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Contents

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Abstract

This paper uses a smooth transition error-correction model (STECM) to model the one-year and five-year mortgage rate changes. The model allows for a non-linear adjustment process of mortgage rates towards their long-run equilibrium. We also introduce time-varying thresholds into the standard STECM specification, to capture the gradual structural changes in the error-correction term. We find that the STECM, whether with fixed or time-varying thresholds, yields better in-sample fit and lower forecast errors than the linear benchmark and univariate models. Our estimation results indicate non-linearities in the adjustment process of mortgage rates towards their long-run equilibria. In particular, we find that mortgage rates respond more significantly to a large than to a small disequilibrium. The improvement of the STECMs in forecasting is statistically significant over the univariate models, but insignificant over the linear model.

JEL classification: C22, C49, E47
Bank classification: Econometric and statistical methods; Interest rates

Résumé

L’auteure recourt à un modèle à correction d’erreurs à transition graduelle (MCETG) pour formaliser l’évolution des taux hypothécaires de un an et de cinq ans. Ce modèle permet un ajustement non linéaire des taux hypothécaires jusqu’à leur niveau d’équilibre à long terme. En outre, l’auteure intègre des seuils variables dans le temps à la formulation type du MCETG, de manière que le terme de correction d’erreurs tienne compte des changements structurels qui sont survenus progressivement. Elle constate que le MCETG, qu’il soit assorti de seuils fixes ou variables dans le temps, se caractérise par une meilleure adéquation statistique sur l’échantillon et par de plus faibles erreurs de prévision que les modèles linéaires de référence et les modèles univariés. Les résultats de l’estimation révèlent des non-linéarités dans le processus d’ajustement des taux hypothécaires vers leur niveau d’équilibre à long terme. En particulier, l’auteure constate que les taux hypothécaires réagissent de façon plus marquée à un déséquilibre important qu’à un faible déséquilibre. Les prévisions que produit le MCETG sont supérieures, du point de vue statistique, à celles des modèles univariés, mais pas à celles du modèle linéaire.

Classification JEL : C22, C49, E47
Classification de la Banque : Méthodes économétriques et statistiques; Taux d’intérêt

1. Introduction

Vector error-correction models (VECMs) are widely used to model economic variables that are non-stationary individually but linked by long-run relationships. A "standard" VECM assumes that these variables follow a linear adjustment process towards their long-run equilibrium. There are, however, economic situations where a non-linear adjustment process may exist. For example, policy intervention may take place only when the economy deviates from equilibrium by a certain margin. The nature of the policy action may also differ, depending on the direction of that deviation. Another example is that arbitrageurs enter the market only if the price deviation of an asset from its no-arbitrage equilibrium is sufficiently large to compensate for transaction costs. Recent literature has described several attempts to construct models that allow for these possible nonlinearities in the error-correction process (e.g., Anderson 1995, Dwyer, Locke, and Yu 1996, and Swanson 1996). This paper examines such possible non-linearities in mortgage rate movements using a smooth transition error-correction model (STECM). We also investigate whether the forecast performance of the model surpasses that of its linear benchmark and univariate models.

In our view, mortgage rates are likely to follow a non-linear adjustment process towards their long-run equilibrium. Given the administrative costs associated with changing posted rates, banks may change mortgage rates only when the cost of mortgage loans, which can be proxied as the mortgage spread (mortgage rate minus government bond rate of the same maturity), deviates from its equilibrium by a certain margin. This potential non-linearity can be captured by a STECM, in which observations switch between regimes as a function of the error-correction term. This specification is particularly useful in models where one variable changes when (and only when) its long-run relationship with the others deviates significantly from equilibrium.

Our study applies a three-regime STECM suggested by van Dijk and Franses (1997) to model the relationship between mortgage and government bond rates. The three regimes are "large" positive disequilibrium, "small" disequilibrium, and "large" negative disequilibrium. In the "small" disequilibrium, or middle regime, the mortgage spread is close to its equilibrium value, and thus mortgage rate changes are less likely to happen. In contrast, mortgage rate changes are more likely to happen in the two "large" disequilibrium, or outer, regimes. The switching between these regimes is determined by a transition function, which depends on the location of the threshold value and the speed of the switching process.

We introduce a time-varying version of the smooth transition mechanism. Existing applications of STECM typically assume that regime switching occurs at the same location throughout the sample period. Our time-varying STECM allows this process to differ based on the mean and variance of the error-correction term across time. This extension is inspired by the fact that the mortgage spread, which is specified as the error-correction term in the model, is slightly upward-sloping and becomes less volatile in the late 1990s (see Graphs 1 and 3).[1] This means that a higher threshold value of the spread and a smaller absolute deviation from the equilibrium are required to trigger a mortgage change in the later part of the sample. The standard STECM that produces a constant threshold value tends to ignore these gradual structural changes.

We find that our STECMs outperform the linear benchmark model in both estimation and out-of-sample forecasting. In particular, the time-varying STECM seems to produce the best estimation and forecast performance among competing models. The results from these non-linear models suggest that mortgage rate changes are relatively more pronounced in response to large than to small deviations of the mortgage spread from its equilibrium. They also imply that a smaller absolute deviation of the spread from its historical average (over the previous six months) is required to trigger a mortgage rate rise than to trigger a rate fall. This finding is consistent with the gradually rising mortgage spreads in the latter half of the 1990s. Furthermore, we find that, across time, the middle regime has narrowed, which implies that a smaller deviation of the mortgage spread from its equilibrium is needed to trigger a mortgage rate change. As a result, mortgage rate changes have become smaller but more frequent.

The rest of this paper is organized as follows. In section 2 we set up a linear benchmark model and discuss its estimation results. In section 3 we describe the specifications and estimations of the two versions of our STECM, and the results of a linearity test. In section 4 we compare the forecast performances between the linear VECM and the STECMs using point estimates of loss functions and forecast-accuracy tests. In section 5 we conclude and suggest avenues for further research.

2. Linear Model

We first establish a backward-looking linear model as a benchmark for explaining mortgage rate changes. A number of variables are found to explain mortgage rate movements. For example, using a two-equation simultaneous model, Clinton and Howard (1994) find that guaranteed

1. There are various explanations for the gradually rising mortgage spreads. One possible explanation is that, under an increasingly competitive market, banks may have been more aggressive in giving discounts (off posted rates) and embedded options to customers in the latter half of the 1990s. As a result, they may have posted rates that were higher than previously required to cover the cost. The reduced volatility in the spread may also have been related to a more competitive mortgage market. However, this paper does not attempt to investigate the validity of these explanations.

investment certificate (GIC) rates, the prime rate, and government bond rates are all statistically significant in explaining changes in the mortgage rate. While the contemporaneous values of the GIC and prime rates are used in their model, the two variables are less useful in a forecasting framework, since banks tend to change all their administered rates at the same time. Therefore, we include only lagged values of the mortgage rate itself and of the government bond rate in our model:

$$\Delta m_t = \beta_0 + \beta_1 \Delta m_{t-1} + \beta_2 \Delta r_{t-1} + \beta_3 \Delta r_{t-2} + \beta_4 \Delta r_{t-3} + \beta_5 (m_{t-1} - r_{t-1}) + \varepsilon_t, \qquad (1)$$

where m is the one-year or five-year weekly (Wednesday) level of the chartered bank closed mortgage rate, and r is the weekly benchmark government bond yield of the same maturity. Government bond rates are included because they are commonly used by chartered banks to set their mortgage rates (see Clinton and Howard 1994). We also add the interest rate swap premium onto the five-year bond rate, to better reflect the cost of funding mortgage loans.[2] The sample period is from 6 January 1993 to 27 September 2000 (404 observations).[3]

The mortgage and government bond rates are modelled in their first differences because a variety of unit root tests suggest that both are I(1) series. In addition, the Johansen and Juselius (1990) trace and rank cointegration tests imply that the two interest rate series are cointegrated. Following Clinton and Howard (1994), we restrict the coefficients on the two variables to be equal in the long run. That is, we use the mortgage spread as the error-correction term. Finally, the lag order of 3 is chosen by minimizing the values of the Akaike (1973) Information Criterion (AIC) based on the model in levels.[4] The coefficients on the second and third lags of mortgage rates are restricted to zero, since the F-tests suggest that these lags are not jointly significant in the regression. Tables 1, 2, and 3 report the results of these pretest procedures.

