BANK OF CANADA
BANQUE DU CANADA

Working Paper/Document de travail
2014-39

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Bank of Canada Working Paper 2014-39

September 2014

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William A. Barnett,[1] Marcelle Chauvet[2] and Danilo Leiva-Leon[3]

[1]University of Kansas
Lawrence, KS 66045-7585
barnett@ku.edu
Center for Financial Stability
New York, NY 1003
wbarnett@the-cfs.org

[2]University of California
Riverside, CA 92521
chauvet@ucr.edu

[3]International Economic Analysis Department
Bank of Canada
Ottawa, Ontario, Canada K1A 0G9
leiv@bankofcanada.ca

ISSN 1701-9397

Acknowledgements

We would like to thank Maximo Camacho, Gabriel Perez-Quiros, Michael Bauer, Hashem Pesaran and Michael McCracken, as well as participants at the 9th Annual CIRANO Workshop and the 2013 CAFE/USC Workshop for helpful comments and suggestions.

Abstract

This paper provides a framework for the early assessment of current U.S. nominal GDP growth, which has been considered a potential new monetary policy target. The nowcasts are computed using the exact amount of information that policy-makers have available at the time predictions are made. However, real-time information arrives at different frequencies and asynchronously, which poses challenges of mixed frequencies, missing data and ragged edges. This paper proposes a multivariate state-space model that not only takes into account asynchronous information inflow, but also allows for potential parameter instability. We use small-scale confirmatory factor analysis in which the candidate variables are selected based on their ability to forecast nominal GDP. The model is fully estimated in one step using a non-linear Kalman filter, which is applied to obtain optimal inferences simultaneously on both the dynamic factor and parameters. In contrast to principal component analysis, the proposed factor model captures the co-movement rather than the variance underlying the variables. We compare the predictive ability of the model with other univariate and multivariate specifications. The results indicate that the proposed model containing information on real economic activity, inflation, interest rates and Divisia monetary aggregates produces the most accurate real-time nowcasts of nominal GDP growth.

JEL classification: C32, E27, E31, E32
Bank classification: Business fluctuations and cycles; Econometric and statistical methods; Inflation and prices

Résumé

Dans cette étude, les auteurs proposent un modèle qui permet d'évaluer de façon précoce la croissance actuelle du PIB nominal des États-Unis, laquelle est considérée comme une possible nouvelle cible de politique monétaire. Les prévisions concernant la période en cours sont établies à partir des mêmes informations dont disposent les décideurs à ce moment précis. Toutefois, les renseignements en temps réel leur parviennent à des intervalles différents et de manière asynchrone, ce qui cause plusieurs problèmes : fréquences diverses, données manquantes, valeurs absentes en fin d'échantillon. Dans cette étude, les auteurs proposent un modèle espace d'états multivarié qui non seulement prend en considération le flux asynchrone d'informations, mais tient compte aussi de l'instabilité éventuelle des paramètres. Ils utilisent une analyse factorielle confirmatoire à échelle réduite, les variables admissibles étant choisies selon leur potentiel de prévision du PIB nominal. L'estimation complète du modèle s'effectue en une seule étape, au moyen d'un filtre de Kalman non linéaire, qui sert à tirer simultanément des inférences optimales aussi bien sur le facteur dynamique que sur les paramètres. Contrairement à l'analyse en composantes principales, le modèle factoriel proposé fait ressortir la corrélation entre les variables plutôt que leur variance sous-jacente. Les auteurs comparent le potentiel prévisionnel du modèle avec d'autres spécifications univariées ou multivariées. D'après les résultats obtenus, le modèle proposé, qui exploite des renseignements sur l'activité de l'économie réelle, l'inflation, les taux d'intérêt et les agrégats monétaires de Divisia, donne les prévisions en temps réel les plus fiables de la croissance du PIB nominal pour la période en cours.

Classification JEL : C32, E27, E31, E32
Classification de la Banque : Cycles et fluctuations économiques; Méthodes économétriques et statistiques; Inflation et prix

1 Introduction

In recent years, U.S. interest rates have reached a technical lower bound level, but the unemployment rate has still remained at high levels. In view of this situation, the Federal Reserve has been using complementary tools to carry out monetary policy. One of them, which is the motivation of our analysis, is "forward guidance." As discussed by Bernanke (2012) and Woodford (2012) at the Annual Jackson Hole Economic Symposium, this tool consists of explicit statements by a central bank about its future medium- and long-run actions, conditional on the developments in the economy, in addition to its announcements about immediate short-run policy. The idea is that, depending on the target and rule that the Fed is committed to follow, pursuing "forward guidance" could lead to changes in expectations by economic agents, which could hasten achievement of the Fed's target.

During the most recent recession, the trend of nominal GDP showed a substantial contraction associated with several large negative shocks, and the gap between the current and pre-crisis trend level is still large (Figure 1). Many economists have suggested that the Fed should start targeting the path of nominal GDP (Hall and Mankiw 1994, Romer 2011, and Woodford 2012, among others), since they consider this would constitute a powerful communication tool. Under this proposal, the funds rate would remain around the lower bound until nominal GDP reaches the pre-crisis level and, once this is achieved, the funds rate would increase as necessary to ensure stable growth in the long run. Since nominal GDP is the output of the economy times the price level, setting the objective of returning nominal GDP to its pre-crisis trajectory could improve expectations about future economic conditions. The conjecture is that such expectations would increase households' incentives to consume more in the present, and firms would be more optimistic regarding their future demand and, therefore, their present investment decisions.[1]

Although the focus of this paper is not to address whether or not the Fed should target nominal GDP, we contribute to this literature by claiming that, under a nominal GDP targeting scenario, monitoring of the output path plays a fundamental role in assessing policy effectiveness and its future direction. The goal of this paper is to provide early real-time nowcasts of nominal GDP growth that can be useful to inform monetary policy and economic agents.[2] The work of Croushore and Stark (2001) was the starting point for a large forecasting literature that emphasizes the use of unrevised real-time data, which allows evaluation of how models performed at the time events were taking place. Accordingly, nowcasts of nominal GDP are computed using only the exact information available at the time predictions are made in order to reproduce the real-time forecasting problem of policy-makers and economic agents, based on a real-time data set for each vintage constructed for this paper.

[1]For an extensive discussion on forward guidance and targeting nominal GDP, see Woodford (2012), Belongia and Ireland (2012), and Del Negro et al. (2012).

[2]Given lags of at least one month in the release of many macroeconomic variables, forecasting the present and even the near past is required to assess the current economic situation. The literature has named this a "nowcast", which is a term that has widespread use, including by the U.S. National Weather Service for current weather.

However, data arrive asynchronously, at different frequencies and, at first, based on preliminary and incomplete information. This creates the challenge of handling mixed frequencies, missing observations and lags in the availability of primary data (ragged edges). Some advances in forecasting methods have been proposed to address these problems. This is particularly the case in the growing literature on short-term forecasting and nowcasting using multivariate state-space models, which rely on the methods of Trehan (1989), Mariano and Murasawa (2003), Evans (2005), Proietti and Moauro (2006), or Giannone, Reichlin and Small (2008). Other mixed-frequency methods have been proposed and applied to univariate and multivariate vector autoregressive (VAR) processes such as the mixed-data sampling (MIDAS) proposed by Ghysels, Santa-Clara and Valkanov (2004) or the mixed-frequency VAR in Kuzin, Marcellino and Schumacher (2011), Gotz and Hecq (2014), and the mixed-frequency Bayesian VAR in Schorfheide and Song (2011).

Our paper combines the multivariate state-space system with the mixed-frequency approach of Mariano and Murasawa (2003) and the small-scale dynamic factor model of Stock and Watson (1989), which is extended in a non-linear version to allow for potential structural breaks. Stock and Watson (1989) propose a widely popular low-dimensional linear dynamic factor model to construct coincident indicators of the U.S. economy. Linear and non-linear extensions of this small-scale dynamic factor model have been successfully used in real-time forecasting. For the United States, see, for example, Chauvet (1998), Chauvet and Hamilton (2006), Chauvet and Piger (2008), Aruoba and Diebold (2010), Aruoba, Diebold and Scotti (2009) and Camacho and Martinez-Martin (2013); for Europe, see Camacho and Perez-Quiros 2010; and for Brazil, see Chauvet (2001). [3]

Several recent papers, such as Bai and Ng (2008a, 2008b), Jungbacker and Koopman (2008), and Doz, Giannone and Reichlin (2012a, 2012b), among several others, find that small-scale factor models estimated through maximum likelihood display desirable properties such as efficiency gains or robustness when fewer but more informative predictors are carefully selected. Boivin and Ng (2006) and Bai and Ng (2008a, 2008b) contend that exploratory large factor models with uninformative data can result in large idiosyncratic error variances and cross-sectional correlated errors, reducing the accuracy of estimates, and the model's predictive content.[4] They argue for the benefits of supervised (confirmatory) factor models - even in a data-rich environment - with pre-screened series based on economic reasoning and their predictive ability for the target variable. More recently, Alvarez, Camacho and Perez-Quiros (2013) show through Monte Carlo analysis that small-scale factor models outperform large-scale models in factor estimation and forecasting.

[3]The indicators for the United States based on these papers are updated on a regular basis and posted on the websites of the Saint Louis Fed: http://research.stlouisfed.org/fred2/series/RECPROUSM156N, the Atlanta Fed: http://www.frbatlanta.org/cqer/researchcq/chauvet_real_time_analysis.cfm, and the Philadelphia Fed:

http://www.philadelphiafed.org/research-and-data/real-time-center/business-conditions-index/. Those for Brazil are updated by the Center for Research on Economic and Financial Cycles: https://sites.google.com/site/crefcus/brazil; and those for the euro area are updated regularly by the Bank of Spain but not posted on its website.

