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Some Explorations, Using Canadian Data, of the *S*-Variable in Akerlof, Dickens, and Perry (1996)

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Abstract

A number of authors have suggested that economies face a long-run inflation-unemployment trade-off due to downward nominal-wage rigidity. This theory has implications for the nature of the short-run Phillips curve when wage inflation is low. Akerlof, Dickens and Perry have developed an empirical model in which a variable (*S*) designed to capture the effect of downward nominal-wage rigidity is constructed as part of the estimation of the short-run Phillips curve. Adding this variable dramatically improves the dynamic out-of-sample inflation forecasts of the curve in both the United States and Canada.

In this paper we perform a variety of tests using both real and constructed data to address whether the addition of *S* truly does provide a better estimate of the short-run Phillips curve, and whether this constitutes evidence that downward nominal-wage rigidity increases the natural rate of unemployment in times of low wage inflation. Our main conclusion is that the performance of the *S*-enhanced Phillips curve in dynamic simulations is independent of whether downward nominal-wage rigidity is an important feature of the macroeconomy.

JEL classifications: C52, E24, E50
Bank classifications: Monetary policy framework; Transmission of monetary policy

Résumé

Selon plusieurs auteurs, les économies seraient aux prises avec un arbitrage entre l'inflation et le chômage à long terme en raison de la rigidité à la baisse des salaires nominaux. La nature de la courbe de Phillips à court terme s'en trouverait alors modifiée en période de faible hausse des salaires. Akerlof, Dickens et Perry ont mis au point un modèle empirique dans lequel une variable (appelée *S*) destinée à saisir l'incidence de la rigidité des salaires nominaux est intégrée à l'estimation de la courbe de Phillips à court terme. L'insertion de cette variable permet d'améliorer considérablement les prévisions dynamiques de l'inflation produites par la courbe de Phillips au delà de la période d'estimation, tant pour les États-Unis que pour le Canada.

Dans leur étude, Seamus Hogan et Lise Pichette appliquent un éventail de tests à des données réelles et artificielles en vue d' établir si l'addition de la variable *S* donne lieu effectivement à une amélioration de l'estimation de la courbe de Phillips à court terme et si cette amélioration, le cas échéant, indique que la rigidité à la baisse des salaires nominaux fait augmenter le taux de chômage naturel en période de faible progression des salaires. Les auteurs concluent que le pouvoir de prévision de la courbe de Phillips augmentée de *S* en simulation dynamique ne varie pas selon qu'il y ait ou non rigidité des salaires nominaux dans l'économie.

Classifications JEL : C52, E24, E50
Classifications de la Banque : Cadre de la politique2 monétaire; Transmission de la politique monétaire

1. Introduction

The short-run (or expectations-augmented) Phillips curve plays a key role in the conduct of monetary policy, particularly in countries where the central bank has an explicit mandate to target the inflation rate. The importance of the short-run Phillips curve is twofold. First, it represents a key step in the transmission mechanism from monetary policy to inflation. As a result, accurate estimates of the Phillips curve and its explanatory variables are crucial to the operational conduct of monetary policy. Second, the nature of the Phillips curve has implications for what monetary policy should be. Many of the policy debates in macroeconomics about the proper objectives for monetary policy concern the parameters and functional form of the short-run Phillips curve. The topics of these debates include the size of the sacrifice ratio, the costs and benefits of low inflation, the importance of explicit inflation targets, and whether monetary policy should act pre-emptively to prevent significant deviations of inflation from the target.

In recent years, estimates of a conventional linear short-run Phillips curve for Canada have tended to underpredict Canadian inflation, a feature that is also true when Phillips-curve models are applied out of sample to the period of the Great Depression in both the United States and Canada. A number of authors have suggested structural features of the economy that would be consistent with this underprediction as well as alternative Phillips-curve models both to test the underlying structural hypotheses and to provide better Phillips-curve estimates. Among these structural hypotheses are the following: the short-run Phillips curve is convex leading to underprediction of linear Phillips curves in times of high unemployment (Dupasquier and Ricketts 1998); the Phillips curve is non-linear so that excess supply has a smaller effect on inflation than does excess demand (Laxton, Rose, and Tetlow 1993); there is hysteresis in unemployment in which short-run movements in unemployment can affect the natural rate in the medium term (Fortin 1991); there have been changes in the process by which inflation expectations are formed such that backward-looking models would misstate the true level of inflation expectations in some periods (Fillion and Léonard 1997); and there is downward nominal-wage rigidity that limits the extent to which unemployment can produce downward pressure on wages, thus increasing the natural rate of unemployment when inflation is low (Akerlof, Dickens, and Perry 1996).

Akerlof, Dickens, and Perry (hereafter, ADP) have suggested an ingenious estimation approach to test the theory of downward nominal-wage rigidity. Their approach draws on a simple wage-price mark-up model to construct a proxy variable designed to capture the extent of downward nominal-wage rigidity. Including this variable in dynamic simulations dramatically improves the out-of-sample performance of the estimated Phillips curve for the United States.

In this paper, we further investigate the estimation approach of ADP. Our main objective is to check whether the performance of their empirical model in dynamic simulations represents some structural feature of the economy or whether it may be simply a statistical artifact. If it is the former, then the question is whether the model has the power to distinguish between downward nominal-wage rigidity and other structural hypotheses that the proxy variable for wage rigidity might instead be capturing.

In the next section, we outline the basic model of a labour-market-based Phillips curve, both with and without downward nominal-wage rigidity. In Section 3, we present the results found when applying the ADP model to Canadian data. These results are qualitatively the same as those of ADP. In Section 4, we examine various measures of the performance of the two models. Finally, in Section 5, we generate artificial data under a range of different structural assumptions to test the power of various empirical methods to distinguish between these assumptions.

2. The Phillips curve with downward nominal-wage rigidity

The model outlined in this section is the one derived by ADP. We give a sketch of only the crucial elements for the derivation of the *S*-variable since more detail is available in ADP. The short-run Phillips curve typically takes the following general form,

$$\pi_t = \pi_t^e + f(Y_t - Y_t^*), \tag{1}$$

where π_t^e is the expectation at the beginning of period t for the rate of inflation over that period, π_t is the actual rate of inflation over the period, and Y_t-Y_t* is the deviation of output from potential (the "output gap") at the start of period t. To consider the effect of downward nominal-wage rigidity empirically, it is more convenient to express the output gap as a deviation of unemployment, u_t, from its "natural rate," u_t* and to impose a linear functional form,

$$\pi_t = \pi_t^e + a(u_t - u_t^*), \tag{2}$$

where $a<0$.

Equation (2) can be derived from a standard model of monopolistic competition in which labour is the only primary input into production. In this model, prices are determined in three steps. First, average desired real wages for period t, ω_t, are determined at the start of period t as a decreasing function of the unemployment rate:

$$\omega_t = g_t(u_t)\,. \tag{3}$$

More specifically, ω_t is the wage that would occur in the absence of any constraint against money wage cuts. It results from a bargain between the firm and its workers. Second, average nominal wages for period t are determined by the expected price level applying in that period:

$$w_t = p_t^e \omega_t\,. \tag{4}$$

Finally, the actual price level is determined as a mark-up on average nominal wages,

$$p_t = \frac{\mu}{\gamma_t} w_t\,, \tag{5}$$

where μ is the mark-up on unit labour costs determined by the elasticity of demand for each firm's product, and γ_t is labour productivity in period t.

It is easily shown that, if the function g_t is log linear, then Equations (3) to (5) combine to give Equation (2). To incorporate downward nominal-wage rigidity into this model, ADP distinguish between the "notional" and the actual wage. The notional wage, w_t^n, is the desired wage determined in Equation (4); the actual wage will be higher than w_t^n if the nominal wages at some firms are constrained by downward nominal-wage rigidity. Let S_t be the difference between actual and notional expected real wages, deflated by labour productivity:

$$S_t = \frac{w_t - w_t^n}{p_t^e \gamma_t}\,. \tag{6}$$

With this gap between actual and notional wages, the Phillips curve becomes

$$\pi_t = \pi_t^e + c + au_t + \ln\left(1 + \frac{\gamma_t S_t}{\omega_t^n}\right), \tag{7}$$

which can be very closely approximated as

$$\pi_t = \pi_t^e + c + au_t + \mu S_t\,. \tag{8}$$

Equations (2) and (8) constitute the two models that ADP estimate, which we shall refer to as the "standard" model and the "ADP" model, respectively. In order to estimate these equations, it is necessary to obtain proxy variables for π_t^e and S_t. ADP adopt a backward-looking (accelerationist) model of inflation expectations:

$$\pi_t^e = \alpha\pi_{t-1} + (1-\alpha)\pi_{t-2}, \tag{9}$$

where α is a parameter to be estimated. By making the assumption that the lagged actual wage and the current notional wage at each firm have a bivariate normal distribution, ADP approximate S_t by the following two equations, which define it recursively,

$$S_t = \phi\left(\frac{v_t}{\sigma}\right)\sigma + \Phi\left(\frac{v_t}{\sigma}\right)v_t, \tag{10}$$

$$v_t = \frac{S_{t-1} - \frac{1}{\mu}(\pi_t^{ee} + \gamma_t - a(u_t - u_{t-1}))}{1 + \pi_t^{ee} + g_t}, \tag{11}$$

where ϕ and Φ are the density and distribution functions, respectively, of the standard normal distribution, σ is the standard deviation of the distribution of $(\omega_{t-1}-\omega_t^n)/\gamma_t$, g_t is the growth of labour productivity, and π^{ee} is the rate of change of price expectations:

$$\pi_t^{ee} = \pi_t^e - \pi_{t-1}^e + \pi_{t-1}. \tag{12}$$

The parameter, σ, is estimated; μ and γ_t are imposed with μ=1.36 being fixed as in ADP without loss of generality; and γ_t resulting from assuming labour productivity has followed a piece-wise linear trend. μ is the inverse of labour's share of income in the national accounts observed in the United States in 1994. For the purpose of this paper, we assume that this ratio is similar in Canada since it should have no major implications for our conclusions.