As Table 4 shows, most of the estimated coefficients from model (1) are statistically significant and consistent with expectations. Both the regression specification error test (RESET) and the autoregressive conditional heteroscedasticity (ARCH) test imply that there might be neglected

2. Banks often engage in interest rate swap contracts to hedge the risk caused by mismatching maturities between their deposits and mortgage loans. Although interest swaps also occasionally occur in one-year mortgage loans, the swap premium is much smaller (approximately 5 to 10 basis points in the sample period) and hence it is ignored in this study.
3. Data from the 1980s and early 1990s are not used, since the results from a Chow test indicate that there is a structural break in the available sample period. The current sample period is chosen based on a brief grid search. The F-statistics of the test for the one-year and five-year equations are 9.92 and 5.81, respectively, and the p-values are both 0.00.
4. An optimal order of four lags in levels means three lags in differences.

non-linearities in the linear model.[5] Given these diagnostic test results, we specify in section 3 a non-linear STECM to help capture the possible non-linearities in the model.

3. Non-Linear STECMs

The most probable cause of non-linearity in this model is that banks may react more significantly to a large than to a small deviation of the mortgage spread from equilibrium. A STECM allows the observations in our model to switch between regimes as a function of this deviation. Thus, the model is a good candidate to consider in our non-linear specification. In this section, we introduce the general specification of a STECM. We then present two versions of our STECM, the results of a linearity test, and the estimation of these models.

3.1 General specification of a STECM

STECMs are extensions of the family of smooth transition autoregressive (STAR) models first introduced by Chan and Tong (1986) and Tong (1990). Both models take the general form

$$y_t = \beta' x_t + (\theta' x_t) F(z_{t-d}; \gamma, c) + \varepsilon_t, \tag{2}$$

where β and θ are vectors of coefficients, x_t is a vector of explanatory variables, F(.) is a bounded transition function that has values between 0 and 1, z_{t-d} is the transition variable, the parameter γ denotes the speed of the transition from 0 to 1 (the larger the γ, the faster the transition), and c is the "threshold parameter" that determines where the transition occurs. When F(.) = 0, y_t is explained by $\beta' x_t$ alone. As F(.) approaches 1, the model is explained more and more by $(\beta' x_t + \theta' x_t)$.

The difference between the STAR and the STECM is the transition variable, z_{t-d}. In a STAR model, z_{t-d} is a lagged dependent variable, y_{t-d}, and in a STECM, z_{t-d} is the error-correction term.

Tong (1990) proves that F(.) can be defined in many ways as long as it is continuous and non-decreasing. A popular function for F(.) is a logistic function proposed by Granger and Teräsvirta (1993):

$$F(z_{t-d}; \gamma, c) = (1 + \exp\{-\gamma(z_{t-d} - c)\})^{-1}, \gamma > 0. \tag{3}$$

5. The ARCH test detects heteroscedastic errors. However, a possible cause for the rejection of homoscedasticity is the presence of neglected non-linearity.

When $\gamma = 0$, F(.) = 1/2, and the model is a linear model; when γ approaches positive infinity, F(.) approaches 1 for $z_{t-d} > c$, and F(.) approaches 0 for $z_{t-d} < c$. Function (3) can be extended to nest three regimes, as follows:

$$F(z_{t-d};\gamma, c) = (1 + \exp\{-\gamma(z_{t-d} - c_1)(z_{t-d} - c_2)\})^{-1}, \gamma > 0, c_2 > c_1. \tag{4}$$

Here, when γ approaches positive infinity, F(.) approaches 0 when $c_1 < z_{t-d} < c_2$, and 1 when $z_{t-d} > c_2$ or $z_{t-d} < c_1$.[6]

3.2 Two versions of our STECM

In our model, the mortgage spread is the error-correction term and thus specified as the transition variable, z_{t-d}. We use the transition function in (4) to allow three regimes in the model: a "large" positive disequilibrium, a "small" disequilibrium, and a "large" negative disequilibrium. In the "small" equilibrium, or middle regime, the mortgage spread is not significantly different from its equilibrium level; thus, mortgage rate changes are less likely to occur. In the two outer regimes, the mortgage spread deviates from equilibrium by enough to warrant either a rate rise or rate cut to restore equilibrium, depending on the direction of the deviation.

Based on the linear model (1), our first STECM takes the following form:

$$\begin{aligned}\Delta m_t = {} & \beta_0 + \beta_1\Delta m_{t-1} + \beta_2\Delta r_{t-1} + \beta_3\Delta r_{t-2} + \beta_4\Delta r_{t-3} + \beta_5(m_{t-1} - r_{t-1}) \\ & + [\theta_0 + \theta_1\Delta m_{t-1} + \theta_2\Delta r_{t-1} + \theta_3\Delta r_{t-2} + \theta_4\Delta r_{t-3} + \theta_5(m_{t-1} - r_{t-1})] \\ & \times [1 + \exp\{-\gamma(z_{t-d} - c_1)(z_{t-d} - c_2)\}]^{-1},\end{aligned} \tag{5}$$

where

$$\frac{\partial\Delta m_t}{\partial\Delta m_{t-1}} = \beta_1 \quad \text{when F(.) = 0}$$

$$\frac{\partial\Delta m_t}{\partial\Delta m_{t-1}} = \beta_1 + \theta_1 \quad \text{when F(.) = 1, etc.}$$

Our second STECM allows c_1 and c_2 to vary across time, based on the mean and variance of the transition variable, z_{t-d}, in different time periods. In (5), c_1 and c_2 are fixed so that regime

6. This model treats the upper and lower regimes the same. A transition function incorporating three or more *different* regimes can be derived from (4). However, the estimation of such a model involves a large number of coefficients, thus introducing degree-of-freedom problems in a relatively small sample like ours. We do not consider this case in this study. For a discussion of a multi-regime STAR or STECM, see Van Dijk and Franses (1999).

switching occurs at the same location throughout the sample period. This extension of the model produces two time-varying "threshold bands" that may better identify the different regimes in the model across time.

We specify *F(.)* as

$$\left[1 + \exp\left\{-\left(\frac{\gamma(z_{t-d} - (MA_{t-1} - d_1 \times STD_{t-1}))(z_{t-d} - (MA_{t-1} - d_2 \times STD_{t-1}))}{VAR_{t-1}}\right)\right\}\right]^{-1}, \quad (6)$$

where MA_{t-1} is the 26-week backward-looking moving average of the mortgage spread, and STD_{t-1} and VAR_{t-1} are the corresponding moving standard deviation and variance, respectively.[7] As evident, c_1 in (5) is replaced by MA_{t-1} - d_1*STD_{t-1}, and c_2 by MA_{t-1} - d_2*STD_{t-1}. The inclusion of VAR_{t-1} is to correct the scaling problem of F(.) arising from the varying volatility of the mortgage spread.