[4]Doz, Giannone, and Reichlin (2012, 2012b) show that misspecification of factors in a two-step estimation using principal components is negligible for a large number of variables and a large sample size. However, they find that, through Monte Carlo analysis, robustness is already achieved with only a handful of predictors.

Following this literature, we use small-scale confirmatory factor analysis in which the candidate variables are carefully selected based on their marginal predictive ability to the target variable nominal GDP growth. In addition, we extend existing frameworks by proposing a multivariate state-space system that considers the possibility of parameter instability in addition to asynchronous information inflow. We propose a single fully specified mixed-frequency dynamic factor model with a potential structural break (MFDFB model). Our model differs from large factor models not only in its scale, but also in its estimation procedure, which yields very different factors. Most large factor models rely on two-step estimations, in which the factors are extracted as principal components. Within this methodology, the resulting factors represent the maximum explained variance of the underlying variables.[5] In contrast, in our paper the factor and model parameters are estimated simultaneously in one step through maximum likelihood. The method yields optimal inferences on the dynamic factor, which captures the common correlation underlying the observable variables. The main difference between these two approaches is that, in the proposed model, the factor does not extract all variance from the variables, but only that proportion that is attributable to the commonality shared by all observable variables (i.e., their common variance). In addition, the set of hypotheses that form the conceptual basis of the fully estimated confirmatory factor analysis enables interpretation of the factor and specification testing.

We compare the predictive ability of the model with alternative univariate and multivariate specifications, which are combined with the best leading indicators of nominal GDP growth. The results indicate that the linear mixed-frequency dynamic factor models containing information on real economic activity, inflation, monetary indicators and interest rates outperform univariate specifications, both linear and non-linear. However, the proposed small-scale mixed-frequency dynamic factor model that allows for structural break outperforms all other specifications considered. The results provide evidence of substantial gains in real-time nowcasting accuracy when allowing for parameter instability.

The structure of the paper is as follows. Section 2 introduces the Mariano and Murasawa mixed-frequency method in a simple-sum naive model. Section 3 presents the linear and the proposed non-linear mixed-frequency dynamic factor model with structural break. Section 4 presents alternative univariate frameworks. Section 5 discusses the timing of forecasts, the real-time data, the variable selection and the empirical results. Section 6 reports the real-time nowcasting findings, and Section 7 concludes.

2 Simple-Sum Mixed-Frequency Naive Model

Nominal GDP (NGDP) is the market value at current prices of all final goods and services produced within a country in a given period of time. It can also be viewed as the real GDP times the price level of the economy. Therefore, letting Z_t, X_t and P_t be nominal GDP, real GDP and the price level, respectively, which are quarterly indicators observable every third period, there is a conceptual link between these

[5] See, e.g., Giannone, Reichlin and Small (2008), Doz, Giannone and Reichlin (2012a, 2012b), Banbura et al.(2013) and Banbura and Modugno (2010).

three variables:

$$\begin{aligned} Z_t &= X_t P_t \\ \ln Z_t - \ln Z_{t-3} &= \ln X_t - \ln X_{t-3} + \ln P_t - \ln P_{t-3} \\ z_t &= x_t + p_t \end{aligned} \tag{1}$$

We can take advantage of the fact that the target variable contains a real activity component and an inflation component and proxy x_t and p_t, which are at quarterly frequency, with indicators available at monthly frequency, such as Industrial Production (IP) and the Consumer Price Index (CPI), respectively. Charts A and B of Figure 2 show real GDP growth and the GDP deflator growth at quarterly frequency, while charts C and D plot IP and CPI growth rates at monthly frequency, respectively. The recessions dated by the National Bureau of Economic Research(NBER) are represented by the shaded areas. The monthly series display similar dynamics to the quarterly ones, but are available in a more timely manner.

We obtain a "naive" monthly index of our target variable NGDP growth by adding IP and CPI growth rates and standardizing that sum with the mean and standard deviation of NGDP. Since the naive index is at monthly frequency and NGDP is at quarterly frequency, we use the transformation in Mariano and Murasawa (2003) to compare both variables in quarterly terms. Let Z_t be the geometric mean of W_t, W_{t-1} and W_{t-2}, then

$$\ln Z_t = \frac{1}{3}(\ln W_t + \ln W_{t-1} + \ln W_{t-2}).$$

Taking three-period differences, for all t,

$$\ln Z_t - \ln Z_{t-3} = \frac{1}{3}(\ln W_t - \ln W_{t-3}) + \frac{1}{3}(\ln W_{t-1} - \ln W_{t-4}) + \frac{1}{3}(\ln W_{t-2} - \ln W_{t-5})$$

or

$$\begin{aligned} z_t &= \frac{1}{3}(w_t + w_{t-1} + w_{t-2}) + \frac{1}{3}(w_{t-1} + w_{t-2} + w_{t-3}) + \frac{1}{3}(w_{t-2} + w_{t-3} + w_{t-4}) \\ &= \frac{1}{3}w_t + \frac{2}{3}w_{t-1} + w_{t-2} + \frac{2}{3}w_{t-3} + \frac{1}{3}w_{t-4}, \end{aligned} \tag{2}$$

where $z_t = \ln Z_t - \ln Z_{t-3}$ and $w_t = \ln W_t - \ln W_{t-1}$. In this way, the quarter-over-quarter growth rates, z_t, can be expressed as month-over-month growth rates, w_t.

Chart A of Figure 3 plots both series at quarterly frequency, as well as the NBER recessions. The naive index yields a relatively good in-sample fit. However, as will be discussed later, the performance of the index is not accurate in real-time nowcasts of NGDP (Figure 3 Chart B, Table 4). In order to obtain more accurate real-time forecasts, we explore the information contained in real and nominal indicators by extracting their underlying co-movement using factor models, rather than relying on their simple sum.

3 Mixed-Frequency Dynamic Factor Model

3.1 Linear Framework (MFDF)

In this section, we specify the linear nowcasting dynamic factor model that allows for the inclusion of both mixed-frequency data and missing observations. We use the approach proposed by Mariano and Murasawa (2003) in equation (2) to express quarterly data in monthly terms. The dynamic factor model extracts the co-movement among the target variable NGDP growth, denoted by $y_{1,t}$, an indicator of real economic activity, $y_{2,t}$, an indicator of inflation dynamics, $y_{3,t}$, and other candidate variables, $y_{h,t}, h = 4, ..., N$. The model separates out common cyclical fluctuations underlying these variables in the unobservable factor, f_t, and idiosyncratic movements not representing their intercorrelations captured by the associated idiosyncratic terms, $v_{n,t}$ for $n = 1, 2, ..., N$. The model is expressed as follows:

$$\begin{bmatrix} y_{1,t} \\ y_{2,t} \\ y_{3,t} \\ \vdots \\ y_{N,t} \end{bmatrix} = \begin{bmatrix} \gamma_1 \left(\frac{1}{3}f_t + \frac{2}{3}f_{t-1} + f_{t-2} + \frac{2}{3}f_{t-3} + \frac{1}{3}f_{t-4}\right) \\ \gamma_2 f_t \\ \gamma_3 f_t \\ \vdots \\ \gamma_N f_t \end{bmatrix} + \begin{bmatrix} \frac{1}{3}v_{1,t} + \frac{2}{3}v_{1,t-1} + v_{1,t-2} + \frac{2}{3}v_{1,t-3} + \frac{1}{3}v_{1,t-4} \\ v_{2,t} \\ v_{3,t} \\ \vdots \\ v_{N,t} \end{bmatrix}, \tag{3}$$

where γ_n are the factor loadings, which measure the sensitivity of the common factor to the observable variables. The dynamics of the unobserved factor and error terms are modelled as autoregressive processes:

$$f_t = \phi_1 f_{t-1} + \ldots + \phi_P f_{t-P} + e_t, \quad e_t \sim i.i.d.N(0,1) \tag{4}$$

$$v_{n,t} = \varphi_{n1} v_{n,t-1} + ... + \varphi_{nQ_n} v_{n,t-Q_n} + \epsilon_{n,t}, \quad \epsilon_{n,t} \sim i.i.d.N(0, \sigma^2_{\epsilon_n}), \quad \text{for } n = 1, 2, ..., N \tag{5}$$

The model also assumes that f_t and v_{nt} are mutually independent at all leads and lags for all N variables. This assumption, together with $\epsilon_{n,t} \sim i.i.d.N(0, \sigma^2_{\epsilon_n})$, is at the core of the definition of the small-scale dynamic factor model, as in Stock and Watson (1989), since it implies that the model separates out common correlation underlying the observed variables from individual variations in each series.

In order to obtain optimal inferences on the unobserved variables f_t and v_{nt}, the system of equations (3) to (5) is cast into a state-space representation, which is estimated using the Kalman filter:

$$y_t = HF_t + \xi_t, \; \xi_t \sim i.i.d.N(0, R) \tag{6}$$

$$F_t = TF_{t-1} + \zeta_t, \; \zeta_t \sim i.i.d.N(0, Q) \tag{7}$$

Equation (6) corresponds to the *measurement equation* that relates observed variables with the unobserved common component and idiosyncratic terms from equation (3). Equation (7) is the *transition equation*, which specifies the dynamics of the unobserved variables in equations (4) and (5).