Equations (2) and (8) to (12) give a system of equations to be estimated to minimize the sum of squared residuals in the Phillips-curve equations. The parameters to be estimated are a, c, α, and σ, and the two estimated Phillips curves are

$$\pi_t = \alpha\pi_{t-1} + (1-\alpha)\pi_{t-2} + c + au_t \tag{13}$$

for the standard model, and

$$\pi_t = \alpha\pi_{t-1} + (1-\alpha)\pi_{t-2} + c + au_t + 1.36S_t \tag{14}$$

for the ADP model.

3. Results from the base model

3.1 The data

ADP estimated both the standard linear Phillips curve and their extended equation with the *S*-variable, using annual U.S. data over the period 1929–95 but excluding the period from 1943–53 during which there were the price controls of World War II and the Korean War. In order to generate out-of-sample forecasts, they also estimated the models for both the pre-war and post-war periods. We adopt a similar strategy, estimating the standard and ADP models using annual Canadian data over the periods 1929–42, 1956–89, and 1990–96. As do ADP, we use the change in the natural logarithm of the GDP deflator as our measure of inflation.

ADP also included the change in profits as a share of GDP in their extended equation to allow for the idea that firms under extreme duress may lower their nominal wages. ADP found that this term was not significant in their regressions. With the Canadian data, it enters with the wrong sign. We therefore have not included it in our estimations.

3.2 Regression results

Table 1 presents our estimates of the standard and ADP models over time periods chosen using the same strategy as ADP. For each period, the first column gives the results for the standard linear Phillips curve, Equation (13), and the second column the results for the ADP model, Equation (14).

Note that in four of the six regressions, the coefficient on unemployment is negative, as required by the hypothesis of a constant natural rate of unemployment. In these cases, the natural rate can be inferred from the estimated coefficients with

$$u^* = \frac{c}{-a} \tag{15}$$

in the standard model, and

$$u^* = \frac{c + \mu S}{-a} \tag{16}$$

in the ADP model. These estimates for the natural rate are given in the final row of Table 1. In the case of the ADP model, the value of *S* in a steady state with constant inflation and unemployment will be a function of that inflation rate. Accordingly, there is no single value for the natural rate in this case, but rather a locus of inflation-unemployment pairs that defines the long-run Phillips curve. This locus also depends on the rate of labour productivity growth, since it is the rate of wage inflation rather than price inflation that determines the effect of downward nominal-wage rigidity.

Table 1: Base model: Canadian Phillips curves with and without *S*

Estimation period	1956–1989		1929–42		1929–42 & 1956–96	
Model (model number)	Standard (1.1a)	ADP (1.1b)	Standard (1.2a)	ADP (1.2b)	Standard (1.3a)	ADP (1.3b)
Constant (c) (H_o: c=0; H_1: c>0)	0.03 (.01)	0.02 (.06)	-0.02 (.18)	-0.05 (.05)	-0.00 (.49)	0.05 (.00)
Inflation(-1) (α) (H_o: α=1; H_1: α ≠ 1)	0.96 (.83)	1.05 (.83)	1.34 (.29)	1.35 (.68)	1.18 (.22)	0.77 (.20)
Inflation(-2) (1–α) (coefficient imposed)	0.04	-0.05	-0.34	-0.35	-0.18	0.23
Unemployment (u) (H_o: a=0; H_1: a<0) (H_o: a=0; H_1: a>0)	-0.40 (.01) 	-0.44 (.01) 	0.24 (.14)	-0.88 (.00) 	0.01 (.46)	-0.84 (.01)
Real-wage shift (S) (coefficient imposed)	—	1.36	—	1.36	—	1.36
s.d. of Δw^n (σ) (H_o: σ=0; H_1: σ>0)	—	0.04 (.01)	—	0.11 (.00)	—	0.04 (.00)
$\bar{R}^2$	0.68	0.69	0.41	0.87	0.66	0.75
Natural rate (u*) (%)	7.3	π=∞ 5.7 π=3% 7.8 π=2% 8.9 π=1% 10.7 π=0% 14.1	n.a.	π=∞ -6.2 π=3% -0.9 π=2% 0.2 π=1% 1.7 π=0% 4.5	n.a.	π=∞ 6.1 π=3% 6.6 π=2% 6.9 π=1% 7.4 π=0% 8.3

The numbers in parentheses under each estimated coefficient are the *p* values for the null and alternative hypotheses specified under the variable name.

In Table 1, the values for the natural rate under different inflation rates are given assuming 1 per cent annual labour productivity growth. The natural rate when inflation equals infinity is what ADP term the “lowest sustainable rate of unemployment” (LSRU). It is the natural rate that applies when inflation is high enough that downward nominal-wage rigidity does not bind. Note that in Model 1.2b, the implied LSRU is negative. This simply reflects the dangers of extrapolating a linear model out of the range of the sample data.

These results closely mirror those obtained by ADP with U.S. data. In all three time periods, the ADP model is characterized by statistically significant negative coefficients on unemployment, whereas the standard model fails to give a negative coefficient in two of the three periods. In all three periods, the ADP model also provides a better in-sample fit as given by a higher adjusted R-squared, although this difference is small for the post-war period. A better test of which model has the most explanatory power, however, is to see which provides the best out-of-sample fit. To this end, ADP use the post-war estimation of the two models to perform both in-sample and out-of-sample dynamic simulations of inflation.

3.3 Dynamic simulations

Figure 1 shows the results of dynamic simulations in which the model that was estimated over 1956–89 is simulated over the entire sample. In these simulations, the ADP model performs better. This happens even if the coefficients on lagged inflation in Table 1 are virtually the same for both models because the *S*-term is not an exogenous variable. Because of the presence of *S*, and by construction, the dynamics of the ADP model is completely different from the standard one. The value of *S* depends on the rate of change of price expectations that are formed with lagged inflation, as seen in Equations 9 to 12. Therefore, *S* is not constant under dynamic simulation; it is re-estimated each period.

In the out-of-sample periods of 1929–42 and 1990–96, the standard model without *S* consistently underpredicts inflation. The model with *S*, on the other hand, provides a far better fit. Again, this closely mirrors the results obtained by ADP using U.S. data. The better performance of the ADP model is particularly striking in the period of the Great Depression. As ADP note: “[T]he Great Depression [is] a period that notoriously defies explanation with conventional natural rate models.” (p. 5)

This impressive performance of *S* in dynamic simulations raises three questions:

1. Is *S* really performing well?
2. If so, is its performance due to a structural economic phenomenon or is it purely statistical?
3. If *S* represents a structural phenomenon, to what extent is that phenomenon downward nominal-wage rigidity?

We address these three questions in the remainder of this paper.

4. Performance indicators

The object of this section is to assess the performance of the *S*-variable. We essentially ask two questions: How revealing are dynamic simulations? and How well does *S* perform as an exogenous variable?

Consider the following general model,

$$y_t = f(y_{t-1}, \boldsymbol{X}_t) + \varepsilon_t, \tag{17}$$

where $\boldsymbol{X}_t$ is a vector of exogenous variables, and ε_t is white noise. In a static simulation, the forecast, $\hat{y}_t$, for the dependant variable is

$$\hat{y}_t = \hat{f}(y_{t-1}, \boldsymbol{X}_t), \tag{18}$$

where $\hat{f}$ is the estimated function from some sample of y and $\boldsymbol{X}$. There are two sources of error in such a static forecast. First, there are errors resulting from the difference between the estimated model, $\hat{f}$, and the true model, f. Second, even if the model has been estimated perfectly, the true y_t depends on the noise term, which, by definition, cannot be forecast.

In a dynamic simulation, the forecast, $\tilde{y}_t$ of the dependant variable is defined recursively with

$$\tilde{y}_1 = \hat{y}_1, \text{ and} \tag{19}$$

$$\tilde{y}_t = \hat{f}(\tilde{y}_{t-1}, \boldsymbol{X}_t) \qquad \forall t > 1\,. \tag{20}$$

Here there is a further source of error due to the misspecification of putting the forecast value rather than the true value of the lagged dependant variable on the right-hand side of the forecast equation. Because of the recursive definition of $\tilde{y}_t$, this misspecification error will tend to increase over time.

In short, the major difference between dynamic and static simulations in equations with lagged dependant variables is that, for dynamic simulations, the lagged values are replaced by their forecasts and for static simulations, actual lagged values are used.

The problem of exploding errors in dynamic simulations is more likely to arise in models with a unit-root specification. This pattern seems to be evident in Figure 1. The standard model fails to predict the sharp reduction in the rate of deflation in 1933 and so generates a large negative static forecast error for that year. In the remaining years of the simulation period of 1929–42, that negative forecast error explodes as the difference between $\tilde{y}_t$ and y_t gets continually larger.

Given the similarity of the coefficients on lagged inflation for both versions of the Phillips curve, a substantial difference in dynamic simulation results between the two models would not be expected. As noted earlier, the explanation lies in the way *S* is calculated for the dynamic simulations. *S* is defined in such a way that it will be high when lagged inflation is low. This negative correlation is illustrated in Figure 2 for the various time periods considered. In the dynamic simulation, the *S*-variable is recalculated each period in the dynamic simulation, using the forecasted inflation term that results from the ADP model itself. The presence of *S* in the Phillips curve reduces the importance of the lagged dependant variable.