3.3 Linearity test

Before estimating the proposed STECMs, we conduct the Lagrange multiplier (LM) type test developed by Luukkonen, Saikkonen, and Teräsvirta (1988) to determine whether our STECM specification is necessary (as opposed to a linear specification). A second purpose of the test is to obtain an estimate of the lag *d* for the transition variable, z_{t-d}, if the test suggests a non-linear error-correction process.

The null hypothesis of the test is Ho: $\gamma = 0$. However, under the null hypothesis, the model is not identified since the form of F(.) is unknown. This would make the usual asymptotic theory inapplicable for deriving the LM tests (Davies 1987). Thus Luukkonen, Saikkonen, and Teräsvirta (1988) suggest that F(.) be replaced by a third-order Taylor approximation, and that (2) be rewritten as follows:

$$\Delta y_t = \beta' x_t + \theta' x_t + \theta_1' x_t z_{t-d} + \theta_2' x_t z^2_{t-d} + \theta_3' x_t z^3_{t-d} + \eta_t. \quad (7)$$

The null hypothesis in favour of a linear model then becomes Ho: $\theta_1 = \theta_2 = \theta_3 = 0$. As Table 5 shows, linearity is rejected in both the one-year rate and five-year rate equations. Granger and

7. There are potentially many ways to estimate a time-varying short-term trend of the mortgage spread. We experiment with backward-looking moving averages for the convenience of constructing out-of-sample forecasts. Another advantage of this approach is that volatility can be incorporated in the estimates of the two "threshold bands." Current lengths of the moving-average window are chosen through a brief grid search based on estimation performance; e.g., significance of coefficients. Results are relatively robust to the choice of the moving-average window.

Teräsvirta (1993) suggest that the transition variable with the lowest *p*-value be used. Thus, both equations use z_{t-1} as the transition variable.

3.4 Estimation results of STECMs

The STECMs are estimated for both the one-year and five-year rate equations using a non-linear least-squares procedure. Table 6 gives the results of the fixed-threshold model and Table 7 gives those of the time-varying threshold model. Because it is difficult to achieve convergence when all parameters are estimated at the same time, γ is chosen based on a grid search procedure. The chosen values of γ minimize the value of the residual sums of squares.

The results from the fixed-threshold model suggest that mortgage rates are more sensitive to changes in government bond rates and the mortgage spread when the latter lies in the outer regime. For example, the one-year mortgage rate changes by 35 ($\beta_5 + \theta_5$) basis points (bp) per week after a 100 bp increase in the mortgage spread in the outer regime, compared to a change of 11 (β_5) bp per week in the middle regime. This non-linearity is also reflected in the coefficients of the lagged changes of government bond rates. For a 100 bp change in each of the previous three consecutive weeks, the one-year rate responds 99 ($\beta_2 + \beta_3 + \beta_4 + \theta_2 + \theta_3 + \theta_4$) bp when the disequilibrium is large enough, while the response is only 49 ($\beta_2 + \beta_3 + \beta_4$) bp otherwise. These non-linearities are also evident in the five-year rate case.

As Graphs 2 and 4 show, the model fails to identify between different regimes in certain periods. For example, in mid-1999 to mid-2000 for the one-year rate, and in 1997–98 for the five-year rate, the indicator function is equal to zero. This means that, during those periods, the mortgage spread remains in the middle regime, where mortgage rates are less sensitive to government bond rate changes. However, mortgage rate changes did take place regularly in those periods. A possible cause of this anomaly is that the fixed width of the estimated threshold bands is inconsistent with the time-varying nature of the mortgage spread. Indeed, the estimated thresholds are 1.30 and 2.16 for the one-year rate and 0.61 and 1.78 for the five-year rate. While these threshold values seem to be reasonable for the earlier part of the sample, they seem to be too wide for the later part of the sample (see Graphs 1 and 3).

Our time-varying threshold specification is designed to help correct this problem. As Graphs 5 and 7 show, the resulting threshold bands from the time-varying model evolve as a function of the mean and variance of the mortgage spread in a certain time period (in our model, the 26 preceeding weeks). This methodology assigns observations to different regimes across time, which is consistent with the fact that mortgage rate adjustments take place throughout the sample period.

The resulting indicator functions in Graphs 6 and 8 switch from 0 to 1 regularly across time, which implies that this model allows regime switching to take place throughout the sample period. One interesting finding is that the lower threshold band is closer to the moving-average series than the upper band. This implies that a smaller deviation of the spread from its average (over the previous six months) is required to trigger a mortgage rate rise than to trigger a rate fall, consistent with the gradually rising mortgage spreads. Another interesting finding is that the range of the middle regime narrows towards the end of the sample, which implies that a smaller absolute deviation of the mortgage spread from its equilibrium is needed to trigger a mortgage rate change. As a result, mortgage rate changes have become smaller but more frequent.[8]

The estimated coefficients in Table 7 suggest more evidence of non-linearities in the adjustment process of mortgage rates towards their equilibria.[9] One-year mortgage rates respond about 2.2 times as much to changes in government bond rates when the spread is large than if it were small.[10] This ratio is 2.7 times in the five-year equation. In the middle regime, the model yields an almost zero coefficient on the mortgage spread, while in the outer regime changes in the mortgage spread result in an almost one-for-one response in the one-year mortgage rate in the following week. Again, this non-linearity is also evident in the five-year rate case.

The $\bar{R}^2$ values are higher in both STECMs than in the linear VECM, implying a better fit of the model. The time-varying STECM also yields a better fit of the data than the fixed-threshold model. With time-varying thresholds, the $\bar{R}^2$ is improved from 0.30 to 0.43 for the one-year equation and from 0.29 to 0.32 for the five-year equation. Also, the coefficients in the linear model are generally larger in absolute value than those in the base regime (βs), and smaller than those from the outer regime (βs + θs) in the STECMs. This implies that the linear model fails to distinguish between these regimes. In addition, the ARCH tests under both STECMs show that the null of homoscedasticity cannot be rejected, suggesting that the heteroscedasticity found in the linear model could be the result of neglected non-linearity.

4. Forecast Performance Comparison

Another way to determine whether these non-linear models contribute to explaining mortgage rate behaviour is to compare the forecast performance between the linear and non-linear VECMs. We also present the forecasts from an AR(1) and a no-change model, since any model should at least

8. This may suggest increased competition in the mortgage lending market.
9. The high volatility of the mortgage rate spread in 1993 creates less satisfactory estimation results in the five-year equation. Thus, we use a sample from 12 January 1994 to 27 September 2000.
10. $(\beta_2 + \beta_3 + \beta_4 + \theta_2 + \theta_3 + \theta_4)/(\beta_2 + \beta_3 + \beta_4)$.

outperform naive univariate benchmarks if it is to be useful as a forecasting tool (Chatfield 1993). We first compare the mean-square errors (MSE) and mean absolute deviations (MAD) of each model. Then we conduct statistical tests to investigate whether the differences between these loss functions are significant.