Using Mariano and Murasawa (2003) and the adaptation of Camacho and Perez-Quiros (2010) we modify the state-space model in equations (6) and (7) to incorporate potential missing observations into the system. The strategy consists of substituting each missing observation with a random draw v_t from a $N(0, \sigma^2_v)$. This substitution keeps the matrices conformable without affecting the estimation of the model parameters. The components of the model (6) and (7) are updated, depending on whether or not $y_{n,t}$ is observed, in the following way:

$$y_{n,t} = \begin{cases} y_{n,t} \text{ if } y_{n,t} \text{ observed} \\ \upsilon_t \text{ otherwise} \end{cases}, \quad H^*_{n,t} = \begin{cases} H_n \text{ if } y_{n,t} \text{ observed} \\ 0_{1\kappa} \text{ otherwise} \end{cases}$$

$$\xi^*_{n,t} = \begin{cases} 0 \text{ if } y_{n,t} \text{ observed} \\ \upsilon_t \text{ otherwise} \end{cases}, \quad R^*_{n,t} = \begin{cases} 0 \text{ if } y_{n,t} \text{ observed} \\ \sigma^2_\upsilon \text{ otherwise} \end{cases}$$

where $H^*_{n,t}$ is the nth row of matrix H which has κ columns, and $0_{1\kappa}$ is a κ row vector of zeros. Therefore, in the model robust to missing observations, the measurement equation (6) is replaced by

$$y_t = H^*_t F_t + \xi^*_t,\ \ \xi^*_t \sim i.i.d.N(0, R^*_t). \tag{8}$$

The Kalman filter is applied to the time-varying state-space model in equations (7) and (8) to obtain in one step optimal linear predictions of the model parameters and the latent state vector F_t, which contains information on the co-movement among the economic indicators, $y_{n,t}$ for $n = 1, 2, ..., N$, collected in the dynamic factor f_t. The filter tracks the course of the dynamic factor, which is calculated using only observations on $y_{n,t}$. It computes recursively one-step-ahead prediction and updating equations of the conditional expectation of the dynamic factor and the associated mean squared error matrices. The output, $f_{t|t}$, is an optimal estimator of the dynamic factor constructed as a linear combination of the variables $y_{i,t}$, using information available through time t. As new information becomes available, the filter is applied to update the state vector on a real-time basis. A by-product of the filter is the conditional likelihood of the observable variables. The filter simultaneously evaluates this likelihood function, which is maximized with respect to the model parameters using an optimization algorithm. These parameters and the observations on $y_{n,t}$ are then used in a final pass of the filter to yield the optimal latent dynamic factor based on maximum-likelihood estimates.

3.2 Mixed-Frequency Dynamic Factor Model with Structural Break (MFDFB)

Over the years, the U.S. economy has experienced different regimes that could have strongly impacted the dynamics of nominal GDP, such as the Great Moderation or the Great Recession (2007M12-2009M06). In order to account for this possibility, we propose a non-linear dynamic factor model that allows nowcasting with mixed frequencies and structural breaks. In particular, this paper extends the model (3)-(5) to allow for potential endogenous breaks in the common factor, which are modelled by considering two independent absorbing Markov processes. Specifically, equation (4) is replaced by[6]

$$f_t = \mu_{S^m_t} + \phi_1 f_{t-1} + \ldots + \phi_p f_{t-P} + e_t, \quad e_t \sim i.i.d.N(0, \sigma_{S^v_t}) \tag{9}$$

$$\mu_{S^m_t} = \mu_0(1 - S^m_t) + \mu_1 S^m_t \tag{10}$$

[6]Note that, in this model, identification of the factor is achieved by setting one of the factor loadings to unity. The choice of normalization does not affect the parameter estimation.

$$\sigma_{S_t^v} = \sigma_0(1 - S_t^v) + \sigma_1 S_t^v, \tag{11}$$

where S_t^m and S_t^v are distinct unobserved two-state Markov variables that capture permanent changes in the factor mean or variance, respectively:

$$S_t^m = 0 \text{ for } 1 \leq t \leq \tau^m \text{ and } S_t^m = 1 \text{ for } \tau^m < t \leq T-1$$
$$S_t^v = 0 \text{ for } 1 \leq t \leq \tau^v \text{ and } S_t^v = 1 \text{ for } \tau^v < t \leq T-1$$

We model the one-time break as an unknown change point, τ^k, for $k = m, v$, which follows constrained unobservable Markov state variables, as in Chib (1998)[7]:

$$\begin{aligned}
\Pr(S_t^k &= 1|S_{t-1}^k = 1) = p_{11}^k = 1 \\
\Pr(S_t^k &= 0|S_{t-1}^k = 1) = 1 - p_{11}^k = 0 \\
\Pr(S_t^k &= 1|S_{t-1}^k = 0) = 1 - p_{00}^k \\
\Pr(S_t^k &= 0|S_{t-1}^k = 0) = p_{00}^k, 0 < p_{00}^k < 1
\end{aligned} \tag{12}$$

On the one hand, in order to capture the structural break, the transition probabilities $p_{ij}^k = \Pr(S_t^k = j|S_{t-1}^k = i)$ are restricted so that the probability that S_t^k will switch from state 0 at the unknown change point τ^k to state 1, at $\tau^k + 1$, is greater than zero. On the other hand, once the economy switches to state 1, it will stay at this state permanently. The corresponding transition probability matrix, for p_{ij}^k with row j, column i is given by

$$P^k = \begin{bmatrix} p_{00}^k & 0 \\ 1 - p_{00}^k & 1 \end{bmatrix}. \tag{13}$$

The motivation for assuming an absorbing state relies on the same intuition as Kim and Nelson (1999) and Chauvet and Su (2013), where the transition probabilities of a Markov-switching univariate model are truncated as in Equation (12) to identify the one-time permanent break in the volatility of U.S. real GDP that occurred in the mid-1980s, the starting point of the Great Moderation. Notice, however, that the assumption of only one break may be restrictive depending on the application of the model. Such as assumption could also be relaxed by including more state variables to allow for multiple breaks; however, in such case, a more complicated issue arises in assessing how many breaks should be assumed. Therefore, in order to avoid unnecessary complexities, we adopt the parsimonious specification of a one-time break for potentially different time spans.

The proposed *mixed-frequency dynamic factor model with structural break (MFDFB)* can be represented in the following state-space form:

$$y_t = H_t^* F_t + \xi_t^*, \; \xi_t^* \sim i.i.d.N(0, R_t^*) \tag{14}$$

$$F_t = \lambda_{S_t^m} + TF_{t-1} + \zeta_t, \; \zeta_t \sim i.i.d.N(0, Q_{S_t^v}). \tag{15}$$

In this case, the model is estimated in one step via maximum likelihood using a combination of the Kalman filter and Hamilton's (1989) algorithm. The non-linear filter forms forecasts of the unobserved

[7] See also Kim and Nelson (1999), McConnell and Perez-Quiros (2000), and Chauvet and Su (2013).

state vector. As in the linear Kalman filter, the algorithm calculates recursively one-step-ahead prediction and updating equations of the dynamic factor and the mean squared error matrices, given the parameters of the model and starting values for the state vector, the mean squared error and, additionally, the probabilities of the Markov states. The updating equations are computed as averages weighted by the probabilities of the Markov states. The conditional likelihood of the observable variables is obtained as a by-product of the algorithm at each t, which is used to estimate the unknown model parameters. The filter evaluates this likelihood function, which is then maximized with respect to the model parameters using a non-linear optimization algorithm. The maximum likelihood estimators and the sample data are then used in a final application of the filter to draw inferences about the dynamic factor and probabilities, based on information available at time t. The outputs are the conditional expectation of the state vector at t given I_t, and the filtered probabilities of the Markov states $\Pr(S_t^k = j|I_t)$, where I_t is the information set at t,based on the observable variables. For details see Kim (1994).

4 Univariate Autoregressive Models

4.1 Linear Autoregressive Model

We compare the real-time nowcasts obtained from the multivariate mixed-frequency model of nominal GDP growth and monthly indicators with those obtained from univariate models based solely on quarterly NGDP growth. Consider the following autoregressive model:

$$y_{t|v} = \phi_{0|v} + \sum_{p=1}^{P} \phi_{p|v} y_{t-p|v} + u_{t|v} \quad u_{t|v} \sim i.i.d.N(0, \sigma^2_{u|v}), \tag{16}$$

where $y_{t|v}$ denotes NGDP growth of quarter t that is observed at monthly vintage v, and $\phi_{p|v}$ are the autoregressive parameters computed with all the available information up to v. At the end of the sample T a forecast for the next period is computed as

$$\hat{y}_{T+1|V} = \hat{\phi}_{0|V} + \sum_{p=1}^{P} \hat{\phi}_{p|V} y_{T-p+1|V}, \tag{17}$$

where V denotes the last available vintage.

4.2 Autoregressive Model with Structural Break

In order to account for potential parameter instability in the autoregressive specifications, we follow the same method proposed for the mixed-frequency dynamic factor model with breaks. That is, the coefficients in equation (16), are subject to potential one-time breaks at unknown date τ, which follow unobserved two-state Markov variables, S_t^m and S_t^v:

$$y_{t|v} = \phi_{0|v,S^m_{t|v}} + \sum_{p=1}^{P} \phi_{p|v,S^m_{t|v}} y_{t-p|v} + u_{t|v}, \quad u_{t|v} \sim i.i.d.N(0, \sigma^2_{u|v,S_t^v}). \tag{18}$$

The dynamics of $S_t^k, k = m, v$ are subject to the same restrictions as in the multivariate approach under structural breaks in Section 3.2. The estimation of the model in equation (18) is performed by maximum likelihood.[8] Out-of-sample nowcasts with real-time data, $\hat{y}_{T+1|V}$, are obtained from

$$\hat{y}_{T+1|V} = E(y_{T+1|V}) = \sum_{j=0}^{1} \Pr(S_{T+1}^k = j|V) y_{T+1|S_{T+1}^k=j,V}, \tag{19}$$

where $\Pr(S_{T+1}^k = j|V)$ can be computed by using the transition probability matrix P^k and $y_{T+1|S_{T+1}^k=j,V}$ can be obtained from equation (16) conditioned on the Markov state variables.