To understand the role of the *S*-variable more clearly, it is useful to consider static simulations, in which the out-of-sample forecasts take the form of Equation (18). Figure 3 presents static simulations for the two models estimated in Table 1. As can be seen by examining these graphs, and confirmed by the root-mean-square forecast errors of the out-of-sample simulations given in Table 2, the relative abilities of the two models to fit the out-of-sample period of the Great Depression reverses when using static rather than dynamic simulations. The ADP model does provide a slightly better fit for the 1990s in static simulations.

Table 2: RMS forecast errors for the base model estimated on 1956–89

Type of simulation	Static		Dynamic	
Model (model number)	Standard (1.1a)	ADP (1.1b)	Standard (1.1a)	ADP (1.1b)
In sample (1956–89)	1.81	1.74	4.51	3.24
Out of sample (1929–42)	4.61	5.55	17.97	3.18
Out of sample (1990–96)	1.13	0.92	1.78	0.86
Out of sample (combined)	3.82	4.57	14.71	2.65

Similar results are obtained when the models are estimated for different sample periods. Figures 4 to 7 present the dynamic and static simulations for the two models estimated on the periods 1929–42 and 1990–97, respectively; Tables 3 and 4 present the static and dynamic RMS forecast errors.

Table 3: RMS forecast errors for the base model estimated on 1929–42

Type of simulation	Static		Dynamic	
Model (model number)	Standard (1.1a)	ADP (1.1b)	Standard (1.1a)	ADP (1.1b)
In sample (1929–42)	3.09	1.38	5.25	2.25
Out of sample (1956–96)	2.40	5.82	35.28	5.08

Table 4: RMS forecast errors for the base model estimated on 1990–96

Type of simulation	Static		Dynamic	
Model (model number)	Standard (1.1a)	ADP (1.1b)	Standard (1.1a)	ADP (1.1b)
In sample (1990–96)	0.74	0.22	0.86	0.19
Out of sample (1929–42)	4.57	5.67	9.33	2.83
Out of sample (1956–89)	2.50	3.07	12.02	4.82
Out of sample (combined)	3.24	4.00	11.30	4.33

These graphs and tables show a consistent pattern. The standard model with the unit root consistently performs better in static simulations, and less well in dynamic, than the model where the persistence of inflation is reduced by the presence of *S*. In Figure 4b, the ADP model consistently underpredicts inflation throughout the sample period, but the presence of *S* prevents the model from exploding. In Figure 6b, the dynamic simulation over 1956–89 produces a sawtooth effect. Of course, as this model was estimated with a very small sample period, the estimates should not be taken too seriously. What is revealing, however, is that the model is capable of producing oscillatory dynamics. This suggests that the net effect of *S* in reducing the persistence of inflation is so strong in this parameterization that lagged inflation enters with a negative sign in the Phillips-curve equation.

Finally, if the *S*-variable is truly proxying some structural aspect of the economy that can explain inflation in a Phillips-curve equation, then it should perform well when the generated values for *S* are treated as an exogenous data series. In Table 5, we re-estimate a Phillips curve both with and without *S* to see how well its presence increases the explanatory power of the equation, and whether its coefficient is significant. In these regressions, we consider the base-case specification and also two extensions of that specification. Model 2.1b imposes the unit-root specification on inflation. It is the same as Model 1.1b with the exception that, in Model 2.1b, *S* is an exogenous

Table 5: *S* exogenous

Estimation period	1956–1989:				
Restriction model number	$\alpha_1+\alpha_2=1$ 2.1b	$b=s=0$ 2.2a	$b=0$ 2.2b	$\alpha_2=s=0$ 2.3a	$\alpha_2=0$ 2.3b
Constant (c) (H_o: c=0; H_1: c>0)	0.02 (0.04)	0.03 (0.01)	0.02 (0.04)	0.0 (0.01)	0.03 (0.01)
Inflation(-1) (π_{t-1}) (H_o: α_1=1; H_1: $\alpha \neq 1$)	1.10 (0.68)	0.94 (0.72)	1.07 (0.76)	1.00 (0.48)	0.93 (0.64)
Inflation(-2) (π_{t-2}) (H_o: α_2=1-α_1; $H_{1:}$ $\alpha_2 \neq 1-\alpha_1$	-0.10 (imposed)	-0.09 (0.44)	-0.10 (0.88)	—	—
Unemployment (u_t) (H_o: a=0; H_1: a<0)	-0.41 (0.03)	-0.28 (0.07)	-0.39 (0.04)	-1.27 (0.01)	-1.30 (0.01)
Unemployment (-1)(u_{t-1}) (H_o: b=0; H_1: b<>0)	—	—	—	0.97 (0.00)	1.08 (0.00)
Real-wage shift (S) (H_o: s=0; H_1: s>0) (H_o: s=1.36; H_1: s<>1.36)	1.44 (0.16) (0.96)	—	1.26 (0.19) (0.94)	—	-0.94 (0.26) (0.12)
$\bar{R}^2$	0.69	0.68	0.68	0.76	0.76

The numbers inside parentheses under each estimated coefficient are the *p* values for the null and alternative hypotheses specified under the variable name.

variable that was created in estimating Model 1.1b. In Models 2.2a and 2.2b, we do not impose the unit root on inflation; in Models 2.3a and 2.3b we add lagged unemployment. In these last two regressions, we dropped the second lag of inflation according to the Akaike Information Criterion (AIC) for optimal lag length.

The notable thing in these regressions is that the presence of *S* has no effect on the goodness of fit according to the adjusted R-squared. Furthermore, the addition of lagged unemployment improves the fit while at the same time reducing the significance of the *S* term and giving it the wrong sign. These results suggest that it might be useful to explore how the *S*-variable performs when the model generating the data is known.

5. Artificial data

Our final way of addressing whether the *S*-variable in the ADP model reflects the degree of downward nominal-wage rigidity in the economy is to generate artificial data from a simulation model where the exact structure of the economy is known, and then to use this data to estimate the standard and ADP Phillips curves.

The model we use to create these data is a modified version of the calibration model used by ADP to model the effect of downward nominal-wage rigidity on the long-run Phillips curve. In their calibration model, agents know the true rate of growth of aggregate demand in the economy and use this to generate perfect-foresight inflation expectations in steady state. We modify this by giving agents purely backward-looking inflation expectations and by introducing shocks to the growth rate of aggregate demand so that unemployment can deviate from its natural rate. The model is structured in such a way that Equations (7) and (9) give the exact functional form for the underlying short-run Phillips curve, other than random noise.

For each simulation, we generated two sets of data, using the same sequence of shocks. In the first data set, we let wages be unconstrained by downward nominal rigidity. In the second, we imposed perfect downward rigidity; that is, we imposed the constraint that nominal wages could never fall at any firm, no matter what the circumstances of the firm. In each simulation, we generated unemployment, inflation, and the true value of *S* (as defined by Equation (6)) for annual data in periods of 14 and 51 years, corresponding to the real-world data used in the previous sections. We then estimated the model over the first 44 years of the second period and performed out-of-sample simulations on the first period and the final 7 years of the second period, as in Figures 1 and 3.

The results for a typical simulation are shown in Figures 8 to 12. Figure 8 shows the estimated values of *S* when the unemployment and inflation data generated by the calibration model are input into ADP's estimation model, both when the calibration model had no nominal rigidity

(NWR=0) and when it had perfect rigidity (NWR=1). It also shows the true value of *S*, generated by the calibration model when under perfect rigidity. Of course, when there is no rigidity, the true value of *S* is 0 in all periods. It is not a criticism of the estimation model that it generates large values for *S* in this case. The method for generating *S* in the estimation model is derived under the assumption that there is perfect downward rigidity. We find that the estimation model heavily overestimates the true value of *S* when the assumption of perfect rigidity is correct, and that there is little difference between the estimated values of *S* for the case where there is perfect rigidity and the case where there is none. This suggests that the estimation model has very little power to distinguish between data generated in a world with downward nominal-wage rigidity and data generated in a world without it.

This conclusion seems to be borne out in Figures 9 to 12, which show the static and dynamic simulations of the artificial data. In each figure, the top graph shows the simulations for the standard model, and the bottom graph those for the ADP model. In Figures 9 and 11, the true model generating the data had no downward nominal-wage rigidity. In Figures 10 and 12, the true model had perfect rigidity. The associated RMS forecast errors for these simulations are given in Tables 6 and 7.

Table 6: RMS forecast errors for model estimated on artificial data (NWR=0)

Type of simulation	Static		Dynamic	
Model	Standard	ADP	Standard	ADP
In sample (1956–89)	0.79	0.79	0.67	1.03
Out of sample (1929–42 & 1990–96)	0.93	1.17	0.89	1.42

Table 7: RMS forecast errors for model estimated on artificial data (NWR=1)

Type of simulation	Static		Dynamic	
Model	Standard	ADP	Standard	ADP
In sample (1956–89)	0.62	0.65	0.50	0.99
Out of sample (1929–42 & 1990–96)	0.74	0.96	0.70	1.33

It might have been interesting to calibrate the model to mimic some of the features of the real-world data, such as the high unemployment during the Great Depression; nevertheless, the above results are quite revealing. Figures 9 to 12 and Tables 6 and 7 demonstrate that, even when downward nominal-wage rigidity is very strong, the ADP model does a poorer job than the standard model in explaining the data.