4.1 Point estimates of loss functions

We obtain out-of-sample forecasts over 138 consecutive weeks for 11 February 1998 to 27 September 2000. We limit our attention to the performance of adaptive one-step-ahead forecasts of the model. That is, each forecast observation is generated by using sample information up to the last period. Table 8 lists the MSE and MAD of both the linear and non-linear models. Both versions of the STECM seem to marginally outperform the linear VECM and the two naive univariate models. The time-varying threshold STECM has the lowest values of both loss functions. In addition, regardless of the model used, the one-year rates seem to have smaller forecast errors than the five-year rates.

4.2 Forecast-accuracy tests

As Diebold and Mariano (1995) suggest, point estimates of forecast accuracy such as MSE and MAD ignore the sampling uncertainties. Thus we apply two formal testing procedures to assess whether differences between the values of these loss functions can be attributed to sample variability, or whether they are "significant."

The first procedure is a general forecast-encompassing test, which involves regressing the forecast errors of Model A on the difference between the predicted values from Model A and Model B. A *t*-ratio test is then applied to the coefficient estimate. The null hypothesis is that the difference between the two models in predicting the dependent variable is not significantly different. We specify the model with a smaller value of the loss function as Model A and its competing model as Model B.

Our second procedure is a variance-based test proposed by Diebold and Mariano (1995). The null hypothesis is Ho: $E[g(e_{At})] = E[g(e_{Bt})]$, where g(.) is the loss function of the forecast errors, namely MSE and MAD in our study. The test statistic is

$$S = \frac{\bar{d}}{\hat{\sigma}_d} \underset{\text{null}}{\overset{\text{asy}}{\sim}} N(0, 1), \tag{8}$$

where $\bar{d}$ is the sample mean of the loss differentials specified as $d_t = e^2_{At} - e^2_{Bt}$ for MSE and $d_t = | e_{At} | - | e_{Bt} |$ for MAD, and $\hat{\sigma}_d$ is a consistent estimator of the standard errors of d_t.

To take care of possible correlation and heteroscedasticity in the d_t series, we adopt the parametric covariance matrix estimation procedure of den Haan and Levin (1996). This procedure involves testing the lag order of an ARMA(p,q) model for

$$d_t = \sum_{j=1}^{p} \alpha_j d_{t-j} + \sum_{k=0}^{q} \beta_k v_{t-k}, \tag{9}$$

where v_t is the t^{th} OLS residual. Assuming stationarity of d_t, a consistent estimator of σ_d is then

$$\hat{\sigma}_d^2 = \frac{\left[1 + \sum_{k=1}^{\bar{q}} \hat{\beta}_k\right]}{\left[1 - \sum_{j=1}^{\bar{p}} \hat{\alpha}_n\right]} \hat{\sigma}^2_v, \tag{10}$$

where $\hat{\sigma}^2_v$ is the estimated variance of the OLS residuals, and $\bar{p}$ and $\bar{q}$ are the estimated ARMA orders for the autoregressive and moving-average terms, respectively.

Table 8 shows the results of these two tests. Since the time-varying STECM yields the lowest point estimates of the two loss functions, we test the forecast performance of this model against that from each of the other models. The results suggest that the improvement of our best-performing STECM is statistically insignificant over either the linear VECM or the fixed-threshold STECM, but significant over the naive univariate models. The STECM seems to forecast the one-year rate slightly better than the five-year rate relative to its linear counterparts.

5. Conclusion

This paper has attempted to model mortgage rate changes in a STECM framework, allowing for a non-linear error-correction process towards their long-run equilibrium. In addition to the standard STECM, we introduce a time-varying version of the smooth transition mechanism. Estimation results from both STECMs suggest that mortgage rate changes are relatively more significant in response to large than to small deviations of the mortgage spread from its equilibrium. The threshold bands from the time-varying STECM imply that a smaller deviation of the spread from its historical average (over the previous six months) is required to trigger a mortgage rate rise than to trigger a mortgage rate fall. These bands also suggest that, across time, the middle regime has narrowed, implying that a smaller absolute deviation of the mortgage spread from its equilibrium is needed to trigger a mortgage rate change. As a result, mortgage rate changes have become smaller but more frequent. In terms of forecasting, we find that the STECM with time-varying thresholds yields the lowest forecast errors among all competing models. This advantage is statistically significant over the univariate models, but insignificant over the linear model and the fixed-threshold STECM.

More work is required to better model the time-varying nature of the error-correction process. It may be worthwhile to modify the time-varying smooth transition autoregressive (TV-STAR) models introduced by Lundbergh, Teräsvirta, and van Dijk (2000) to incorporate the error-correction process. However, these models involve a large number of the parameter estimates and many pretesting and specification issues. They may be more applicable to higher-frequency data such as stock prices. Further efforts may also be devoted to exploring more alternative estimates of the time-varying threshold bands; e.g., using alternative estimates of the short-term trend of the error-correction term, instead of its moving average. Finally, while we use asymptotic forecast-accuracy tests, future research can generate Monte Carlo simulated distributions of these tests to better compare the forecast performance of these models.

The smooth transition mechanism in STECMs supplements traditional linear VECMs with non-linear features that often occur in economic series. Compared to simple threshold models, STECMs allow for possible “stickiness” in the adjustment process by specifying the “threshold” as a continuous function. Unlike non-parametric models, this methodology yields coefficient and function estimates that are tractable and interpretable. Thus it provides a more transparent analytical framework and is more useful when the purpose of a study is to do more than forecasting. The extension to allow for time-varying threshold values seems to be useful when the location of the adjustment process also changes across time. This paper serves as an example of the many possible applications of this methodology. Researchers may also find it useful in modelling other non-linear economic relationships, such as a convex Phillips curve and nominal rigidities in the transmission mechanism.

Bibliography

Akaike, H. 1973. "Information Theory and an Extension of the Maximum Likelihood Principle." In *2nd International Symposium on Information Theory*, 267–81, edited by B.N. Petrov and F. Csaki. Budapest: Academiai Kiado.

Anderson, H.M. 1995. "Transaction Costs and Nonlinear Adjustment Towards Equilibrium in the US Treasury Bill Market." Working paper, University of Texas, Austin.

Chan, K.S. and H. Tong. 1986. "On Estimating Thresholds in Autoregressive Models." *Journal of Time Series Analysis* 7: 179–90.

Chatfield, C. 1993. "Neural Networks: Forecasting Breakthrough or Passing Fad?" *International Journal of Forecasting* 9: 1–3.

Clinton, K. and D. Howard. 1994. *From Monetary Policy Instruments to Administered Interest Rates: The Transmission Mechanism in Canada.* Technical Report No. 69. Ottawa: Bank of Canada.

Davies, R.B. 1987. "Hypothesis Testing when a Nuisance Parameter is Present Only Under the Alternative." *Biometrika* 64: 33–43.

den Haan, W.J. and A. Levin. 1996. "A Practitioner's Guide to Robust Covariance Matrix Estimation." NBER Working Paper No. 197.

Diebold, F.X. and R.S. Mariano. 1995. "Comparing Predictive Accuracy." *Journal of Business and Economic Statistics* 13: 253–63.

Dueker, M. 1999. "Conditional Heteroscedasticity in Qualitative Response Models of Time Series: A Gibbs-Sampling Approach to the Bank Prime Rate." *Journal of Business and Economic Statistics* 17: 466–72.

Dwyer, G.P., P. Locke, and W. Yu. 1996. "Index Arbitrage and Nonlinear Dynamics between the S&P 500 Futures and Cash." *Review of Financial Studies* 9: 301–32.