5 Empirical Results

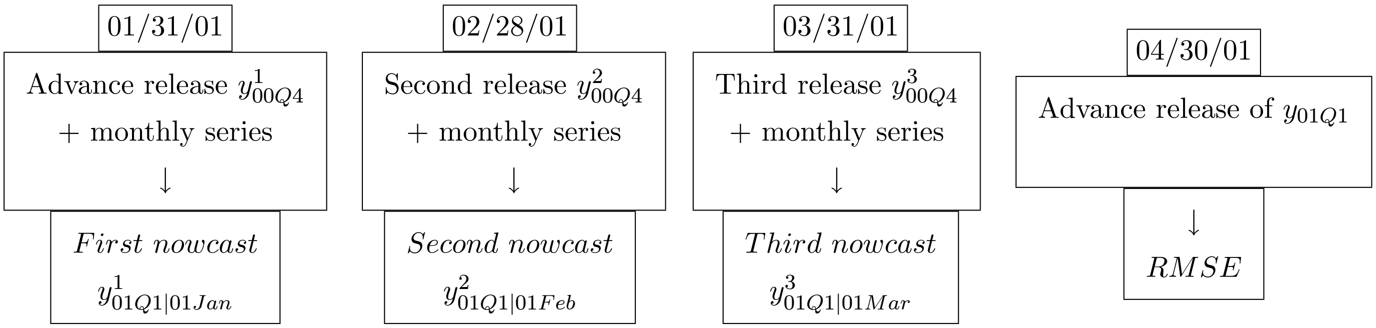
5.1 Timing of Forecasts

The U.S. nominal GDP (NGDP) series is first released by the Bureau of Economic Analysis (BEA) based on timely but incomplete information. Subsequent releases may involve large revisions that may yield inconsistencies caused by lags in the data availability. There are three main releases of NGDP for a quarter, which occur in the three subsequent months following that quarter. For example, the first release of NGDP ("advance" estimate) for the fourth quarter of a year occurs at the end of January of the following year. The "second estimate" is released at the end of February, and the "third estimate" is released at the end of March. This allows us to compute three monthly inferences of NGDP for each quarter.

Our interest is in an assessment based solely on information that was available at each date, reproducing the real-time forecasting problem for monetary policy monitoring at the time events were unfolding. We have collected vintages of NGDP and many other macroeconomic and financial time series as they would have appeared at the end of each month. For each vintage, the sample collected begins in 1967M1. The models are estimated with data from 1967M1 to 2000M12, and then recursively estimated for the period starting in 2001M1 and ending in 2012M12, using only collected real-time vintages as released at each period to generate nowcasts of NGDP growth.[9] For example, the first prediction is for nominal GDP growth in the first quarter of 2001, y_{01Q1}, which uses monthly indicators and NGDP growth up to its "advance release" of 2000Q4, y_{00Q4}^1, based on information up to January 2001. The first nowcast is $y_{01Q1|01Jan}^1$. The second nowcast of y_{01Q1} is $y_{01Q1|01Feb}^2$ obtained at the end of 2001M2, using monthly information up to 2001M2 and NGDP growth up to its second release of 2000Q4, y_{00Q4}^2. The third nowcast of y_{01Q1} is $y_{01Q1|01Mar}^3$, obtained at the end of 2001M3, using monthly information up to 2001M3 and NGDP growth up to its third release of 2000Q4, y_{00Q4}^3. Notice that this is the last prediction of y_{01Q1} since, at the end of April, its "advance release" is published by the BEA. The timing of the nowcasts is as follows:

[8] Since the probabilities to initialize the filter are unknown, and the ergodic ones are not suitable because of the truncation in the transition probabilities, we treat the initial probabilities as additional parameters to be estimated in the maximization.

[9] The sample is determined by the availability of data.



Given our interest in reproducing real-time forecasts for monetary policy monitoring, we use the "advance release" real-time data on GDP for each quarter as our target variable, since its publication dates closely match the Federal Open Market Committee (FOMC) meetings of January, April, July/August and October/November. By the time the FOMC meets on those months, most of the information on the "advance release" of NGDP is available and used in their conjectures. The nowcasting accuracy of the models is then assessed with the root mean squared errors (RMSE) associated with this release.

5.2 Confirmatory Factor Analysis

Several recent papers find that small-scale dynamic factor models can produce more accurate nowcasts than large-scale models, such as in Chauvet (2001), Boivin and Ng (2006), Bai and Ng (2008a, 2008b), and Alvarez, Camacho and Perez-Quiros (2013). One reason is the potential misspecification of the factor and idiosyncratic error autoregressive dynamics. Another reason is that most economic series can be classified into a small number of categories. Thus, large models that include all available variables without pre-screening can lead to large cross-correlation in the idiosyncratic errors of the series. However, Bai and Ng (2008a) find that, even when there is weak cross-correlation, models that include carefully selected variables display higher signal-to-noise ratios and outperform large-scale models.

The approach in this paper is "confirmatory" dynamic factor analysis, in which models are specified based on prior knowledge of the economic variables' dynamics and relationships. The proposed single fully estimated framework allows diagnostic tests that enable assessment of the reliability of the nowcasts. The variables included in the models are selected based on whether they represent similar economic or financial sectors, on their marginal predictive contribution to nowcast NGDP growth, and on model specification tests. The nowcasts are then compared using Diebold and Mariano's test for non-nested models (DM 1995), and Clark and McCracken's test for nested models (CM 2001).

5.2.1 Data

The series were obtained from the Federal Reserve Bank of Philadelphia and from the Federal Reserve Bank of Saint Louis real-time data archives, and from data collected by the authors for the papers Chauvet (1998), Chauvet and Hamilton (2006), and Chauvet and Piger (2008).

We note that, although there is a large database of series available, only a smaller subset of monthly real-time vintage series have a sample long enough to allow reasonable estimation inferences. We collected all available National Income and Product Account (NIPA) series at the monthly frequency, nominal variables from the product side, industrial production and capacity utilization, consumption expenditures, labour market variables, all price indices from the production and consumption sides, and monetary and financial series. All variables were transformed to growth rates, with the exception of those already expressed in rates.

5.2.2 Variable Selection

Several selection criteria were implemented to find series that display simultaneous co-movements with NGDP growth. The underlying guidelines were the economic significance of the variables, their statistical adequacy, and their overall conformity to the U.S. business cycle and inflation fluctuations. First, the series were ranked according to their marginal predictive content for NGDP growth, similarly to Chauvet (2001), Camacho and Perez-Quiros (2010), and Bai and Ng (2008a), and their ability to Granger-cause NGDP growth. Second, we evaluate their contemporaneous and cross-correlation with NGDP growth. The confirmatory dynamic factor model captures the common cyclical co-movements underlying the observable variables. Thus, it is important that the series selected display a strong contemporaneous correlation with the target variable. If the series have offset cycles, the upturn in NGDP growth may be offset by the downturn in the other variables, which will generate a latent dynamic factor with a lower signal-to-noise representation of common cyclical movements. Another important criterion used is the availability of real-time vintages of the series and their availability at a reasonable sample length, which allows for testing the reliability of the NGDP nowcasts in real time.

From these procedures, we classified and ranked the top variables. These series represent different measurements of real economic activity, inflation, and monetary and financial activities.

5.3 Multivariate Mixed-Frequency Dynamic Factor (MFDF) Model

5.3.1 Benchmark Model

Chauvet (2001), Boivin and Ng (2006), Bai and Ng (2008a) and Alvarez, Camacho, and Perez-Quiros (2013) find that small-scale dynamic factor models that use one representative indicator of each classification outperform large-scale dynamic factor models that include all economic indicators, since they minimize cross-correlation in the idiosyncratic errors of series from the same classification.

We also find that fewer pre-selected variables lead to more accurate nowcasts. We thus start with the construction of a three-variable benchmark model, based on the definition of nominal GDP, which incorporates information about our target variable NGDP growth, one real-activity indicator, and one inflation indicator. This benchmark is then enlarged with additional variables that were ranked highly in the procedure described above, and based on diagnostic tests.

Among all variables, the top three representative indicators of real economic activity concur with the traditional coincident indicators used by the NBER business cycle dating committee: Industrial Production (IP), Real Personal Income Less Transfer Payments (PILT) and an employment measurement, which in our case is Nonfarm Labor (NFL). The three most representative indicators of U.S. inflation dynamics are the Consumer Price Index (CPI), the Producer Price Index (PPI) and the Personal Consumption Expenditures Price Index (PCEP). The three best real activity indicators and three best inflation indicators yield nine possible pairwise (one real, one inflation) combinations, which will constitute the new set of potential benchmark models to nowcast NGDP growth. We estimate these nine models as in equations (7) and (8), always using NGDP growth and one of the pairwise combinations in the benchmark set to obtain an index based on the common component among the variables. We then compute the RMSE with respect to the "advance release" of NGDP growth for the corresponding quarter.

The results are reported in Table 1. The combination that displays the best predictive performance is Model A:{NGDP, IP, CPI}, with an $RMSE = 0.297$, which is significantly lower than the RMSE for all other combinations, based on the DM test (1995). Figure 4 plots the best three-variable MFDF benchmark Model A and the target variable, as well as NBER recessions. Nowcasts from Model A closely match NGDP growth, and show a substantial improvement with respect to the naive simple-sum model (Figure 3).

