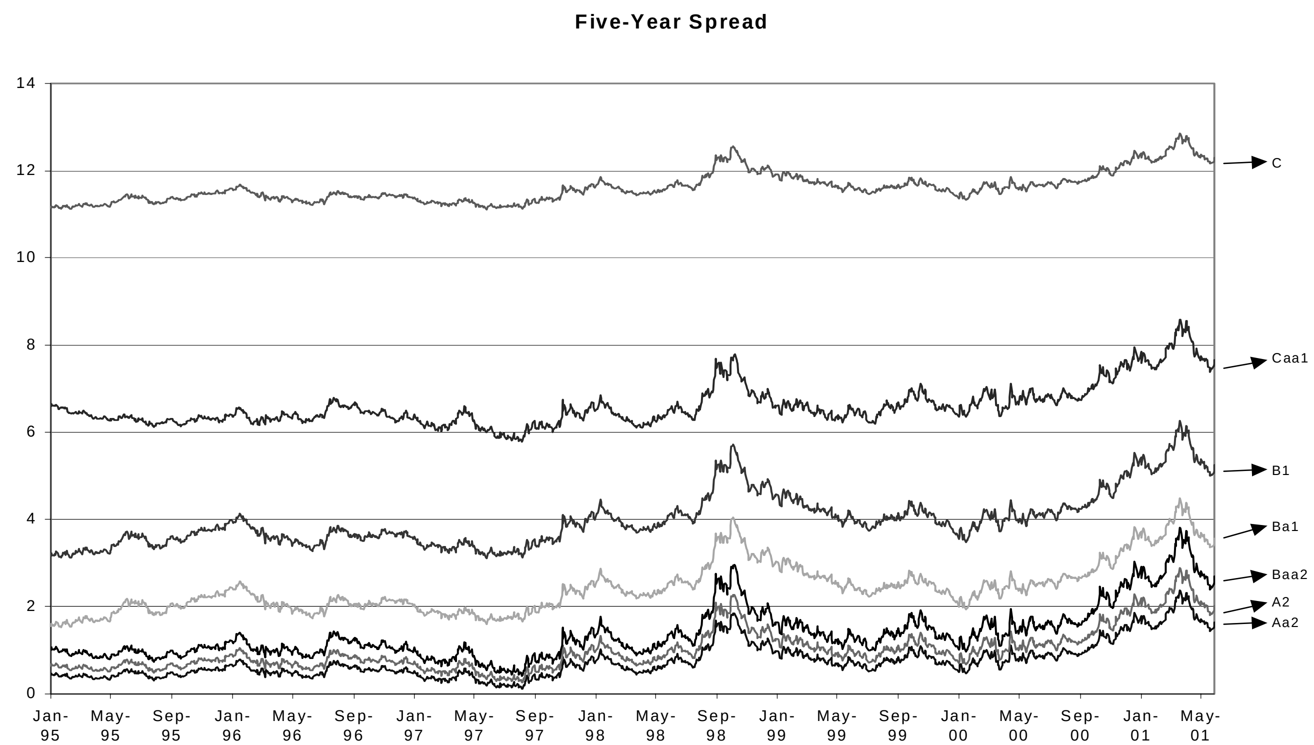

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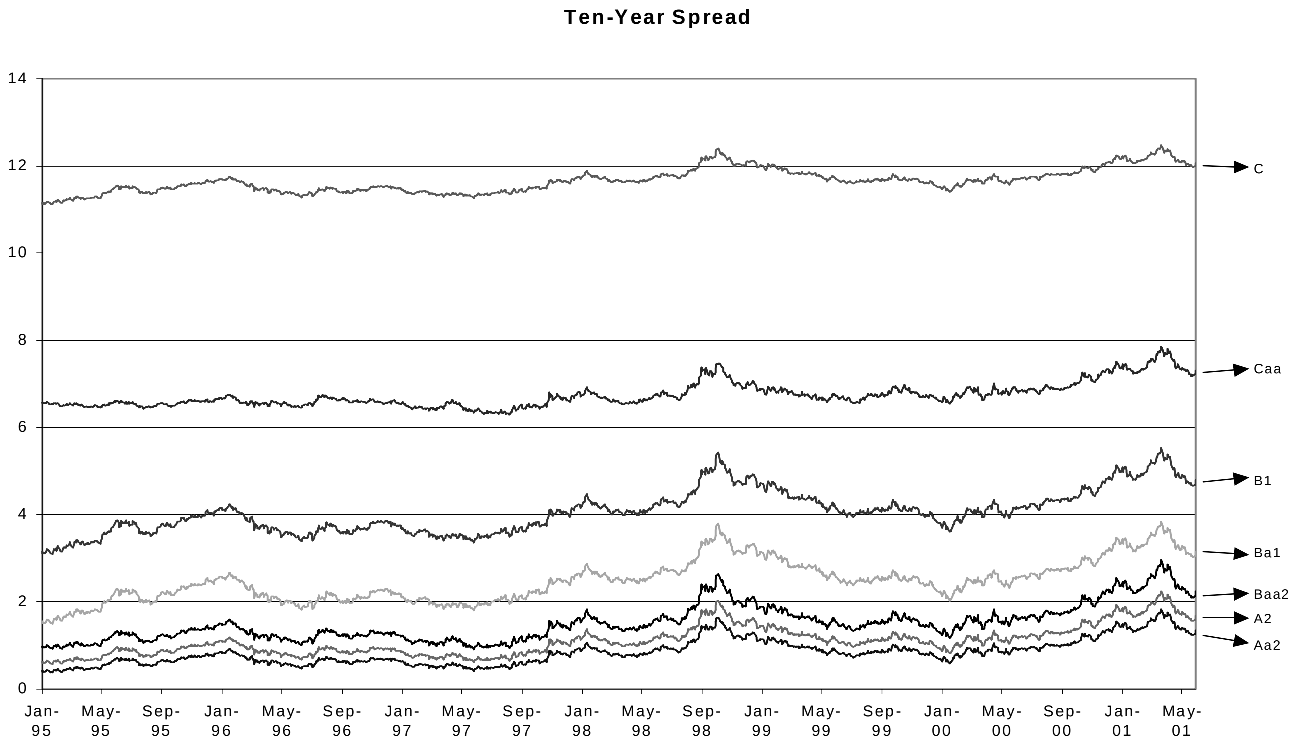
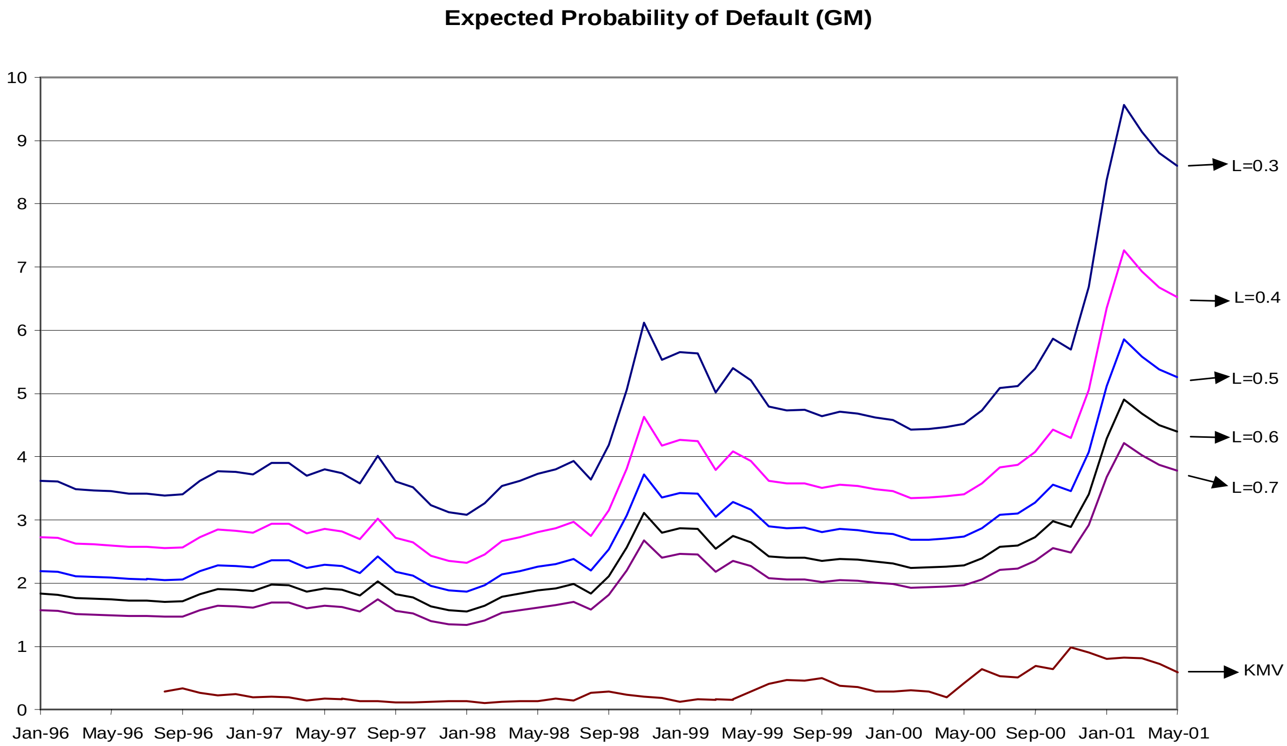


Figure 8. Sensitivity of Estimated