Moreover, Figure 8 suggests that the *S*-variable does not really capture the effect of the downward nominal-wage rigidity. That could mean that the *S* term might be picking up a misspecification. We have not investigated this possibility extensively; we have tried to add other variables to the Phillips-curve equation, such as the employment insurance (EI) disincentives (measured with an index proposed by Sargent (1995)) or oil prices in order to represent a supply shock. Although these attempts were not successful in improving the performance of the standard model, examining alternative specifications might be useful for future work.

6. Conclusion

ADP suggest an interesting way to improve the standard Phillips curve. Before adopting this suggestion into the process for formulating monetary policy, it is useful to understand its strengths and weaknesses. This paper reports various empirical investigations, using Canadian data, of the *S*-variable proposed by ADP.

We draw a number of conclusions from the analysis in this paper. The most important one is that the performance of the ADP model in dynamic simulations should not be taken as evidence in favour of the hypothesis of downward nominal-wage rigidity. This performance seems to reflect the fact that the specification of the ADP model, in particular its use of the *S*-variable, heavily reduces the inflation persistence imposed on the standard model by the unit-root assumption. This therefore reduces the problem of accumulated errors that arise in dynamic simulations. Static simulation results support this interpretation.

The intuition for the *S*-variable, which is defined as the difference between the actual wage and the wage that would occur without any downward rigidity, is interesting. However, the way this variable is constructed in the model does not seem to be consistent with this definition. Our experiments with artificial data suggest that the ADP model is unable to distinguish between an economy with and without downward nominal-wage rigidities. This is perhaps because *S* represents another structural economic phenomenon.

Our results do not address the question of why the ADP model performs so much better in-sample for the period of the Great Depression than does the standard model. To investigate this, we would like to model alternative specifications of the Phillips curve to try to encompass some of the other structural hypotheses we outlined in the introduction. This remains for future work.

Figure 1 : Dynamic Simulations of Inflation
Sample : 1956-1989

(b) With Downward rigidities

Figure 2 : Correlation of S with lagged inflation

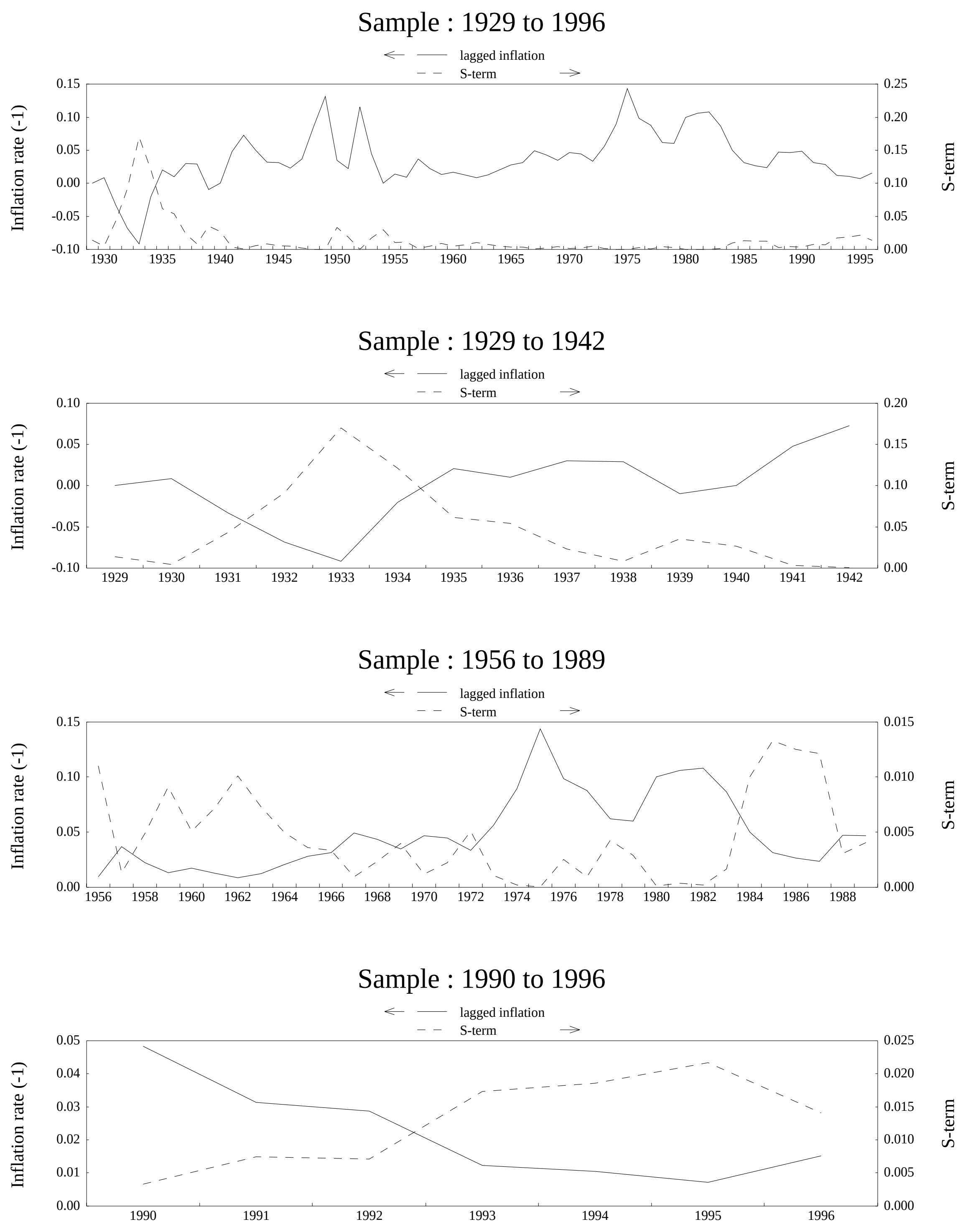
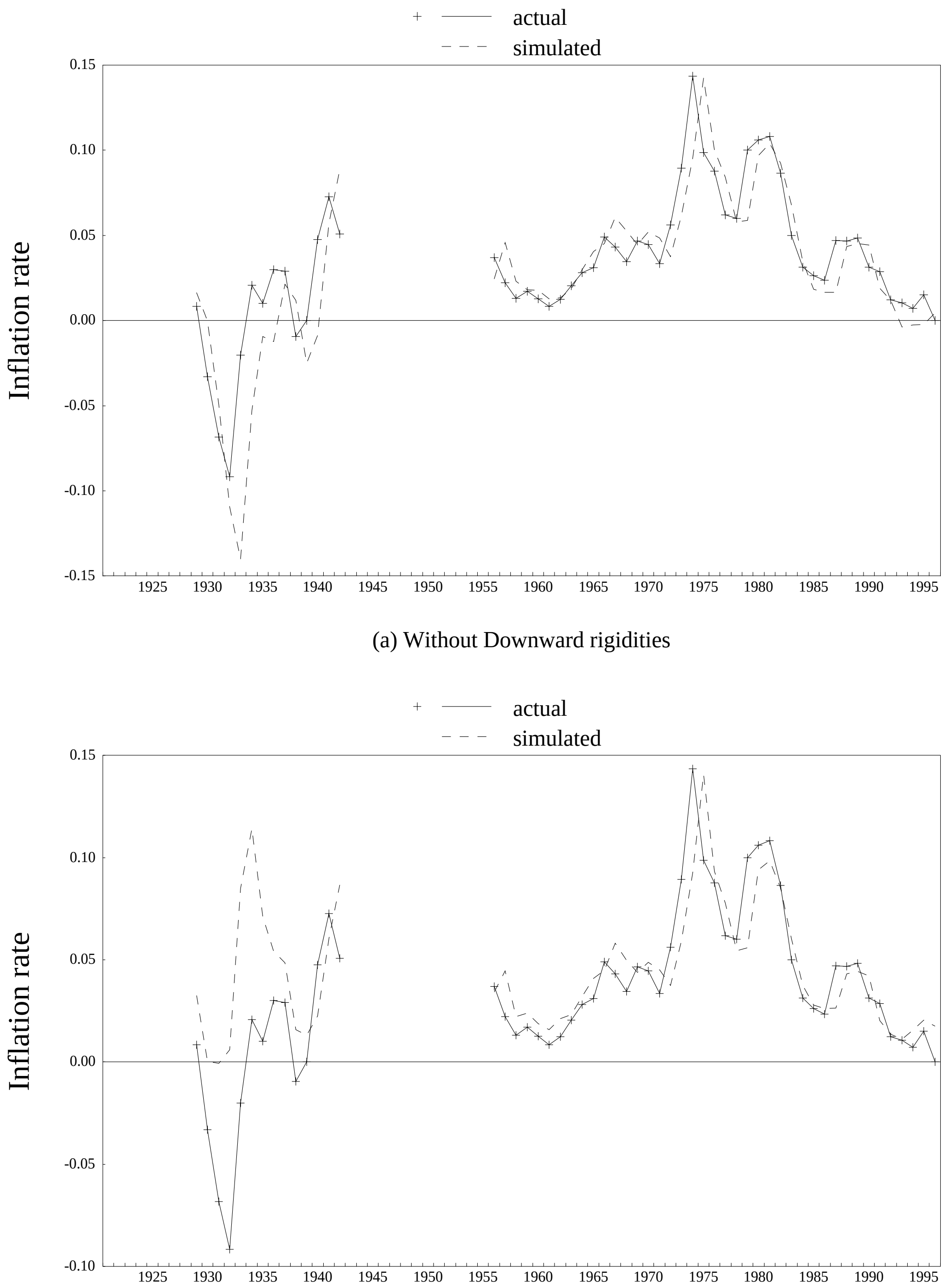