5.3.2 Augmented Multivariate Mixed-Frequency Factor Models

We augment the basic three-variable benchmark dynamic factor model that includes NGDP growth, an indicator of real economic activity and an indicator of inflation by including additional highly ranked series. We assess the contribution of these additional indicators in several ways. First, since the small-scale dynamic factor structure captures cyclical co-movements underlying the observable variables, we test whether the resulting augmented dynamic factor is highly correlated with the series used in its construction. This indicates whether or not the structure was simply imposed on the data by assuming large idiosyncratic errors. Second, since the model assumes that the factor summarizes the common dynamic correlation underlying the observable variables, this implies that the idiosyncratic errors should be uncorrelated with the observed variables. In order to test this assumption, the disturbances are regressed on lags of the observable variables. The additional series are kept if the parameters of the equations are found to be insignificantly different from zero. Third, we adjust the number of lags based on maximum likelihood tests; on Bayesian Information criteria; and on whether the one-step-ahead conditional forecast errors, obtained from the filter described in section 3, are not predictable using lags of the observable variables, as implied by the model. Finally, the i.i.d. assumption of the residuals from equations (4) and (5) or (9) is tested using Ljung-Box statistics on their sample autocorrelation.

Additional Indicators We now consider four-variable mixed-frequency dynamic factor models. Table 2 reports the RMSE of the best four-variable models. Some interesting findings emerge from the results. In particular, the RMSE increases substantially if the additional fourth series is another measure of real activity or inflation. That is, once one real and one inflation indicator have been already incorporated into the model, any additional indicator in the same category (real or inflation) yields substantial decreases in the accuracy of the enlarged model. This corroborates the results of Chauvet (2001) and Alvarez, Camacho, and Perez-Quiros (2013), and substantiates the arguments of Boivin and Ng (2006) and Bai and Ng (2008a, 2008b).

The next step is to assess the marginal predictive ability of additional indicators from a category other than inflation and real activity, which could improve the fit between our index and NGDP growth. We consider a large number of series. However, we find that most of the larger models display an inferior performance in terms of RMSE compared with the best three-variable benchmark (Model A). The exceptions are when some monetary and financial variables are considered. From these exceptions, the best performing indicators are the 3-Month Treasury Bill (TBILL), the S&P500 index, and Divisia measures of M3 and M4 computed by the Center for Financial Stability (CFS), which relies on the methodology proposed by Barnett (1980).[10] The results are shown in Table 2. The best four-variable combinations correspond to Model B: {NGDP, IP, CPI, M3}, Model C: {NGDP, IP, CPI, M4} and Model D: {NGDP, IP, CPI, TBILL}. The difference between the nowcasts of these models and the ones from the others (all non-nested) is significantly different at the 1% or 5% level based on DM tests.

We next estimate models with five and six variables.[11] The results are reported in Table 3. Once again, we find that including more than one series from the same category (e.g., interest rates, monetary aggregates, stock market indices, etc.) substantially reduces the models' predictive performance. The best five-variable models are Model E: {NGDP, IP, CPI, M3, TBILL} and Model F: {NGDP, IP, CPI, M4, TBILL}. Notice, however, that the RMSE of these larger models are not substantially different from the benchmark Model A based on the DM (1995) test and the CM (2001) test.

The combined results indicate that the top-ranked variables and specifications that have the best predictive performance for the target variable NGDP growth are different combinations of real activity, inflation, Divisia monetary aggregates and interest rates.

6 Real-Time Nowcasting

6.1 Nowcasting with Linear Models

In this section, we discuss the results of the mixed-frequency dynamic factor (MFDF) models estimated over real-time recursive samples from 2001M1 to 2012M12, as described in subsection 5.1.[12] We use the six MFDF models that yield the best predictive performance so far to assess their ability to predict current growth of NGDP, using the exact amount of data available at the time of the prediction, and by taking into account all possible revisions in previous releases of variables. For comparison, we also

[10] The Divisia monetary aggregates for the United States, including the broad measures M3 and M4 (both quantity and dual-user cost aggregates), are made available to the public by a program directed by William A. Barnett at the Center for Financial Stability at http://www.centerforfinancialstability.org/amfm.php. For an explanation of the methods underlying the data, see Barnett et al. (2013).

[11] We have also estimated larger models. However, since this requires including series that represent similar economic and financial sectors, we find that these models are not top ranked, because they (1) fail diagnostic and specification tests and (2) display lower predictive performance to NGDP growth than the smaller-scale factors considered. We do not report their results because of space considerations, but they are available upon request.

[12] There are two recessions in the time period studied. The NBER dated the 2001 recession as starting in 2001M3 and ending in 2001M11. The "Great Recession" started in 2007M12 and ended in 2009M6.

estimate the naive simple-sum model (section 2) and the autoregressive models (section 3) over real-time recursive samples.[13]

The RMSE for these models are reported in Table 4. The MFDF models show relatively similar performance compared with each other over the full real-time sample. However, there are substantial differences between the nowcasts from the MFDF models and those from the alternative models. The best-performing specification for this period is the MFDF Model B ($RMSE = 0.512$), followed closely by Models C and F. The RMSE of Model B is approximately 24% and 45% lower than the ones from the autoregressive model and the naive simple-sum model, respectively. These differences are statistically significant at the 5% level based on the DM test. The worst-performing model is the naive simple-sum model with $RMSE = 0.740$.

The real-time nowcasts of the MFDF models and NBER recessions are plotted in Figure 5 and the nowcasts for the autoregressive model are shown in Figure 6 for $p = 1, 2, 3$ in equation (16), from the left chart to right, respectively. The nowcasts from the naive model are shown in Chart B of Figure 3. NBER recessions are represented as shaded areas. As can be seen, the performance of the autoregressive models is generally not accurate, with actual NGDP growth being overestimated during most of the period. This is also the case for nowcasts from the naive simple-sum model, which also overestimates NGDP growth but to a much lesser extent. Chauvet and Potter (2013) study several univariate and multivariate models and find, in contrast, that the univariate autoregressive model performs well for real GDP growth compared with other more sophisticated models. However, we find that this is not the case when considering NGDP growth as the target. The nowcasts from the linear mixed-frequency dynamic factor model show a substantially better fit compared with the other models, although they also overestimate NGDP growth after the Great Recession.

In effect, we notice that the performance of all models seems to change over sub-periods. Chauvet and Potter (2013) find that it is more difficult to predict real output growth during recessions than during expansions. We also find that this is the case for NGDP growth. The autoregressive models miss the two recessions in the sample, the 2001 and the Great Recession, since they predict only a small decrease in growth. Although the 2001 recession was a mild downturn, the Great Recession was characterized by largely negative NGDP growth. The nowcasts from the simple-sum naive model and from the MFDF model display much better performance in predicting the timing and intensity of the decline in NGDP growth during these recessions, although the simple-sum model overestimates the severity of the downturns.[14]

We investigate the potential changes in predictive performance across different samples, by computing the RMSE for the periods before, during and after the Great Recession. The results are also reported in Table 4. The MFDF models exhibit lower RMSE in the period before the Great Recession compared with

[13] As in Leiva-Leon (2014), we have also estimated a model with two dynamic factors, one that uses information on real activity indicators and the other based on inflation indicators. We find that the model with two separate factors performed similar to the naive model. These results are available from the authors upon request.

[14] This is related to the fact that the simple-sum model uses monthly Industrial Production, which displays a much larger decline (Figure 2, Chart C) than NGDP during recessions (Figure 1, Chart B).

the autoregressive models and the naive simple-sum model. The differences in accuracy are even more substantial during the Great Recession, with the values of the RMSE from the MFDF models generally around half of those from the alternative models. In contrast, all models show a more similar performance following the most recent recession, in the period from 2009M7 to 2012M12. The autoregressive models show a slightly better performance but the difference compared with the best MFDF model for this period (Model C) is not statistically significant at the 5% level using the DM test. Interestingly, Figures 3, 5 and 6 show that the nowcasts of all models tend to overestimate NGDP growth since the Great Recession.

6.2 Real-Time Nowcasting under Parameter Instability

Although linear MFDF models have shown marked improvements in nowcasting performance in comparison to univariate and naive models, there are differences in the predictive performance over subsamples across all models. These results could be due to instability in the models' parameters. Over the years, the U.S. economy has experienced some abrupt changes that could have strongly impacted nominal GDP dynamics, such as the Great Moderation or the Great Recession. In this section, we account for this possibility by reporting the results of the proposed non-linear mixed-frequency dynamic factor model that produces nowcasts in the presence of structural breaks (MFDFB).

We first estimate the MFDFB model using the same variables from the linear MFDF model B {NGDP, IP, CPI, M3} in the previous section, which presented the overall best nowcasting performance for the full real-time sample compared with all other linear models (Table 4). The estimation is based on equations 8 and 9 to 12, as explained in section 3.2.

Figure 7, Chart A shows the estimated factor along with the probabilities of regime change. The probabilities of a break show some increases during periods that have been discussed in the previous literature as potential permanent breaks, such as in 1970-71 (productivity slowdown), 1975 (oil crisis), and 1982 (the end of the Great Inflation period). However, the probability of a permanent break reached the value of 1 in 1990M12, after which the model imposes that it remains in this regime until the end of the sample. This period is related to a change in volatility of NGDP growth, as shown in Figure 1, Chart B, which is associated with the Great Moderation. It is interesting to note that an extensive literature has found that real GDP displays a break in volatility around 1984 (e.g., Kim and Nelson 1999, McConnell and Perez-Quiros 2000, and Chauvet and Potter 2001), while other authors find that inflation volatility shows a break in the late 1980s (Chauvet and Popli 2003). The breaks in NGDP growth seem to reflect abrupt changes in its components, specifically, inflation and real GDP growth. Figure 7, Chart B plots the estimated factors and the probabilities of a break in their mean, which clearly reflects the abrupt change in the economy associated with the Great Recession. The probabilities of a break switched to regime one and reached the value of 1 in 2008M10, which is associated with the impact of the Lehman Brothers' crisis.