Eichengreen, B., M.W. Watson, and R.S. Grossman. 1985. "Bank Rate Policy under the Interwar Gold Standard." *Economic Journal* 95: 735–45.

Goldberg, M.A. 1984. "The Sensitivity of the Prime Rate to Money Market Conditions." *The Journal of Financial Research* 7: 269–80.

Granger, C.W. and T. Teräsvirta. 1993. *Modelling Nonlinear Economic Relationships.* New York: Oxford University Press.

Hendry, S. 1992. "Determinants of the Prime Rate: 1975–1989." Bank of Canada Working Paper No. 1992–2.

Johansen, S. and K. Juselius. 1990. "Maximum Likelihood Estimation and Inference on Cointegration—With Application to the Demand for Money." *Oxford Bulletin of Economics and Statistics* 52: 160–210.

Lundbergh, S., T. Teräsvirta, and D. van Dijk. 2000. "Time-Varying Smooth Transition Autoregressive Models." SSE/EFI Working Paper Series in Economics and Finance, No. 376.

Luukkonen, R., P. Saikkonen, and T. Teräsvirta. 1988. "Testing Linearity Against Smooth Transition Autoregressive Models." *Biometrika* 75: 491–99.

Manouchehri, A. and G. Proulx. 1997. "Can Mortgage Rate-Bond Yield Spreads Predict Mortgage Rate Changes?" CMHC, *Trends,* 3rd Quarter.

Swanson, N.R. 1996. "Finite Sample Properties of a Simple LM Test for Neglected Nonlinearity in Error-Correcting Regression Equations." University of Pennsylvania Working Paper.

Teräsvirta, T. 1994. "Specification, Estimation, and Evaluation of Smooth Transition Autoregressive Models." *Journal of the American Statistical Association* 89: 208–18.

Tong, H. 1990. *Nonlinear Time Series—A Dynamical System Approach*. Oxford: Clarendon Press.

van Dijk, D. and P.H. Franses. 1997. "Nonlinear Error-Correction Models for Interest Rates in the Netherlands." Erasmus University, Rotterdam, Econometric Institute Report 9704/A.

van Dijk, D. and P.H. Franses. 1999. "Modelling Multiple Regimes in the Business Cycle." *Macroeconomic Dynamics* 3: 311–40.

Table 1: Unit Root Test Results, 6 January 1993 to 27 September 2000

Variable	ADF	Phillips-Peron	Critical value	Decision
m1	-1.45	-1.08	-2.86	I(1)
r1	-1.87	-2.27	-2.86	I(1)
m1-r1	-6.85	-8.65	-2.86	I(0)
m5	-1.69	-1.47	-2.86	I(1)
r5	-1.53	-1.67	-2.86	I(1)
m5-r5	-5.39	-6.37	-2.86	I(0)

Table 2: Cointegration Test Results, 6 January 1993 to 27 September 2000

Equation	Cointegrating vector	Trace statistic[a]	Rank statistic[b]	Lags[c]	Decision
m1	1.00	86.92	83.44	4	system cointegrated
r1	-0.92				
m5	1.00	68.62	65.19	4	system cointegrated
r5	-0.93				

a. Critical value to reject the null of no cointegration is 13.31 at 90 per cent confidence interval.
b. Critical value to reject the null of no cointegration is 10.60 at 90 per cent confidence interval.
c. Lag length for the VAR is chosen by minimizing the AIC criterion.

Table 3: Results of F-tests on Restricted Lagged Coefficients

Equation	Variable	F(2, 393)	*p*-value
$\Delta m1$	$\Delta m1_{t-2}$, $\Delta m1_{t-3}$	2.05	0.13
$\Delta m5$	$\Delta m5_{t-2}$, $\Delta m5_{t-3}$	1.45	0.24

Table 4: Estimation Results of Linear Model, 6 January 1993 to 27 September 2000

Variable	Equation[a]	
	Δm1	Δm5
β_0, Constant	0.13 (3.82)	0.14 (3.83)
β_1, Δm_{t-1}	-0.09 (-1.63)	-0.12 (-2.31)
β_2, Δr_{t-1}	0.27 (6.33)	0.26 (5.08)
β_3, Δr_{t-2}	0.23 (5.58)	0.25 (5.09)
β_4, Δr_{t-3}	0.09 (2.39)	0.15 (3.31)
β_5, m_{t-1}-r_{t-1}	-0.09 (-3.81)	-0.09 (-3.89)
$\overline{R}^2$	0.24	0.22
ARCH Chi-squared(1) test statistics[b]	4.50 (0.03)	3.57 (0.06)
RESET 2nd order F(1, 394) test statistics[c]	54.03 (0.00)	25.55 (0.00)
RESET 3rd order F(1, 393) test statistics	257.64 (0.00)	215.78 (0.00)
RESET 4th order F(1, 392) test statistics	189.28 (0.00)	179.02 (0.00)

a. T-ratios in parentheses.
b. Ho: homoscedastic errors. *p*-values in parentheses.
c. Ho: linear specification is valid. *p*-values in parentheses.

Table 5: Results of Luukkonen, Saikkonen, and Teräsvirta (1988) Linearity Tests, 6 January 1993 to 27 September 2000 [a]

Transition variable	Δm1		Δm5	
	Chi-squared(24) test statistic	*p*-value	Chi-squared(24) test statistic	*p*-value
$(m-r)_{t-1}$	85.21	0.00	75.28	0.00
$(m-r)_{t-2}$	63.25	0.00	68.25	0.00
$(m-r)_{t-3}$	45.17	0.15	59.47	0.00
$(m-r)_{t-4}$	39.54	0.18	41.07	0.03

a. Ho: Model is linear. Ha: in favour of a STECM specification.

Table 6: Estimation Results of STECM: Fixed Thresholds, 6 January 1993 to 27 September 2000[a]

Variable	Δm1	Δm5
β_0, constant	0.18 (2.54)	0.07 (1.14)
β_1, Δm_{t-1}	-0.15 (-2.04)	-0.05 (-0.86)
β_2, Δr_{t-1}	0.29 (5.27)	0.18 (3.09)
β_3, Δr_{t-2}	0.13 (2.52)	0.17 (3.08)
β_4, Δr_{t-3}	0.07 (1.62)	0.11 (2.25)
β_5, m_{t-1}-r_{t-1}	-0.11 (-2.61)	-0.05 (-1.09)
θ_0, constant	0.20 (1.88)	0.31 (2.72)
θ_1, Δm_{t-1}	0.04 (0.33)	-0.38 (-2.16)
θ_2, Δr_{t-1}	0.07 (0.80)	0.29 (2.17)
θ_3, Δr_{t-2}	0.29 (3.36)	0.31 (2.42)
θ_4, Δr_{t-3}	0.14 (1.83)	0.18 (1.48)
θ_5, m_{t-1}-r_{t-1}	-0.24 (-2.80)	-0.15 (-2.30)
c_1	1.30 (61.18)	0.61 (6.59)
c_2	2.16 (60.03)	1.78 (32.34)
γ	70.00	50.00
$\overline{R}^2$	0.30	0.29
ARCH Chi-squared(1) test statistics[b]	0.45 (0.50)	0.57 (0.45)

a. T-ratios in parentheses.
b. Ho: homoscedastic errors. *p*-values in parentheses.