Figure 3 : Static Simulations of Inflation

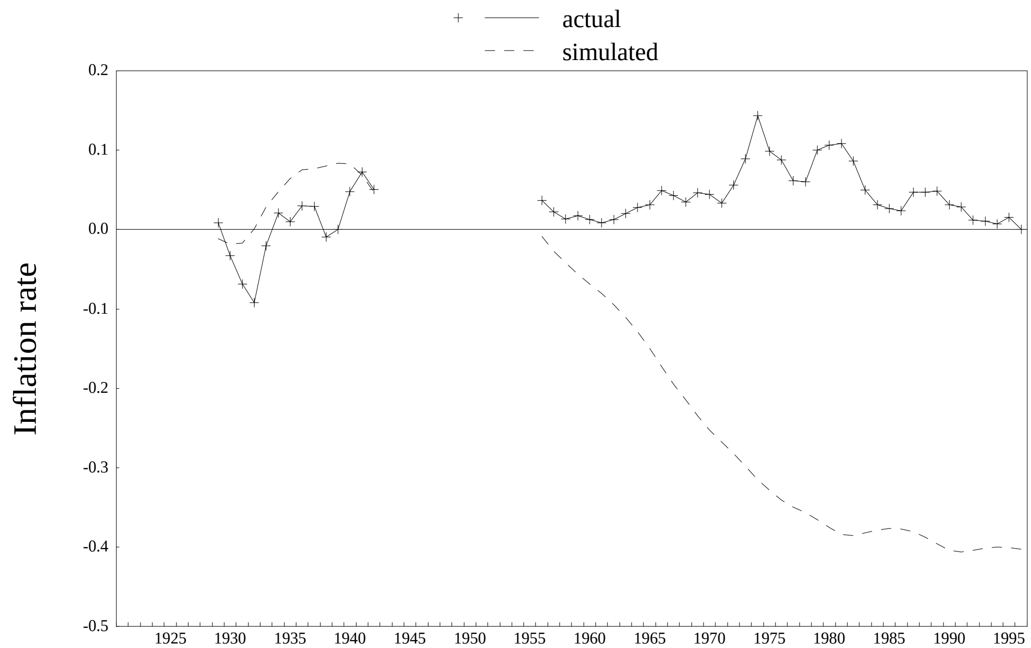
Sample : 1956-1989



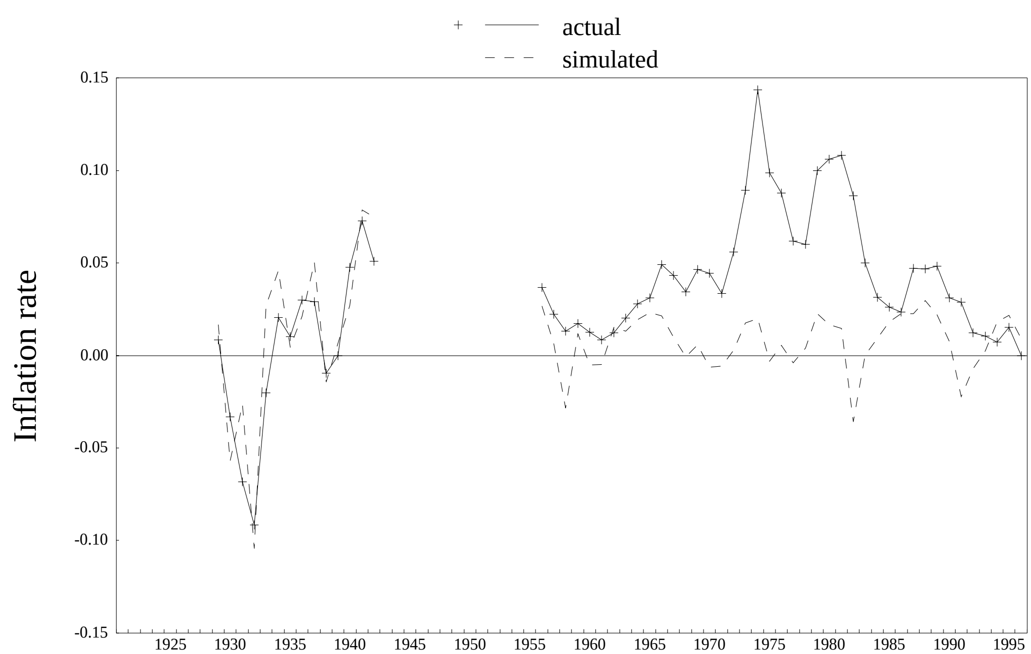
(a) Without Downward rigidities

(b) With Downward rigidities

Figure 4 : Dynamic Simulations of Inflation
Sample : 1929-1942

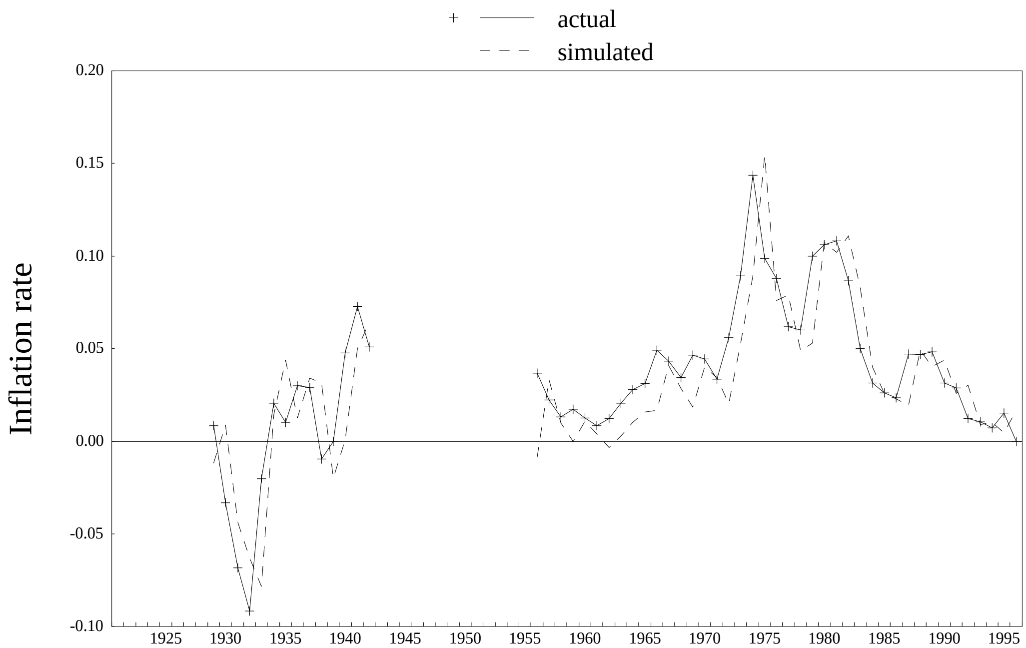


(a) Without Downward rigidities

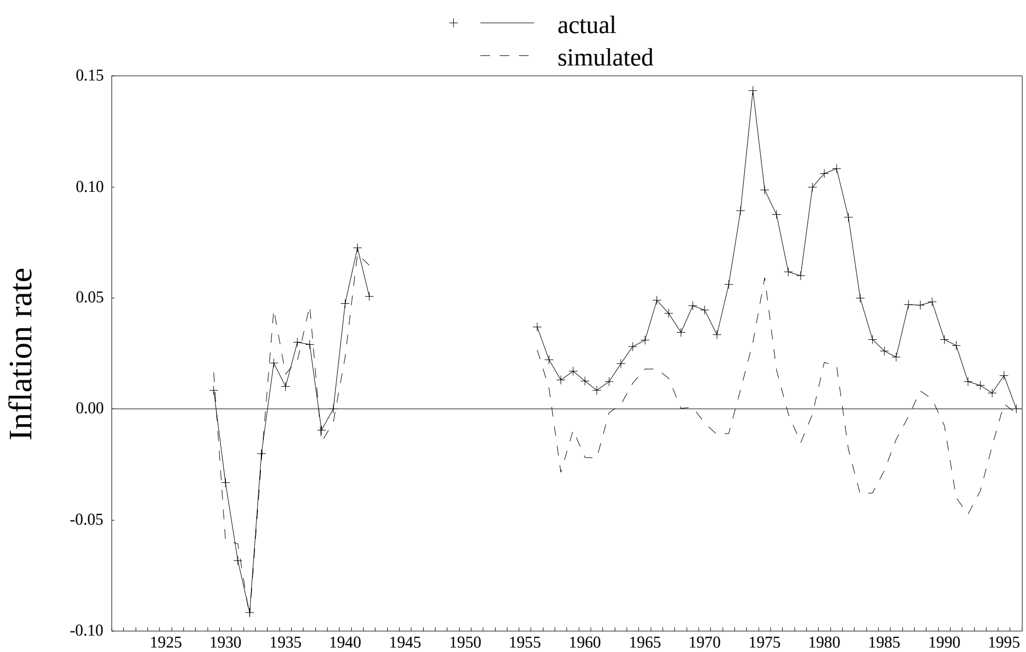


(b) With Downward rigidities

Figure 5 : Static Simulations of Inflation
Sample : 1929-1942

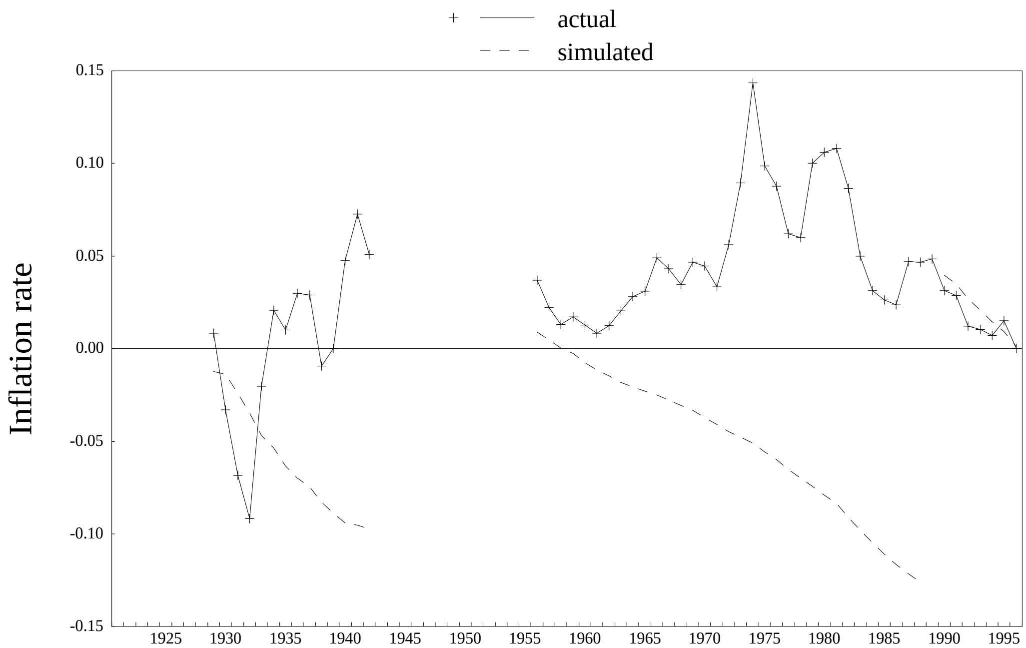