We also examine whether univariate models of NGDP growth also display structural breaks. We first obtain the recursive autoregressive parameters from the real-time estimation of the AR models using equation (16), which are plotted in Figure 8 for $p = 1, 2, 3$, from left to right, respectively. The parameter of the AR(1) model shows evidence of parameter instability during the Great Recession. This is also the case for the AR(2) and AR(3) models but to a lesser extent. This result could be the origin of the overestimation of the nowcasts of the real-time AR models during this period (Figure 6).

We next estimate the univariate autoregressive model under structural breaks, based on equation 18. We focus on the AR(1) specification, since it shows the largest parameter instability. Figure 9, Chart A plots nominal GDP growth and the probabilities of a break from the autoregressive model, which are similar to the ones obtained for the MFDFB model. The probabilities also indicate a structural change at around the same time as the MFDFB model, switching to state one in 1989Q3. Figure 9, Chart B plots the corresponding probabilities of a break in the mean, which reach the value of 1 in 2008Q4, coinciding with the date of the break found with the multivariate model in 2008M10. Thus, both univariate and multivariate approaches unveil structural breaks in nominal output growth around 1989-90 and in the midst of the Great Recession in 2008.

Based on these findings, we estimate the proposed MFDFB model presented in section 3.2 over real-time recursive samples to obtain NGDP growth nowcasts. Notice that we use only information available at the time of the predictions. Thus, assessments of potential breaks in the sample are endogenously recursively estimated for every vintage. The predictions associated with this model along with the NGDP growth data are plotted in Figure 10. The results show marked improvements compared with the performance of the linear version of this model (Figure 7). In particular, the previous finding of overestimated nowcasts is substantially reduced with this framework. This can also be seen in Table 5, which reports the RMSE associated with the MFDFB models and the AR(1) specification subject to breaks across different subsamples. The MFDFB specification that generally displays the best performance across periods is Model D, which contains information on NGDP, IP, CPI and TBILL. However, for the period since the Great Recession, the model with the best performance is the smaller benchmark, which contains only the NGDP, IP and CPI series. The reason might be that the TBILL is not informative at very low values and almost no volatility, as it has been since the most recent recession.

Overall, the nowcasting performance of the MFDFB (i.e., models subject to breaks) is more accurate than the nowcasting from the linear MFDF models, with the exception of the time of the Great Recession. Specifically, during the full sample period and before the Great Recession, all MFDFB non-linear models present considerably lower RMSE than the linear MFDF models. This is also the case for most non-linear models after the Great Recession. The RMSE for the best MFDFB for this period (Model A) is significantly lower than that of the best linear MFDF (Model F) at the 5% level using the CM test (2001). Compared with the autoregressive model (Figure 11 and Table 5), the nowcasts from the MFDFB models show clear improvements. This is more accentuated for the period since the Great Recession, for which all linear models generate overestimated nowcasts. The RMSE of the MFDFB is less than half that of the linear AR(1) model and around half that of the AR(1) with a break.

In summary, the results provide evidence of substantial gains in the nowcasting ability of the proposed mixed-frequency multivariate models when allowing for potential structural breaks in parameters.

7 Conclusions

Given the non-conventional situation that the U.S. Federal Reserve faces regarding the zero lower bound of the interest rate, many economists have suggested that alternative strategies should be adopted to decrease the unemployment rate. One of the proposals is forward guidance through targeting nominal GDP. This paper proposes a non-linear nowcasting dynamic factor model that includes mixed-frequency and parameter instability that can be helpful in the assessment of current economic conditions.

We evaluate the performance of univariate and multivariate, linear and non-linear econometric models that can be useful for earlier assessments of current nominal GDP growth, under real conditions that policy-makers face at the time the predictions are made. The univariate analysis shows that classical autoregressive models provide poor performance regarding real-time nowcasts of the target variable. However, when allowing for parameter instability, the performance of the univariate model substantially increases.

We then estimate the proposed small-scale non-linear mixed-frequency dynamic factor models. We find the presence of two breaks in NGDP growth dynamics: the first in the late 1980s, associated with the Great Moderation, and the second in the midst of the Great Recession in 2008. The multivariate models that allow parameter instability outperform linear multivariate as well as linear and non-linear univariate specifications, yielding the best nowcasting performance. The best specifications are parsimonious and include economic activity, inflation, monetary indicators and/or interest rates.

Appendix

The state space representation in equations (6) and (7) for the case of $P = 6$, $Q_1 = 6$ and $Q_n = 2$, for $n = 2, 3, ..., N$, is given by the Measurement Equation, $y_t = HF_t + \xi_t$, defined as

$$
\begin{bmatrix} y_{1,t} \\ y_{2,t} \\ \vdots \\ y_{N,t} \end{bmatrix}
=
\begin{bmatrix}
\frac{1}{3}\gamma_1 & \frac{2}{3}\gamma_1 & \gamma_1 & \frac{2}{3}\gamma_1 & \frac{1}{3}\gamma_1 & 0 & \frac{1}{3} & \frac{2}{3} & 1 & \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & \dots & 0 & 0 \\
\gamma_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & \dots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
\gamma_N & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 1 & 0
\end{bmatrix}
\begin{bmatrix} f_t \\ f_{t-1} \\ f_{t-2} \\ f_{t-3} \\ f_{t-4} \\ f_{t-5} \\ v_{1,t} \\ v_{1,t-1} \\ v_{1,t-2} \\ v_{1,t-3} \\ v_{1,t-4} \\ v_{1,t-5} \\ v_{2,t} \\ v_{2,t-1} \\ \vdots \\ v_{N\ ,t} \\ v_{N,t-1} \end{bmatrix}
+
\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix},
$$

and the Transition Equation, $F_t = TF_{t-1} + \zeta_t$, defined as

$$
\begin{bmatrix} f_t \\ f_{t-1} \\ f_{t-2} \\ f_{t-3} \\ f_{t-4} \\ f_{t-5} \\ v_{1,t} \\ v_{1,t-1} \\ v_{1,t-2} \\ v_{1,t-3} \\ v_{1,t-4} \\ v_{1,t-5} \\ v_{2,t} \\ v_{2,t-1} \\ \vdots \\ v_{N,t} \\ v_{N,t-1} \end{bmatrix}
=
\begin{bmatrix}
\phi_1 & \phi_2 & \dots & \phi_6 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & \dots & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \dots & 0 & \varphi_{11} & \varphi_{12} & \dots & \varphi_{16} & 0 & 0 & 0 & 0 \\
0 & 0 & \dots & 0 & 1 & 0 & \dots & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \dots & 0 & 0 & 1 & \dots & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & \varphi_{21} & \varphi_{22} & 0 & 0 \\
0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & 1 & 0 & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & 0 & 0 & \varphi_{N1} & \varphi_{N2} \\
0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix} f_{t-1} \\ f_{t-2} \\ f_{t-3} \\ f_{t-4} \\ f_{t-5} \\ f_{t-6} \\ v_{1,t-1} \\ v_{1,t-2} \\ v_{1,t-3} \\ v_{1,t-4} \\ v_{1,t-5} \\ v_{1,t-6} \\ v_{2,t-1} \\ v_{2,t-2} \\ \vdots \\ v_{N\ ,t-1} \\ v_{N,t-2} \end{bmatrix}
+
\begin{bmatrix} e_t \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \epsilon_{1,t} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \epsilon_{2,t} \\ 0 \\ \vdots \\ \epsilon_{N,t} \\ 0 \end{bmatrix}.
$$

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Table 1. RMSE for 3-Variable MFDF Model Benchmark: NGDP, Real Activity and Inflation

Inflation	**Real Activity Indicators**		
Indicators	IP	PILT	NFL
CPI	**0.297**$^{**\cdot\cdot}$	0.822	0.810
PPI	0.370	0.449	0.810
PCEP	0.341	0.862	0.810

Table 2. RMSE for 4-Variable MFDF Models: NGDP, Real Activity, Inflation and Others

	NFL	PILT	PPI	PCEP	PI	PCE	M3	M4	SP500	TBILL
IP, CPI	0.814	0.805	0.986	0.958	0.435	0.327	**0.298**$^{**\cdot}$	**0.295**$^{**\cdot}$	0.330	**0.297**$^{*\cdot}$
NFL, CPI	—	0.812	0.968	0.971	0.534	0.423	0.809	0.809	0.809	0.809
PILT, CPI	—	—	0.995	0.959	0.762	0.420	0.805	0.805	0.823	0.918
IP, PPI	0.814	0.769	—	0.960	0.421	0.318	0.375	0.364	0.380	0.799
NFL, PPI	—	0.812	—	0.960	0.784	0.418	0.810	0.810	0.810	0.811
PILT, PPI	—	—	—	0.960	0.760	0.385	0.476	0.477	0.443	0.442
IP, PCEP	0.815	0.809	—	—	0.463	0.368	0.869	0.865	0.338	0.909
NFL, PCEP	—	0.812	—	—	0.529	0.433	0.809	0.809	0.809	0.810
PILT, PCEP	—	—	—	—	0.762	0.452	0.858	0.854	0.865	0.904

Table 3. RMSE for 5- and 6-Variable MFDF Models: NGDP, Real Activity, Inflation and Others

Variables	**RMSE**
IP, CPI ,M3, M4	1.261
IP, CPI, M3, TBILL	**0.298**$^{\cdot\cdot}$
IP, CPI, M4, TBILL	**0.294**$^{\cdot\cdot}$
IP, CPI, M3, M4, TBILL	1.494

Note: RMSE stands for root mean squared errors, and MFDF is the mixed-frequency dynamic factor model. $^{(*)}$ and $^{(**)}$ stand for statistically significant at the 5% and 1% level, respectively, based on DM test used to compare non-nested models. $^{(\cdot)}$ and $^{(\cdot\cdot)}$ stand for statistically significant at the 5% and 1% level, respectively, based on CM test used to compare nested models.