Table 7: Estimation Results of STECM: Time-Varying Thresholds[a]

Variable	Δm1 6 Jan. 1993– 27 Sep. 2000	Δm5 6 Jan. 1993– 27 Sep. 2000
β_0, constant	0.03 (0.81)	0.01 (0.29)
β_1, Δm_{t-1}	-0.06 (-1.25)	-0.07 (-1.19)
β_2, Δr_{t-1}	0.18 (4.33)	0.21 (3.48)
β_3, Δr_{t-2}	0.14 (3.54)	0.18 (3.10)
β_4, Δr_{t-3}	0.06 (1.96)	0.08 (1.54)
β_5, m_{t-1}-r_{t-1}	-0.02 (-0.91)	-0.01 (-0.32)
θ_0, constant	0.39 (2.92)	0.58 (3.88)
θ_1, Δm_{t-1}	-0.17 (0.74)	-0.08 (0.32)
θ_2, Δr_{t-1}	0.24 (2.67)	0.26 (1.23)
θ_3, Δr_{t-2}	0.18 (3.32)	0.19 (2.56)
θ_4, Δr_{t-3}	0.04 (1.49)	0.23 (1.51)
θ_5, m_{t-1}-r_{t-1}	-0.89 (-5.95)	-0.41 (-4.01)
d_1	1.92 (22.97)	1.70 (8.80)
d_2	2.46 (40.61)	1.91 (12.04)
γ	1.70	2.00
$\bar{R}^2$	0.43	0.32
ARCH Chi-squared(1) test statistics[b]	0.21 (0.65)	0.05 (0.82)

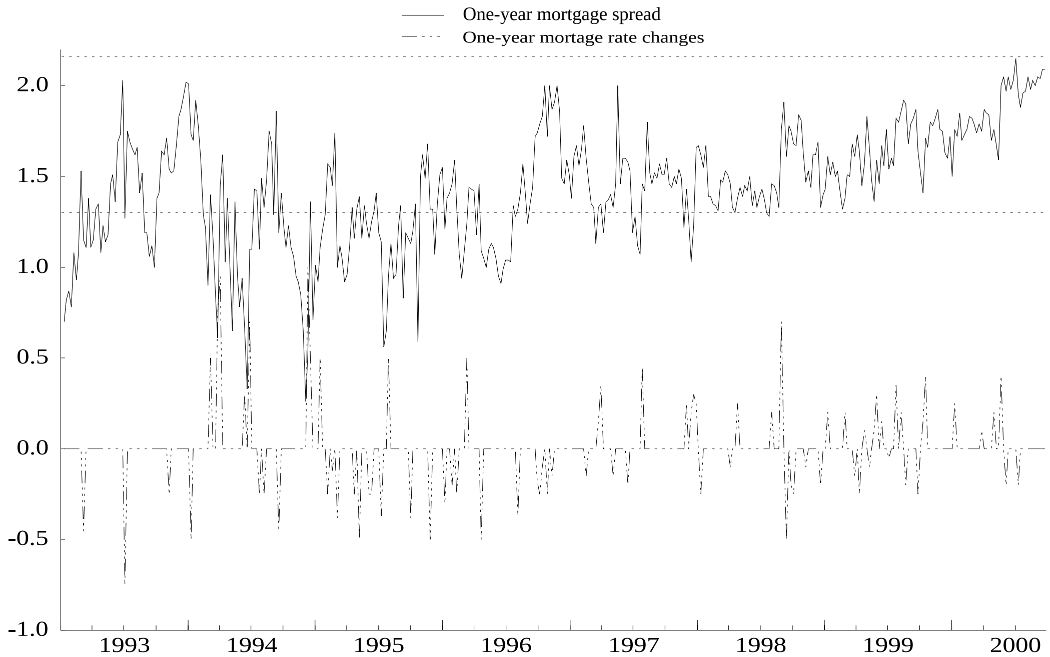
a. T-ratios in parentheses.
b. Ho: homoscedastic errors. *p*-values in parentheses.

Table 8: Comparison of One-Step-Ahead Out-of-Sample Forecast Performance (Forecast Sample: 11 February 1998 to 27 September 2000)[a]

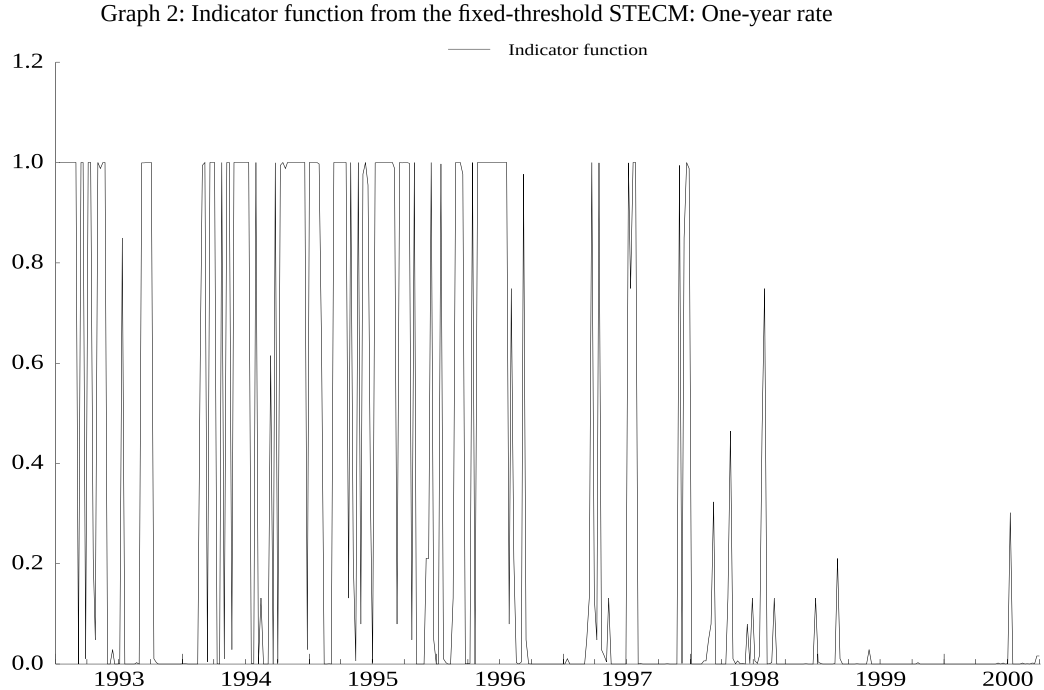
Equation	Model	MSE	MAD	Forecast encompassing test T(136)	Diebold and Mariano (1995) sign test N(0,1)	
					MSE	MAD
One-year rate	STECM (fixed-threshold)	0.0125	0.0745	0.51 (0.61)	0.68 (0.50)	0.35 (0.73)
	STECM (time-varying threshold)	0.0114	0.0689	-	-	-
	Linear VECM	0.0135	0.0786	-1.12 (0.26)	1.45 (0.15)	-1.51 (0.13)
	AR(1)	0.0365	0.1152	1.98 (0.05)	3.12 (0.00)	3.08 (0.00)
	No change	0.0432	0.1391	4.02 (0.00)	3.59 (0.00)	4.25 (0.00)
Five-year rate	STECM (fixed-threshold)	0.0182	0.0741	0.49 (0.62)	0.86 (0.39)	-0.73 (0.47)
	STECM (time-varying threshold)	0.0175	0.0691	-	-	-
	Linear VECM	0.0194	0.0801	-1.51 (0.13)	1.41 (0.16)	1.09 (0.28)
	AR(1)	0.0414	0.1527	2.34 (0.02)	3.98 (0.00)	3.36 (0.00)
	No change	0.0490	0.1473	4.98 (0.00)	4.44 (0.00)	3.12 (0.00)

a. *p*-values in parentheses. The time-varying STECM is specified as Model A in all tests, as it yields the lowest loss functions.