(a) Without Downward rigidities

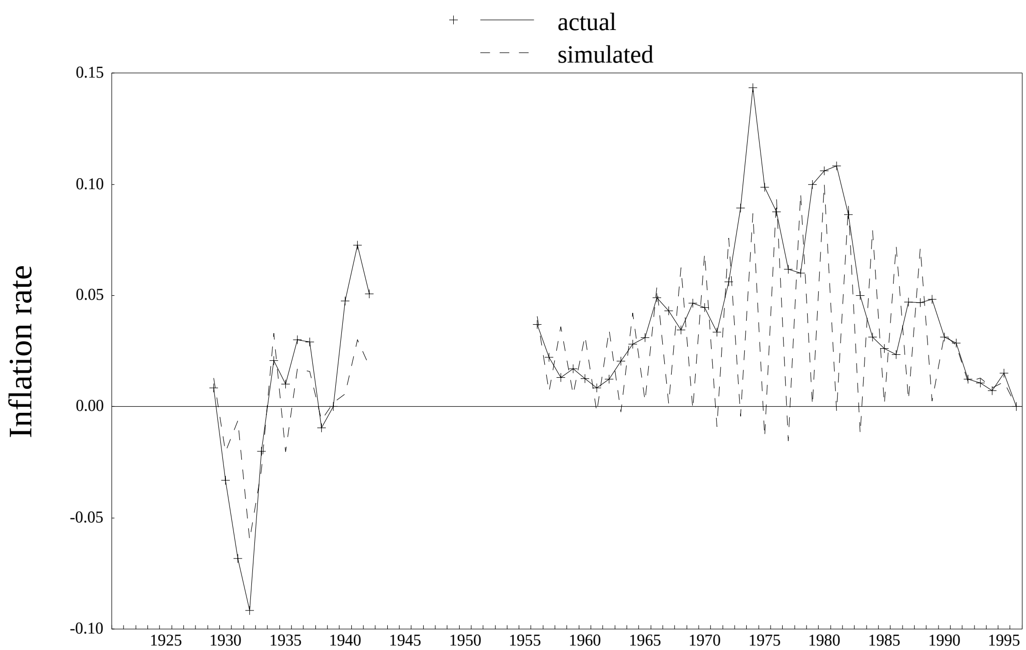


(b) With Downward rigidities

Figure 6 : Dynamic Simulations of Inflation
Sample : 1990-1996



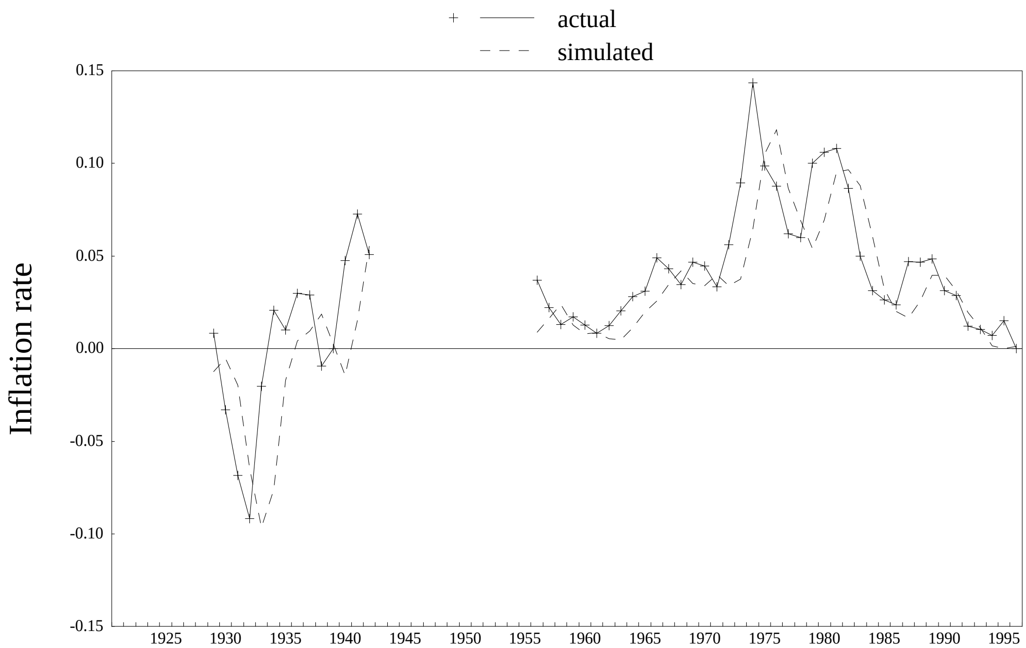
(a) Without Downward rigidities



(b) With Downward rigidities

Figure 7 : Static Simulations of Inflation

Sample : 1990-1996



(a) Without Downward rigidities

(b) With Downward rigidities

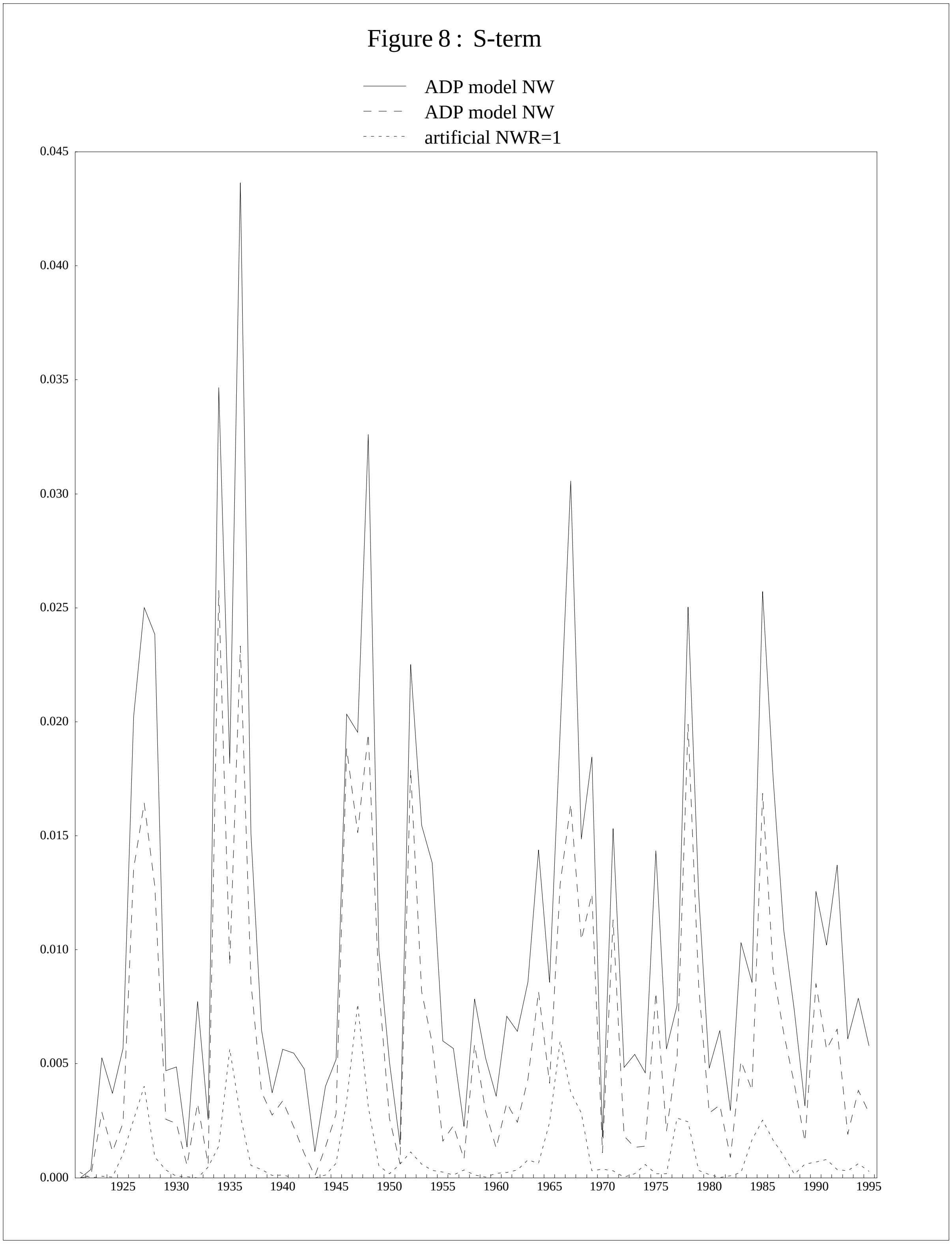
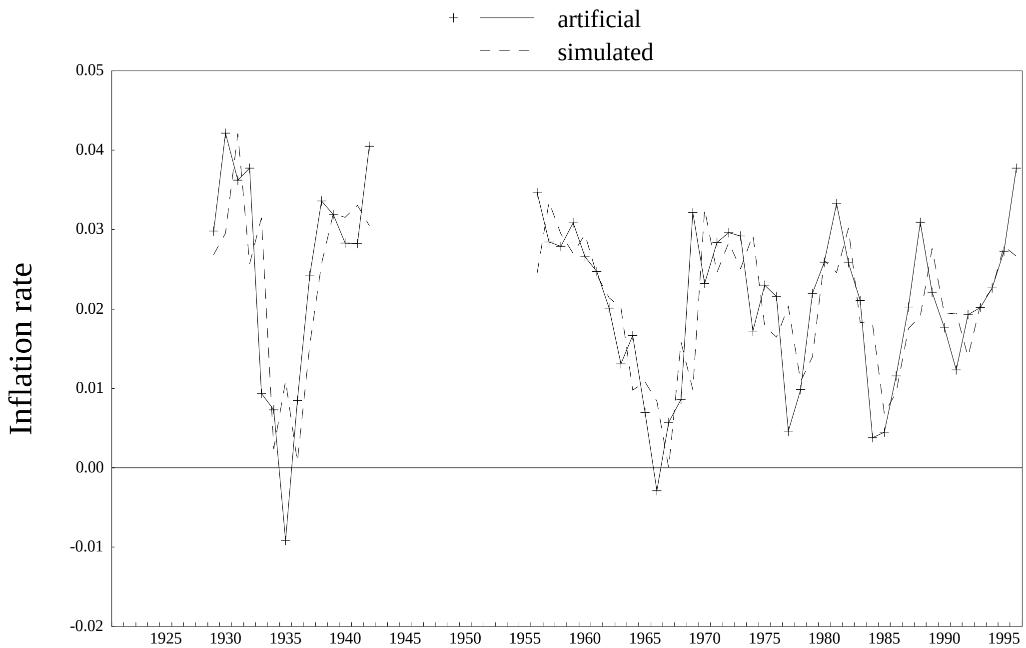