Table 4. RMSE for Real-Time Nowcasts from Best MFDF Models

Model	Variables	Full Sample		Great Recession	
		RMSE	RMSE Before	RMSE During	RMSE After
A	IP,CPI	0.524	0.532	0.584	0.472
B	IP, CPI, M3	**0.512****	**0.530****	0.589	0.429
C	IP, CPI, M4	0.514	0.531	0.603	0.422
D	IP, CPI, TBILL	0.560	0.532	0.777	0.489
E	IP, CPI, M3, TBILL	0.521	0.546	**0.576****	0.423
F	IP, CPI, M4, TBILL	0.514	0.532	0.608	0.418
AR(1)		0.700	0.628	1.259	**0.401***
AR(2)		0.671	0.570	1.235	0.440
AR(3)		0.719	0.568	1.336	0.561
Naive		0.740	0.647	1.035	0.752

Table 5. RMSE for Real-Time Nowcasts with Break

Model	Variables	Full Sample		Great Recession	
		RMSE	RMSE Before	RMSE During	RMSE After
A	IP,CPI	0.546	0.447	1.031	**0.365**$^{**\cdot\cdot}$
B	IP, CPI, M3	0.476	0.400	0.812	0.400
C	IP, CPI, M4	0.509	0.410	0.944	0.382
D	IP, CPI, TBILL	**0.449****	**0.377****	**0.737****	0.402
E	IP, CPI, M3, TBILL	0.505	0.393	0.792	0.536
F	IP, CPI, M4, TBILL	0.545	0.503	0.755	0.506
AR(1)	with Break	0.630	0.536	0.916	0.643

Note: RMSE stands for root mean squared errors, MFDF is the mixed-frequency dynamic factor model and MFDFB is the mixed-frequency dynamic factor model with break. The full sample in the third column refers to the real-time period from 2001M1 to 2012M12. The fourth column refers to the real-time sample from 2001M1 to 2007M11, the fifth column from 2007M12 to 2009M6, and the sixth column from 2009M7 to 2012M12. $^{(*)}$ and $^{(**)}$ stand for statistically significant at the 5% and 1% level, respectively, based on DM test used to compare non-nested models. $^{(\cdot)}$ and $^{(\cdot\cdot)}$ stand for statistically significant at the 5% and 1% level, respectively, based on CM test used to compare nested models.

Figure 1. Nominal GDP (NGDP)

Chart A

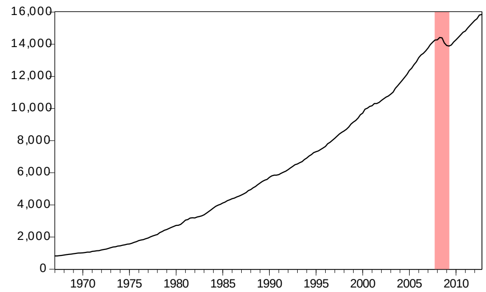
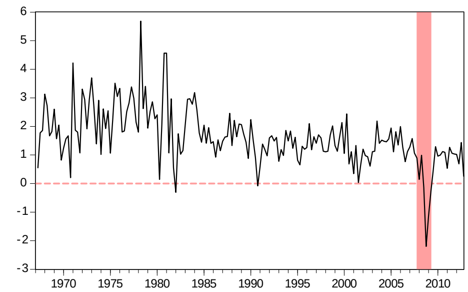


Chart B



Note. Chart A plots NGDP in levels and Chart B plots the growth rate of NGDP.

Figure 2. Real Activity and Inflation

Chart A

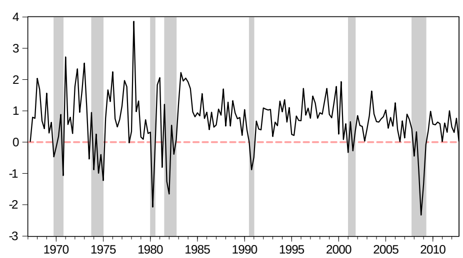


Chart B

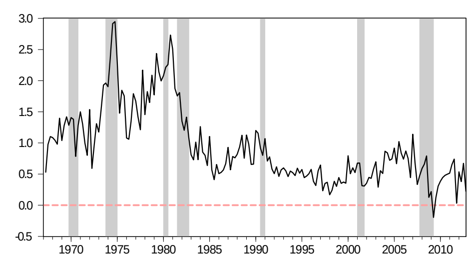


Chart C

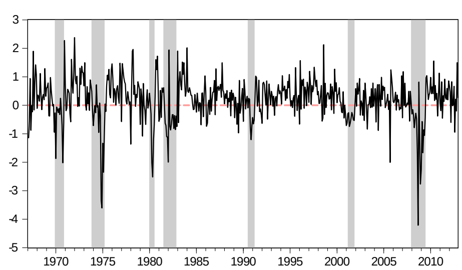
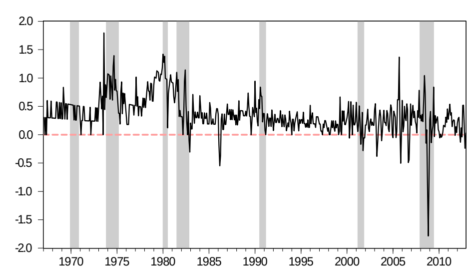


Chart D



Note. Chart A plots real GDP growth rate, Chart B plots the GDP Deflator growth rate, Chart C plots IP growth rate, and Chart D plots CPI growth rate.

Figure 3. Naive Simple-Sum Model

Chart A

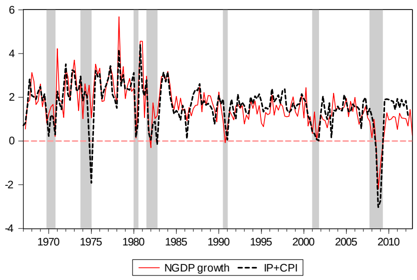
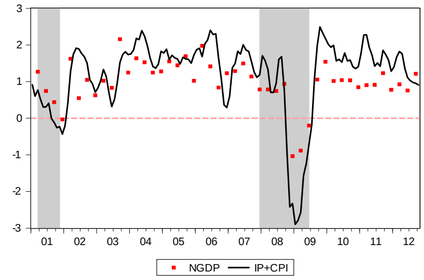
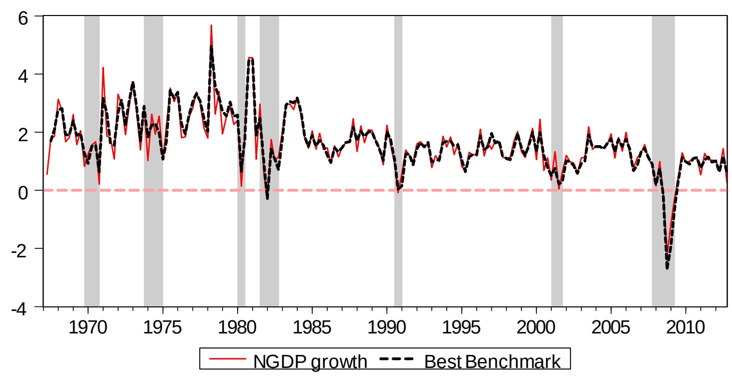


Chart B



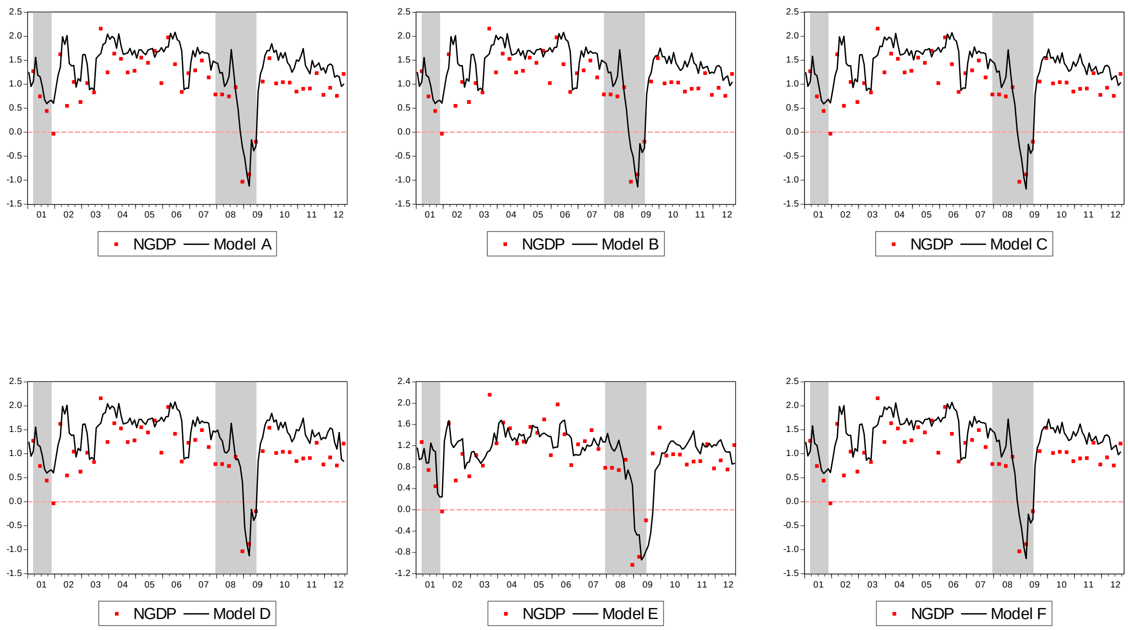
Note. Chart A plots NGDP growth rate along with the in-sample predictions from the naive simple-sum model. Chart B plots NGDP growth rate along with the out-of-sample nowcasts from the naive simple-sum model.