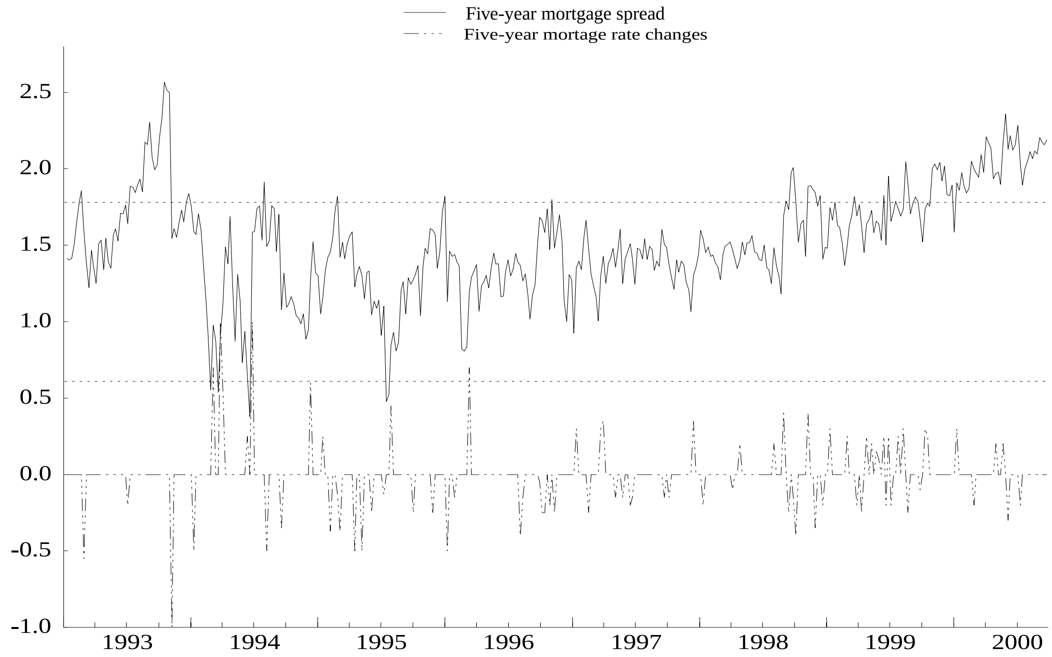
Graph 1: One-year mortgage spread and estimated thresholds from the fixed-threshold STECM



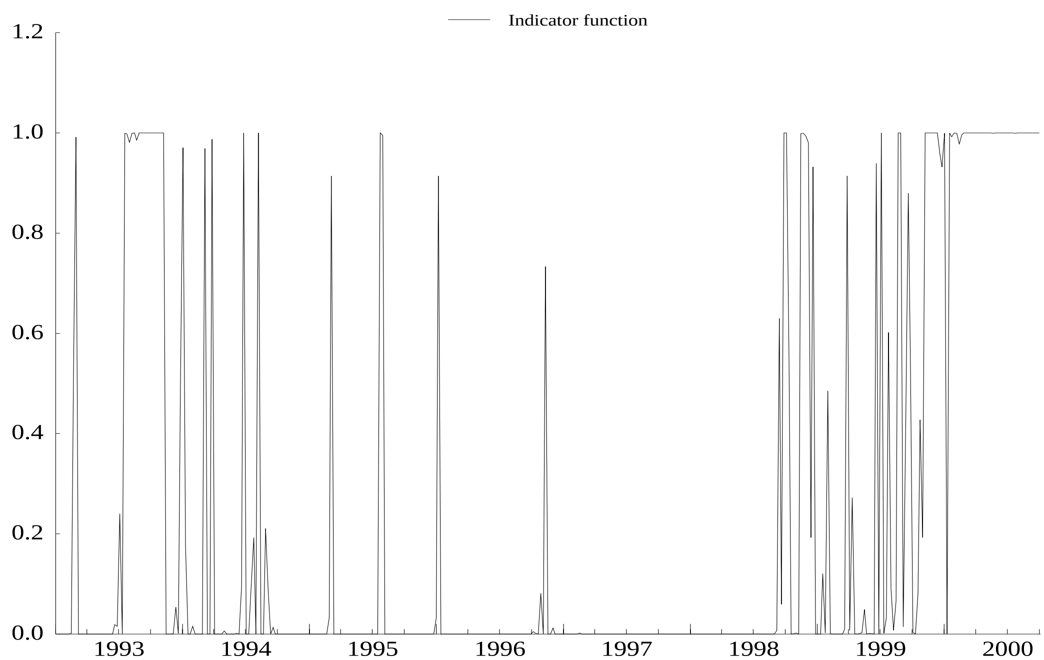
Graph 2: Indicator function from the fixed-threshold STECM: One-year rate



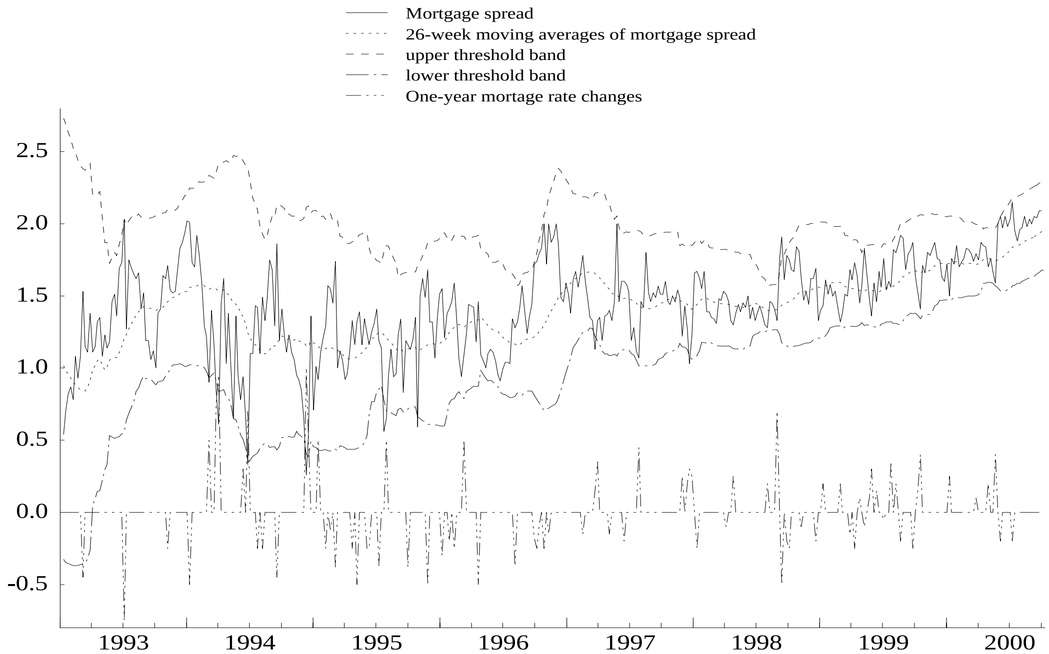
Graph 3: Five-year mortgage spread and estimated thresholds from the fixed-threshold STECM