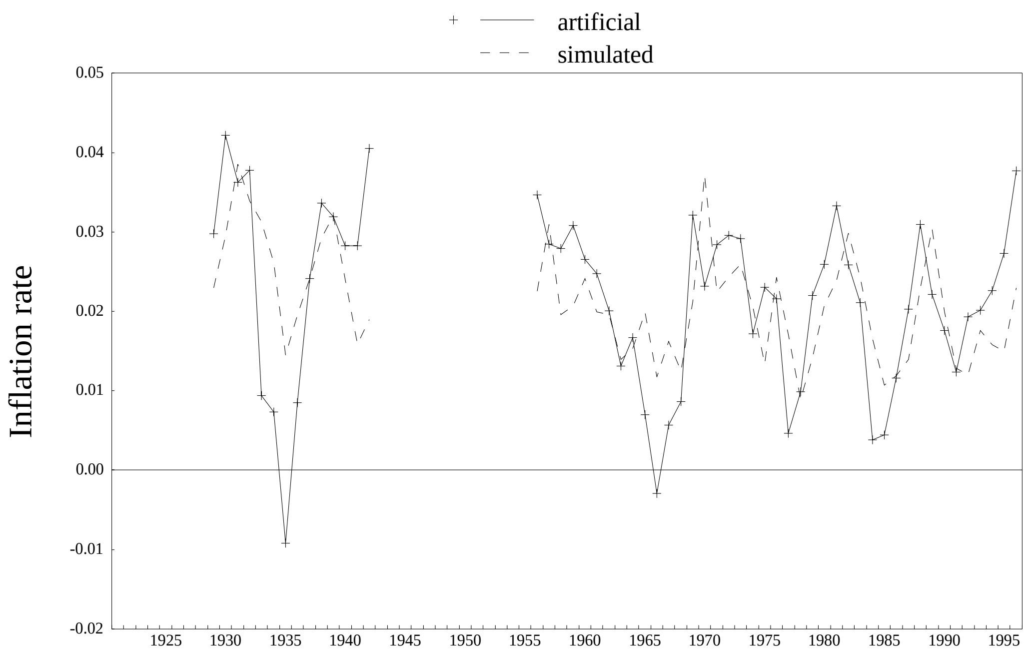
Figure 8 : S-term
ADP model NW
ADP model NW
artificial NWR=1
0.045
0.040
0.035
0.030
0.025
0.020
0.015
0.010
0.005
0.000
1925
1930
1935
1940
1945
1950
1955
1960
1965
1970
1975
1980
1985
1990
1995

Figure 9 : Static Simulations of Inflation

Sample : 1956-1989 (NWR=0)



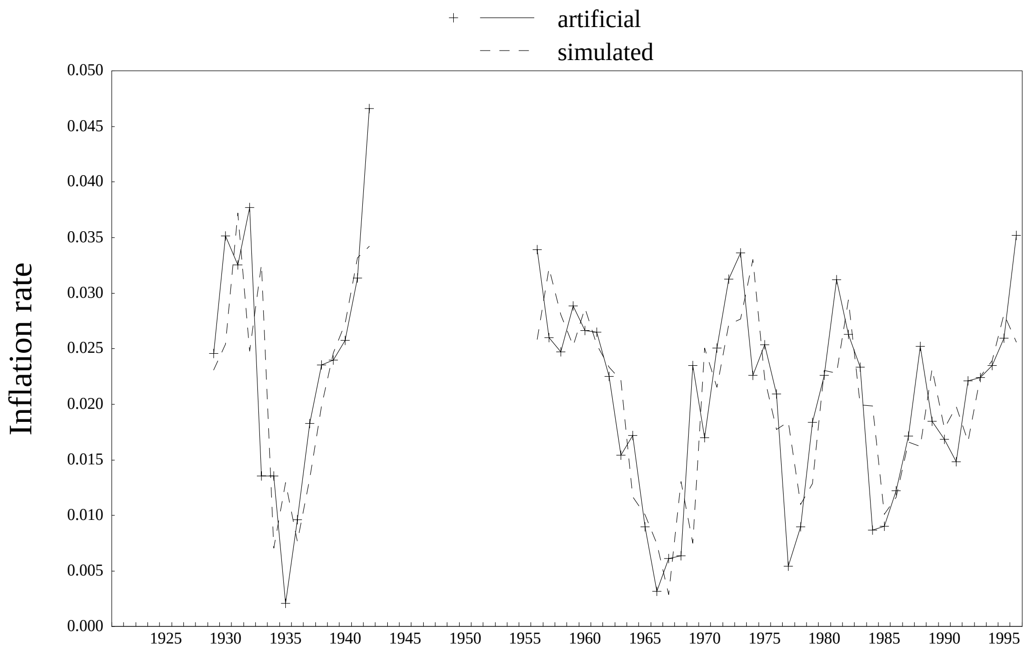
(a) Without Downward rigidities



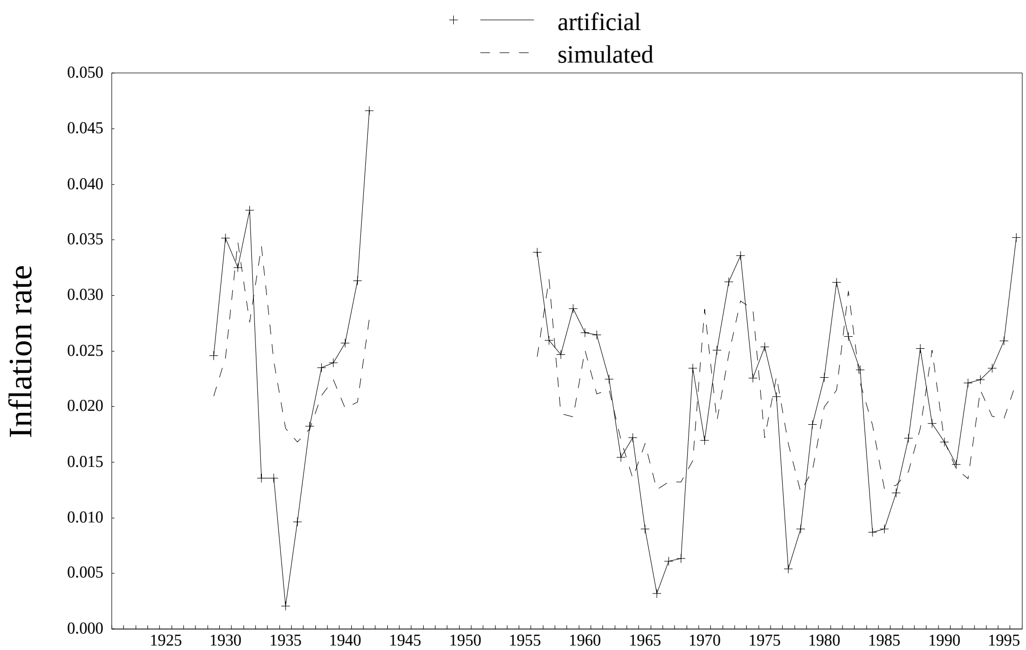
(b) With Downward rigidities

Figure 10 : Static Simulations of Inflation

Sample : 1956-1989 (NWR=1)