Figure 4. Nowcasts from Best Benchmark Linear MFDF Model



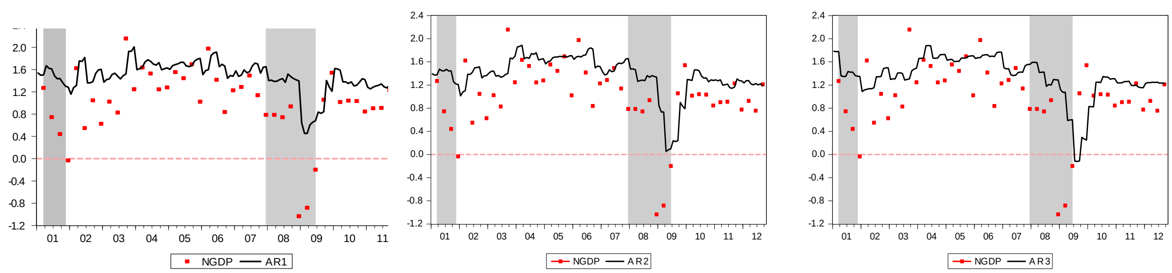
Note. The figure plots NGDP growth rate along with the in-sample predictions from the benchmark MFDF model containing IP, CPI and NGDP.

Figure 5. Real-Time Nowcasts from Best Augmented Linear MFDF Models



Note. The figure plots the real-time nowcasts obtained from each linear mixed-frequency dynamic factor model specified in Table 4 (black line) along with the actual NGDP growth rate (red dots).

Figure 6. Real-Time Nowcasts from Univariate Autoregressive Models



Note. The figure plots the real-time nowcasts obtained from each linear autoregressive model specified in Table 4 (black line) along with the actual NGDP growth rate (red dots).

Figure 7. Probability of a Break - MFDF with Structural Break Model

Chart A

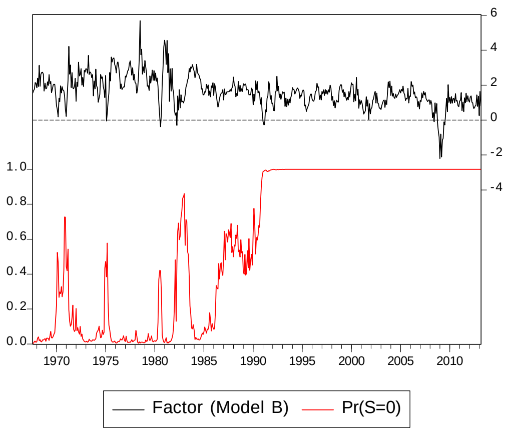
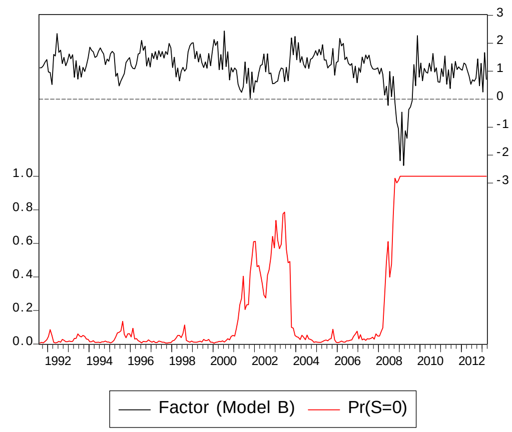


Chart B



Note. Chart A and Chart B plot the factors estimated from the non-linear mixed-frequency dynamic factor models along with their corresponding probability of a break for the periods 1967M08-2013M03 and 1991M08-2013M03, respectively.

Figure 8. Recursive Autoregressive Parameters from Univariate Models

Chart A

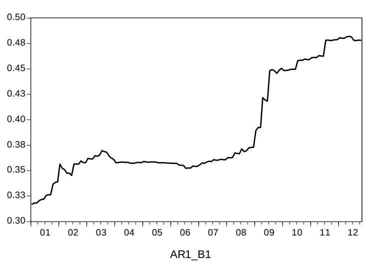


Chart B

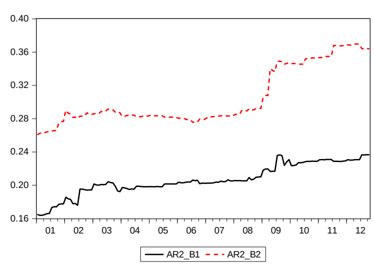
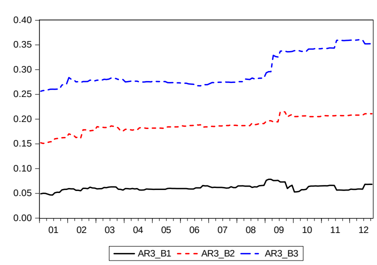


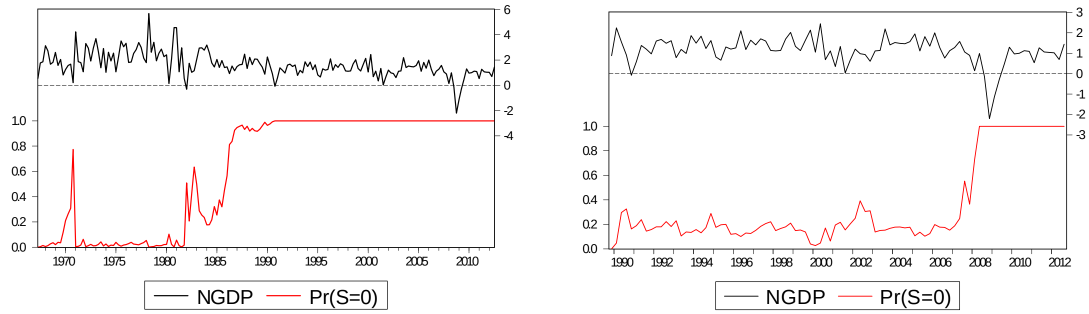
Chart C



Note. Chart A, Chart B and Chart C plot the autoregressive parameters of AR(1), AR(2) and AR(3) models, respectively, which are estimated recursively as new data arrive.

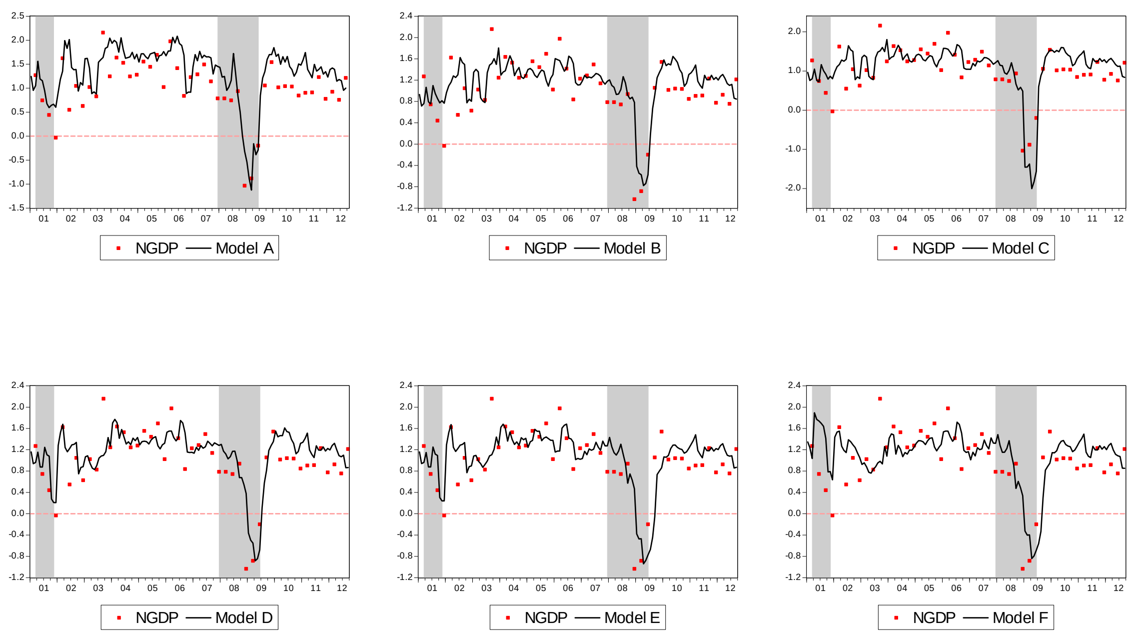
Figure 9. Break Probability: Univariate Autoregressive Model

Chart A Chart B



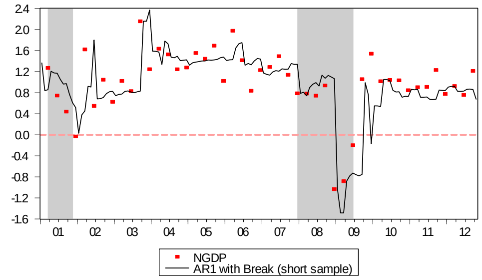
Note. Chart A and Chart B plot the NGDP growth rate along with its corresponding probability of a break for the periods 1967Q2-2012Q3 and 1989Q4-2012Q3, respectively.

Figure 10. Real-Time Nowcasts from MFDFB (Structural Break) Model



Note. The figure plots the real-time nowcasts obtained from each non-linear mixed-frequency dynamic factor model specified in Table 5 (black line) along with the actual NGDP growth rate (red dots).

Figure 11. Real-Time Nowcasts from Univariate Autoregressive Model with Structural Break



Note. The figure plots the real-time nowcasts obtained from the non-linear autoregressive model specified in Table 5 (black line) along with the actual NGDP growth rate (red dots).