Probability of Default (Percentage) to Fractional Loss

Given the estimated intensity function and liquidity function, and the market prices of risk for the state variables, we compute the expected probability of default (EDF) in one year for different levels of fractional loss using data from two individual companies: General Motors (GM) and Merrill Lynch (MER). The sample period is January 1995 to May 2001.



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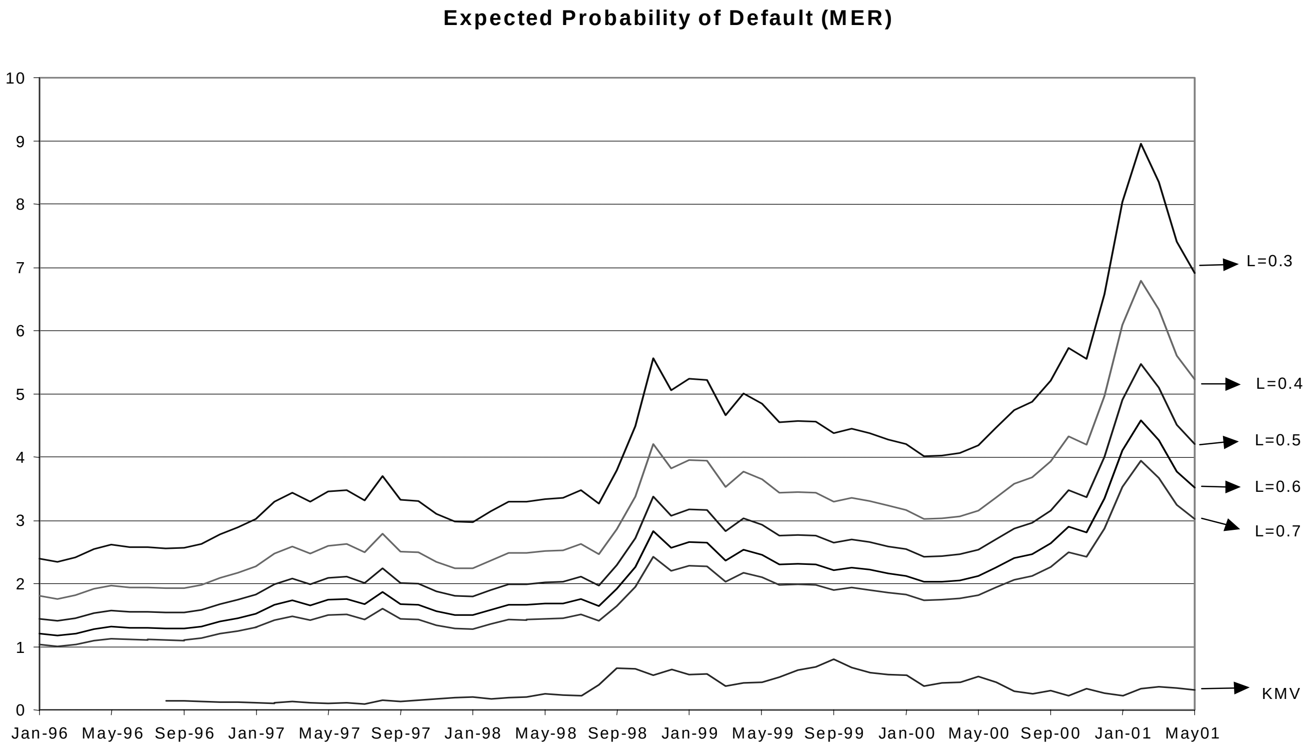

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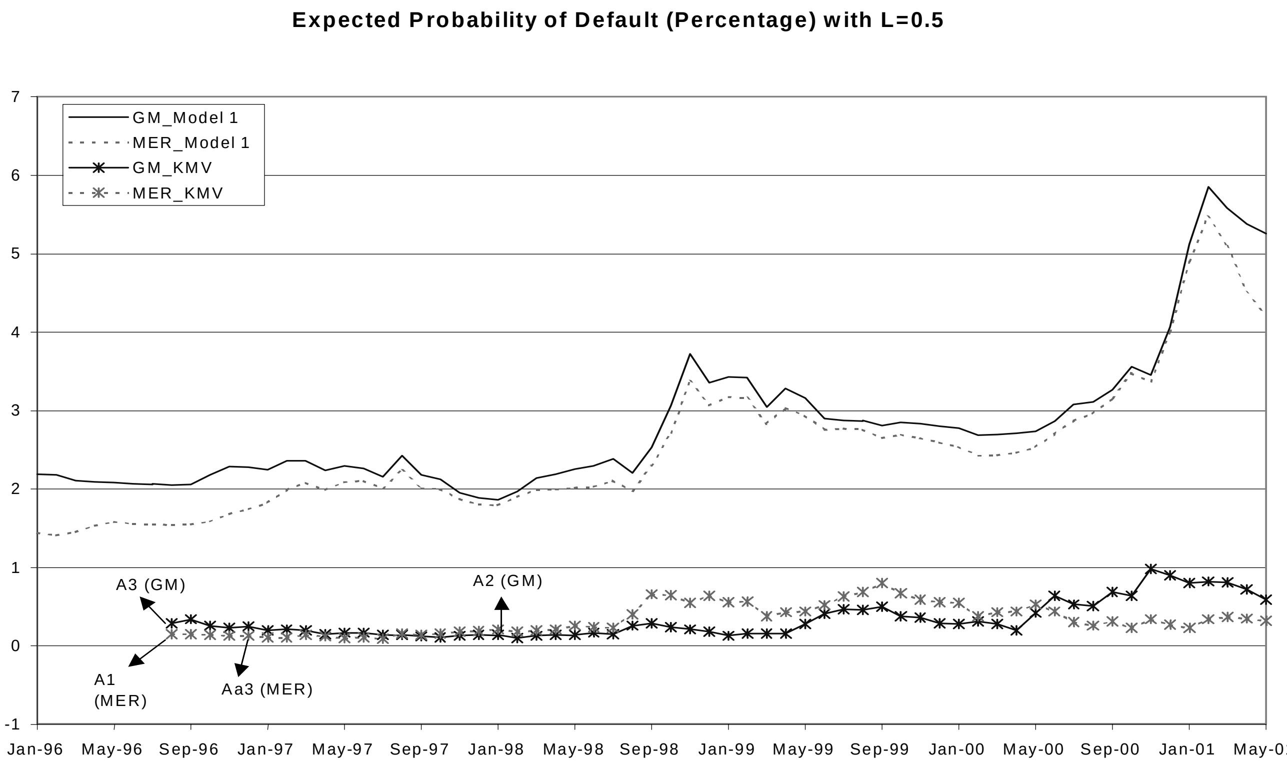

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