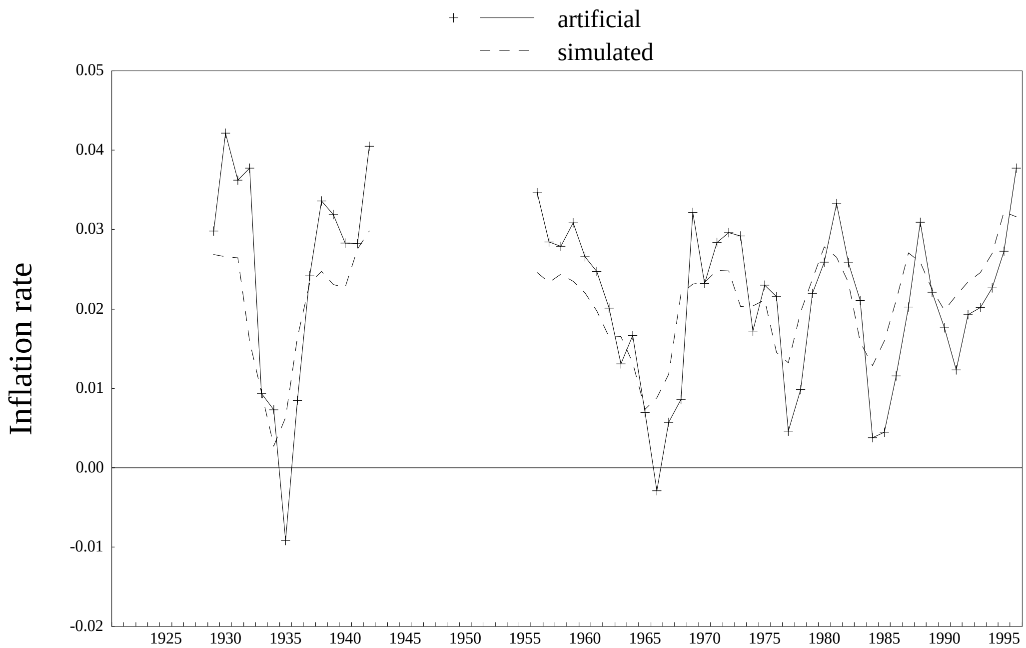
(a) Without Downward rigidities



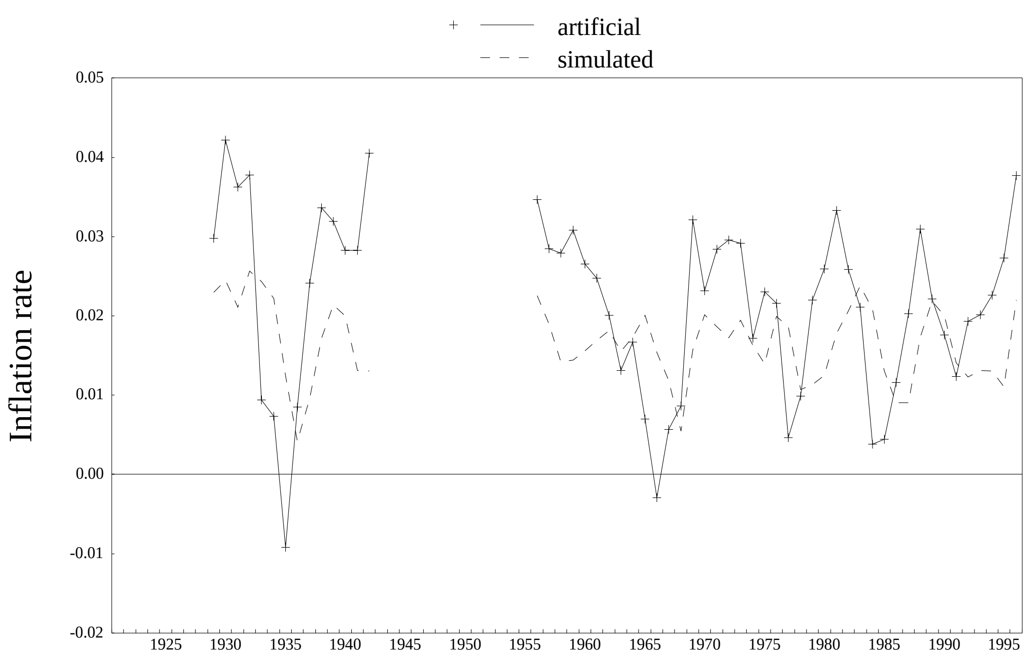
(b) With Downward rigidities

Figure 11 : Dynamic Simulations of Inflation

Sample : 1956-1989 (NWR=0)



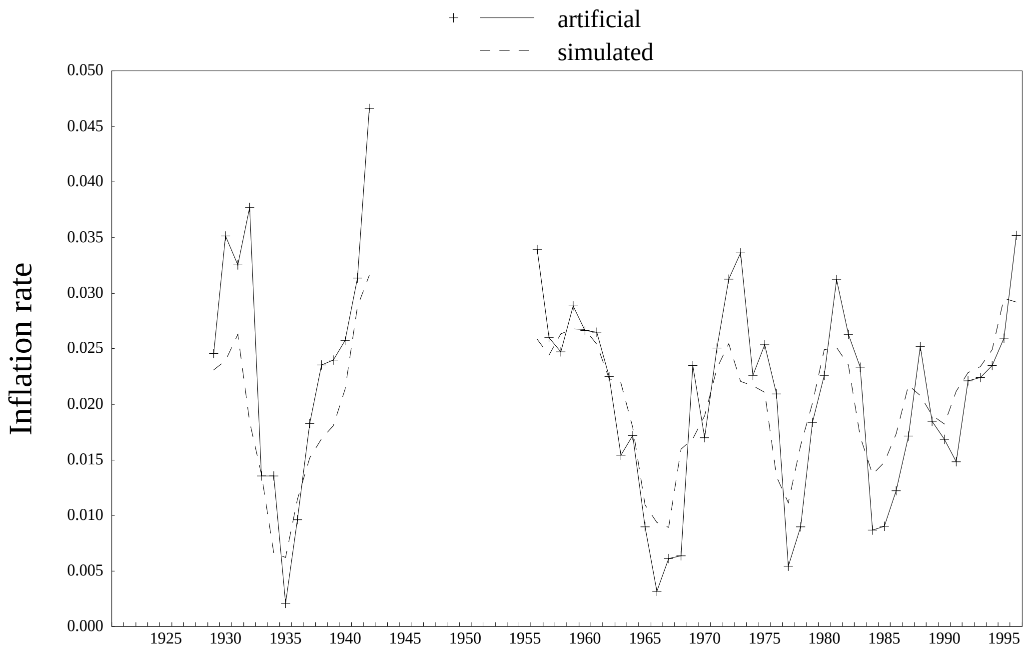
(a) Without Downward rigidities



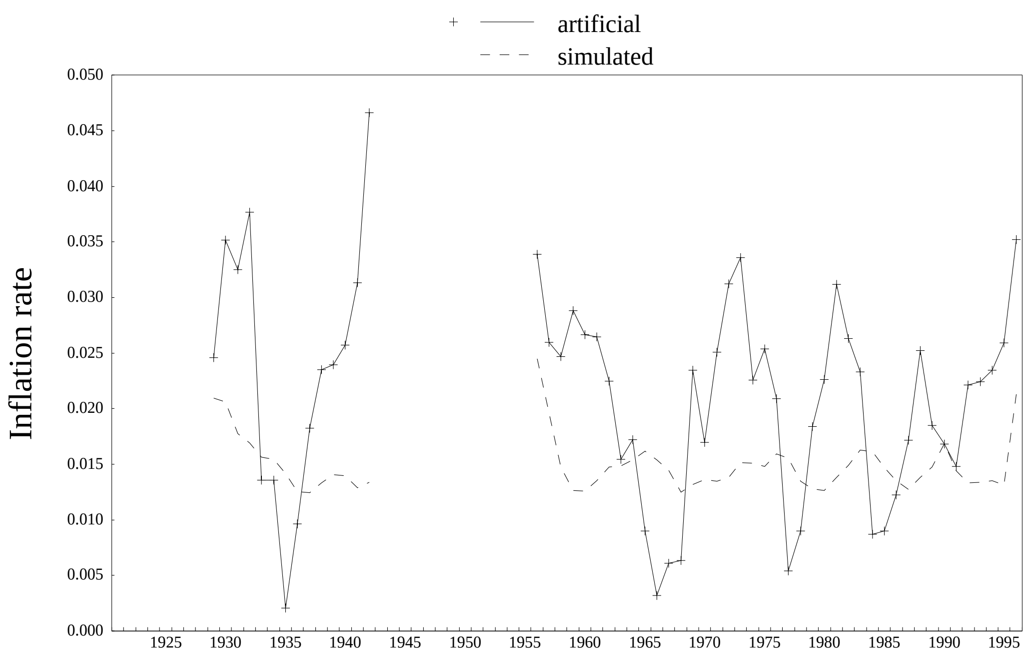
(b) With Downward rigidities

Figure 12 : Dynamic Simulations of Inflation

Sample : 1956-1989 (NWR=1)



(a) Without Downward rigidities



(b) With Downward rigidities

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