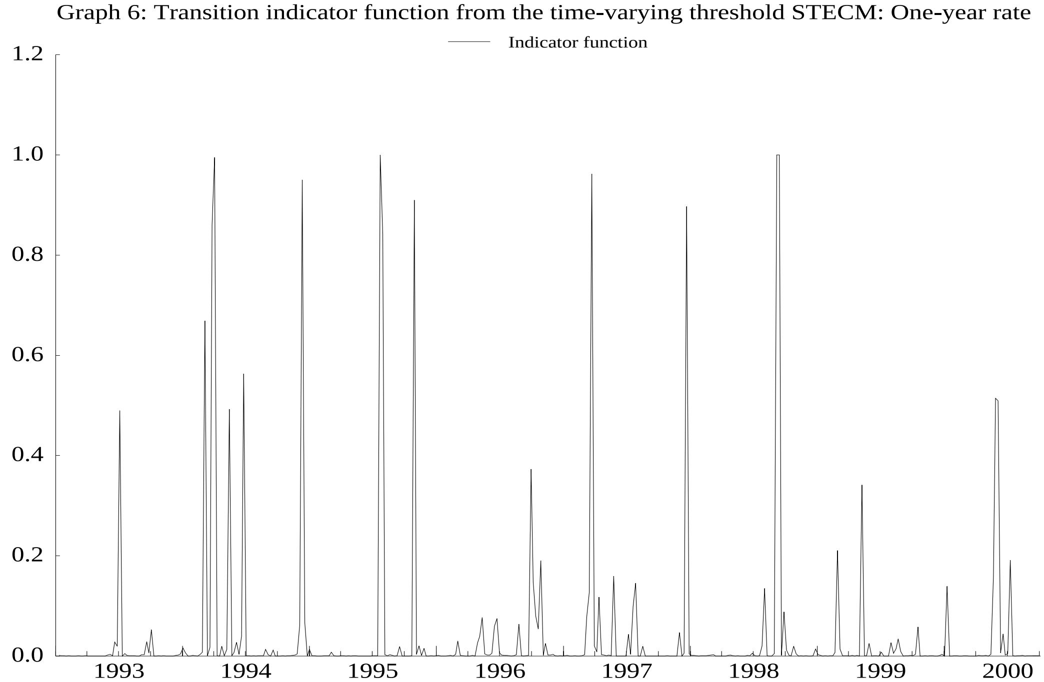
Graph 4: Transition indicator function from the fixed-threshold STECM: Five-year rate



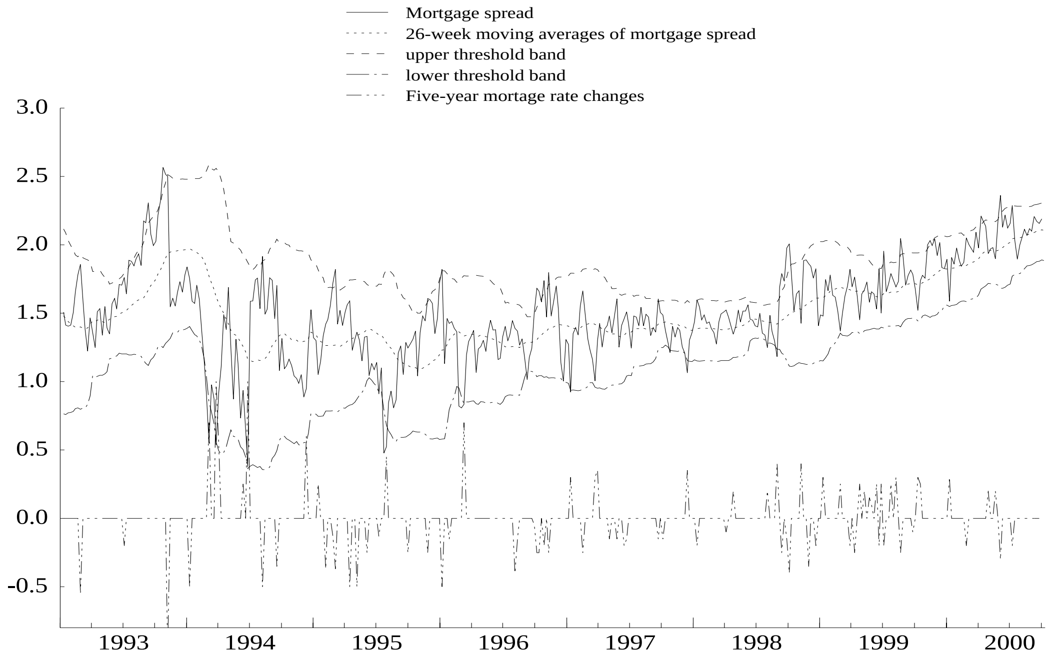
Graph 5: One-year mortgage spread and the estimated threshold bands from the time-varying threshold STECM



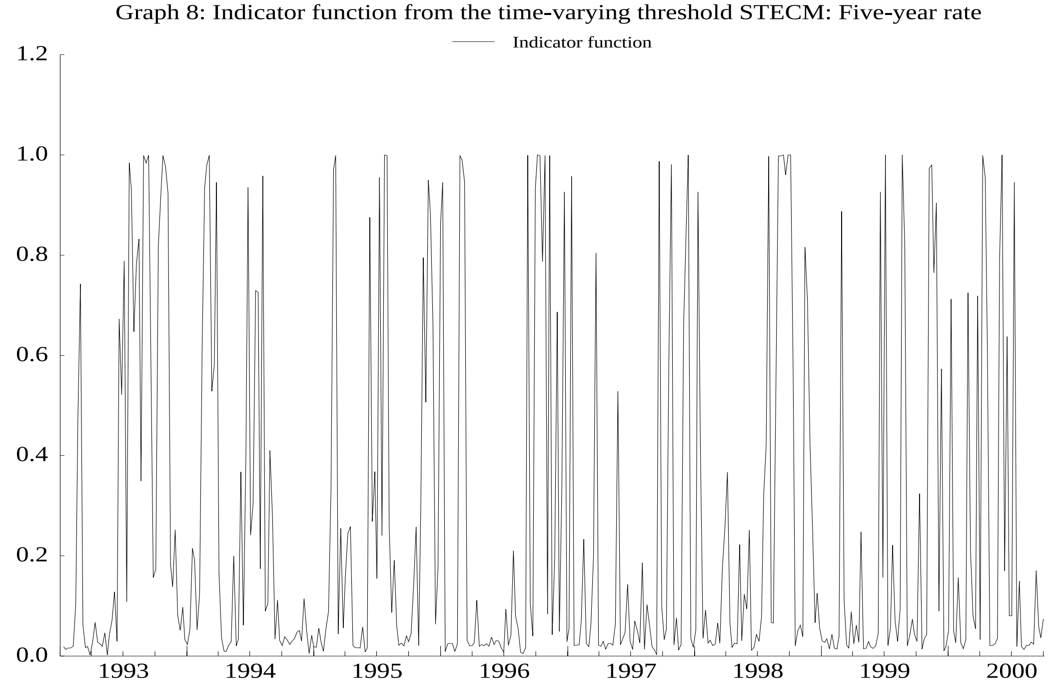
Graph 6: Transition indicator function from the time-varying threshold STECM: One-year rate



Graph 7: Five-year mortgage spread and the estimated threshold bands from the time-varying threshold STECM



Graph 8: Indicator function from the time-varying threshold STECM: Five-year rate



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